CRANBROOK
SCHOOL

Year 11 (2U) Mathematics

Preliminary Yearly Examination

Wednesday September 1, 2010

Time Allowed: 2 hours *plus* 5 minutes reading time

Total Marks: 84

There are 7 questions, each of equal value.

Start a new booklet for each question.

All necessary working should be shown in every question.

Full marks may not be awarded if work is careless or badly arranged.

Only approved calculators may be used.

MARKED BY BG

Question 1 **Marks**

(a) Factorise completely:

(i) m^2-4m+4 **1**

(ii) $ab-3bc+4a-12c$ **1**

(b) Express $\frac{2}{\sqrt{5}-1}$ in simplest form with rational denominator. **2**

(c) Evaluate $\sqrt{3.142^2-0.858^2}$ correct to 3 significant figures. **2**

(d) Simplify: $\sqrt{a^3}+\sqrt{a}-\sqrt{9a}$ **2**

(e) Solve: $x^2-8x=20$ **2**

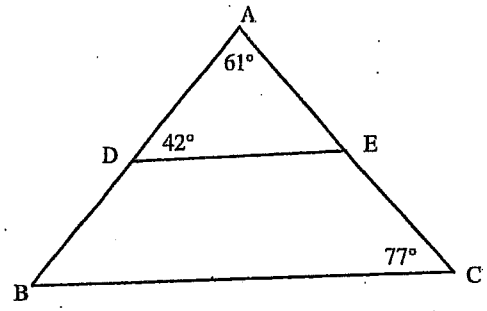
(f) The Selling Price of an article is calculated by adding a Goods and Services Tax of 10% of the Pre-Tax Price to this Pre-Tax Price. If it was decided to increase the Goods and Services Tax to 15% of the Pre-Tax Price, what would be the percentage increase in the Selling Price? **2**

MARKED BY CRA

Question 2 *Start a new page* **Marks**

(a) Simplify: $\dfrac{x^2-144}{x^3+8} \div \dfrac{x+12}{x+2}$ **3**

(b) **2**



Not to Scale

In the diagram above $\angle ADE = 42°$, $\angle ACB = 77°$ and $\angle DAE = 61°$.

Show that DE is parallel to BC.

(c) Solve; $|2x-1| \leq 7$. **2**

(d) Express $\frac{5}{110}$ as a recurring decimal. **1**

(e) Solve: $\sqrt{2}\sin\beta - 1 = 0$ for $0° \leq \beta \leq 360°$. **2**

(f) Draw on a number plane the region defined by the inequalities **2**

$y \geq x^3$, $x \geq 0$ and $y \leq 1$

MARKED BY CRA

Question 3 *Start a new page* **Marks**

(a) From an observation window, which is 52 metres above sea level, in a vertical lighthouse, the angle of depression to a yacht is 27°.

(i) Draw a neat sketch showing this information. **1**

(ii) How far is the yacht from the base of the lighthouse? **2**

(b) Differentiate the following

(i) $\dfrac{5-x^3}{3x-5}$ **2**

(ii) $\left(x^2-3x\right)^5$ **2**

(iii) $\dfrac{2}{\sqrt{x}} - 3x^2$ **2**

(c) For what values of p will $x^2-(p+3)x+4p=0$ have:

(i) Equal roots? **2**

(ii) No real roots? **1**

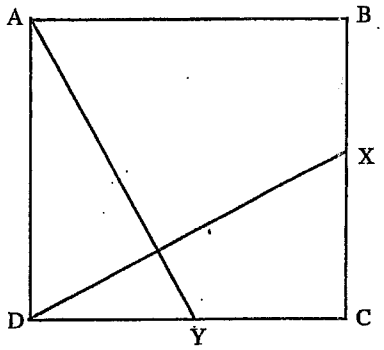
MARKED BY RABS

Question 4 *Start a new page* **Marks**

(a) (i) Find the equation of the tangent to the parabola $y = 3x - x^2$ at the point (2,2) on it. **2**

(ii) Draw a neat sketch showing the parabola and its x and y intercepts. Draw also the tangent to the parabola. **2**

(b)



Not to scale

In the diagram. above ABCD is a square. X and Y are the midpoints of BC and DC respectively.

(i) Prove that $\Delta ADY \equiv \Delta CDX$. **3**

(ii) Hence, or otherwise, show that $AY = DX$. **1**

(c) The quadratic equation $x^2 - 3x + 7 = 0$ has roots α and β.
Find the value of:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\frac{2}{\alpha} + \frac{2}{\beta}$ **1**

(iv) $(\alpha - 1)(\beta - 1)$ **1**

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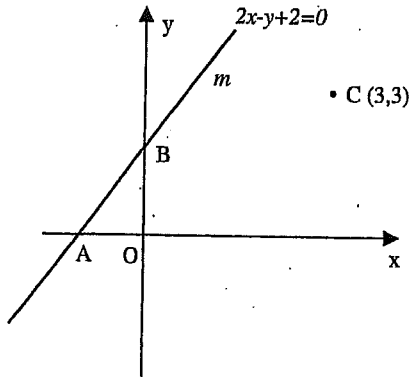
Question 5 *Start a new page* **Marks**

(a)



Not to Scale

In the diagram above, the line m with equation, $2x - y + 2 = 0$ cuts the x and y axes in A and B respectively. C is the point (3,3).

Copy or trace the diagram onto your worksheet.

(i) Find the coordinates of A and B. **1**

(ii) Find the gradient of AB. **1**

(iii) Show that the equation of the line n, through C perpendicular to AB has equation $x + 2y - 9 = 0$. **2**

(iv) Find the size of the angle between line n and the positive x-axis. **1**

(v) By solving the appropriate pair of simultaneous equations, find the coordinates of P, the point of intersection of lines m and n. **2**

(vi) Find the length of AP. **1**

(vii) Find the coordinates of M, the midpoint of BP. **1**

(b) A triangle has sides of length $x^2 - 1, x^2 + 1$ and $2x$ where x is an integer greater than 1.

(i) Show that the triangle is right-angled with $x^2 + 1$ as the hypotenuse. **2**

(ii) Explain briefly why $x \neq 1$. **1**

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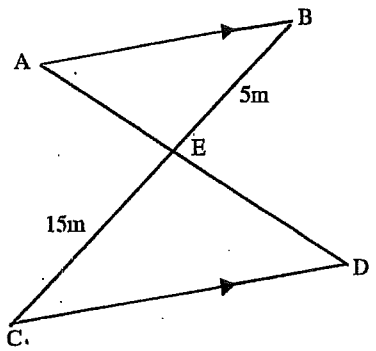
Question 6 *Start a new page* **Marks**

(a) State the domain for function $f(x)$, given that $f(x)=\sqrt{9-x^2}$ **1**

(b)



Not to Scale

In the diagram above, AB is parallel to CD. BE = 5m, CE = 15m,

(i) Prove that ΔABE is similar to ΔDCE. **3**

(ii) Find the ratio AE : AD. **1**

(c) A function is partly defined as $f(x)=3x-1$ for $x>0$. It is given that $f(x)$ is an odd function.

(i) Complete the definition of $f(x)$ for $x\leq 0$ **1**

(ii) Draw a neat sketch of the graph of $y=f(x)$ **2**

(d) A parabola has its vertex at the point (3,1) and its directrix has equation $y=-1$

(i) What is the focal length? **1**

(ii) State the coordinates of the focus. **1**

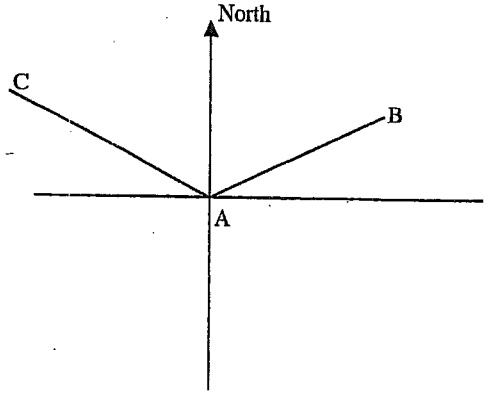
(iii) Find the equation of the parabola. **1**

(iv) What is the equation of the axis of symmetry. **1**

MARKED BY GHW

Question 7 *Start a new page* **Marks**

(a) The diagram below shows three Fire Observation Towers A, B and C. From Tower A, Tower B bears 057°T and is at a distance of 7 kilometres. Tower C bears 296°T from Tower A and is 9 kilometres from it.



Not to scale

(i) Copy the diagram onto your worksheet and mark on it the information given above **1**

(ii) Calculate the distance from Tower B to Tower C (give your answer correct to one decimal place). **2**

(iii) Find the bearing of Tower B from Tower C. (give your answer correct to the nearest degree) **2**

(b) Solve: $x^6+7x^3-8=0$. **3**

(c) ΔABC is an isosceles triangle in which $AB = BC = 13$ cm and $AC = 7$cm.

(i) Find the size of $\angle ABC$ (correct to the nearest degree) **2**

(ii) Calculate the area of ΔABC (correct to one decimal place). **1**

(d) Write down the equation of the circle with centre (3,-2) and radius 11. **1**

Question 1

1 a) i) $m^2 - 4m + 4 = (m-2)^2$ (1)

ii) $ab - 3bc + 4a - 12c = b(a-3c) + 4(a-3c)$

$= (b+4)(a-3c)$ (1)

b) $\frac{2}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1} = \frac{2\sqrt{5}+2}{4}$ (1)

$= \frac{\sqrt{5}+1}{2}$ (1)

c) $\sqrt{3.142^2 - 0.858^2} = 3.02$ (3 sf)

(1) (1)

One mark for correct answer

One mark for 3sf regardless of ans (1)

d) $\sqrt{a^3} + \sqrt{a} - \sqrt{9a} = a\sqrt{a} + \sqrt{a} - 3\sqrt{a}$

$= a\sqrt{a} - 2\sqrt{a}$

$= \sqrt{a}(a-2)$ (1)

a variety of answers were accepted.

e)

$x^2 - 8x = 20$

$x^2 - 8x - 20 = 0$ This was well done.

$(x-10)(x+2) = 0$

$x = -2, 10$

f) $\frac{115-110}{110} \times 100 = \frac{5}{110} \times 100$ Many just wrote 5%.

$= 4.5\dot{4}\%$

QUESTION 2 - CRA

WORKING

(2) (a) $\frac{x^2-144}{x^3+8} \div \frac{x+12}{x+2}$

$\frac{(x-12)(x+12)}{(x+2)(x^2-2x+4)}$ ✓① $\times \frac{(x+2)^2}{(x+12)} = \frac{x-12}{x^2-2x+4}$ ✓③

✓②

(b) $\angle AED = 180 - 61 - 42$

$= 77° = \angle ECB$

$\therefore DE \parallel BC$ as corresponding angles

$\angle AED = \angle ECB$

(c) $|2x-1| \leq 7$

$2x - 1 \leq 7$ $\quad -2x + 1 \leq 7$

$2x \leq 8$ $\quad -2x \leq 6$

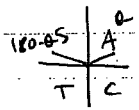
$x \leq 4$ ✓① $\quad x \geq -3$ ✓②

(d) $\frac{5}{110} = 0.0\dot{4}5$

(e) $\sqrt{2}\sin\beta - 1 = 0$ $\quad 0 \leq \beta \leq 360$

$\sqrt{2}\sin\beta = 1$

$\sin\beta = \frac{1}{\sqrt{2}}$

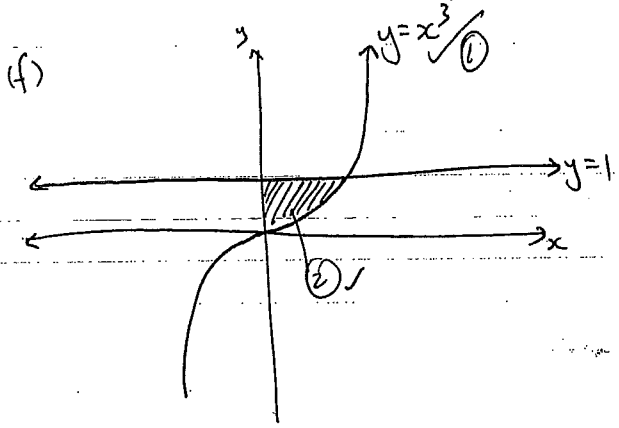


$\beta = 45$

i.e. at $180 - 45$

$\therefore \beta = 45, 135°$

(f)

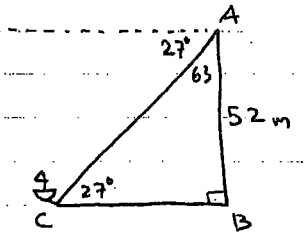


MARKS	COMMENTS
✓✓ ③	• One mark for factorising top, one mark for bottom. • One mark for cancelling.
✓ ✓②	• There were a variety of ways to prove, corresponding, alternate, co-interior angles to prove parallel lines.
✓✓ ②	
✓①	
✓ ✓②	• One mark awarded for solving β. Then another for 180 - β. E.C.F. (error, carried forward) was applied if first β was wrong.
✓	
✓②	• One mark awarded for correctly drawing curve. • Second mark awarded for shading correct area. • Graphs need to be neat! Axes need to be labelled!
(12)	

QUESTION 3 - CONT

WORKING

(a) (i)



(ii) $\tan 63 = \frac{CB}{52}$

$\therefore CB = 102$ $\therefore$ Yacht is 102 m from the lighthouse.

(b) (i) $y' = \frac{(3x-5)(-3x^2) - (5-x^3)(3)}{(3x-5)^2}$

$= -9x^3 + 15x^2 - 15 + 3x^3$

$= \frac{-6x^3 + 15x^2 - 15}{(3x-5)^2}$

(ii) $y' = 5(x^2-3x)^4 \times (2x-3)$

$= (10x-15)(x^2-3x)^4$

(iii) $y = 2x^{-1/2} - 3x^2$

$y' = -x^{-3/2} - 6x$

$= -\frac{1}{\sqrt{x^3}} - 6x$

(c) $x^2 - (p+3)x + 4p = 0$

(i) Equal roots when $\Delta = 0 = b^2 - 4ac$

$(-p-3)^2 - 4(1)(4p) = 0$

$p^2 + 6p + 9 - 16p = 0$

$p^2 - 10p + 9 = 0$

$(p-1)(p-9) = 0$

$\therefore p = 1, 9$

(ii) $1 < p < 9$

MARKS	COMMENTS
✓①	• Many students need to revise bearings!
	• Ensure diagrams are very clear!
✓	• E.C.F. was applied if diagrams were wrong but correct calculations were carried out.
✓②	
✓	• This was done very poorly.
✓②	• Look over differentiation rules!
	• Many students lost marks for silly
✓	• errors with +ve's and
✓②	-ve's.
✓	
✓②	
✓②	• Most problems with part (c) again came from poor algebra, ad expanding brackets.
✓½	
✓②	
✓①	

QUESTION 4.

9. (i) $y = 3x - x^2$

$\frac{dy}{dx} = 3 - 2x$ (sub in $x = 2$)

$m = 3 - 2 \times 2$

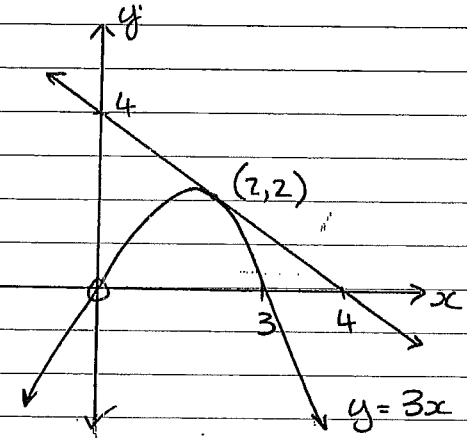
$= -1$ ✓

$y - 2 = -1(x-2)$ (pt./grad. formula)

$y = -x + 4$ or $x + y - 4 = 0$ ✓

- Mostly correct. Well done!

(ii)



1 mark: correct parabola & x/y ints ✓

1 mark: correct tangent with intersection or x/y ints ✓

- Could have been answered much better.
- Many could not factorise to find x-intercepts.

b.

(i) In $\triangle ADY$ & $\triangle XDC$

✓✓ $AC = DC$ (sides of $\square$ are =)
$\angle ADC = \angle XCD$ (all $\angle$s in a $\square$ are 90°)
$XC = YD$ (midpoints of equal sides in a $\square$)

$\therefore \triangle ADY \equiv \triangle XDC$ (SAS) ✓

- Very poorly answered.
- Too many students are using vague, incorrect or unnecessary reasons.
- Attention to detail... make sure you name sides, angles or even triangles correctly.

(ii) $AY = DX$ (corresponding sides on congruent $\triangle$s are =) ✓

- Again, poor reasons were a prob.

c.

(i) $\alpha + \beta = \frac{-b}{a} = \frac{-(-3)}{1} = 3$ ✓

(ii) $\alpha\beta = \frac{c}{a} = \frac{7}{1} = 7$ ✓

(iii) $\frac{2}{\alpha} + \frac{2}{\beta} = \frac{2\alpha}{\alpha\beta} + \frac{2\beta}{\alpha\beta}$

$= \frac{2(\alpha+\beta)}{\alpha\beta}$

$= \frac{6}{7}$ ✓

(iv) $(\alpha-1)(\beta-1) = \alpha\beta - \alpha - \beta + 1$

$= \alpha\beta - (\alpha+\beta) + 1$

$= 7 - 3 + 1 \quad = 5$ ✓

- Mostly correct. Well done.
- As soon as you see α & β, etc.. remember the rules from the Quadratic Function topic... DON'T use quadratic formula.

QUESTION 5

a.

(i) $2x - y + 2 = 0$

A: $y = 0$, $2x + 2 = 0$
$x = -1$
A: $(-1, 0)$

B: $x = 0$, $-y + 2 = 0$
$y = 2$
B: $(0, 2)$ ✓ (both had to be correct)

(ii) $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 0}{0 + 1}$

$= 2$ ✓

(iii) $m_1 = 2$

$\therefore m_2 = -1/m_1 = -1/2$ ✓

$y - 3 = -1/2 (x - 3)$

$y = -1/2 x + 4\,1/2$

or

$x + 2y - 9 = 0$ ✓

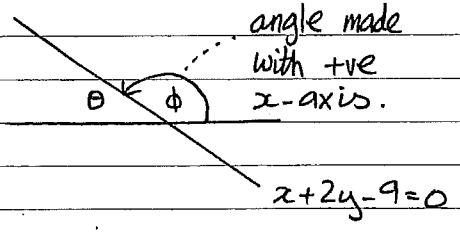
- Answered quite poorly

- Quite a few students chose to prove that the given equation had a gradient of $-1/2$. This is not what the question was asking!

(iv) $m = -1/2$

$m = \tan\theta$

$-1/2 = \tan\theta$



$\theta = -26.56°$...

$= 26.56°$ (tan is +ve in Q2)

$\phi = 180° - 26°56°$

$= 153°\,26'$ ✓

- Answered quite poorly.

- A lot of students did not recognise they had to find the obtuse angle ϕ.

(v) $2x - y + 2 = 0$ —— (1)

$x + 2y - 9 = 0$ —— (2)

(1)×2: $4x - 2y + 4 = 0$ —— (3)

(2)+(3): $5x \quad - 5 = 0$

$\therefore x = 1$

sub $x = 1$ into (2): $1 + 2y - 9 = 0$

$2y = 8$

$y = 4$

$\therefore$ co-ords: $(1, 4)$ ✓✓

(vi) $A(-1, 0)$ $P(1, 4)$

$d = \sqrt{(-1-1)^2 + (0-4)^2}$

$= \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5}$ ✓

(vii) $B(0, 2)$ $P(1, 4)$

$M = \left(\frac{0+1}{2}, \frac{2+4}{2}\right)$

$= (1/2, 3)$ ✓

- Overall, most parts answered correctly.

- Take care to remember formulae, etc...

b.

(i) Using pythagoras' theorem:

$$(x^2+1)^2 = (2x)^2 + (x^2-1)^2$$

$$x^4+2x^2+1 = 4x^2 + x^4 - 2x^2 + 1 \checkmark$$

$$x^4+2x^2+1 = x^4+2x^2+1 \checkmark$$

LHS = RHS

∴ triangle is right-angled with x^2+1 as hypotenuse.

- Answered well.
- Remember $(2x)^2 \neq 2x^2$.

(ii) If $x = 1$, the side $x^2 - 1$ will equal:

$$1^2 - 1 = 0.$$

You cannot have a side with length 0. ✓

- Could have been answered better.
- Make sure your answers & reasons are clear. Stop & think before you write!

Year 11 Prelim Final Exam 2010.

Mathematics

Question 6

a) $-3 \leq x \leq 3$

① Mark for answer.

b) i) Prove $\triangle ABE \,|||\, \triangle DCE$

$\angle AEB = \angle DEC$ (vertically opposite $\triangle$'s are equal)

$\angle BAE = \angle CDE$ (Alternate $\triangle$'s are equal, in parallel lines $AB \| CD$)

$\angle ABE = \angle DCE$ ($\triangle$ sum of a triangle)

$\therefore \triangle ABE \,|||\, \triangle DCE$ equiangular

② for reasons.
- Alot of people missed a mark by not putting enough information.

eg (Alternate $\triangle$s are =) is not ~~enough~~ enough info.

① final reason.

ii) If $AE = 1$ then $ED = 3$ (ratio of sides in similar $\triangle$'s)

$\therefore AD = 4$

$AE : AD = 1 : 4$ ①

c) $f(x) = 3x-1 \quad x>0$

If $f(x)$ is odd then

$-f(x) = f(-x)$

or

$f(x) = -f(-x)$

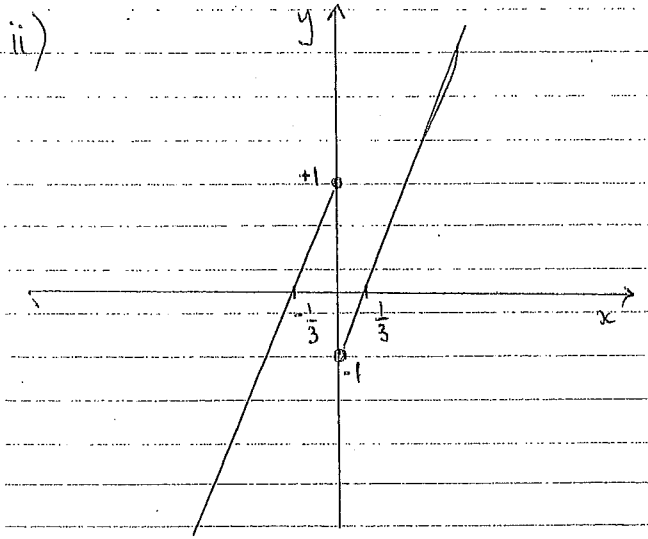
$= -(3(-x)-1)$

$= -(-3x-1)$

$= 3x+1$

$\therefore$ for $x \leq 0$, $f(x) = 3x+1$ ①

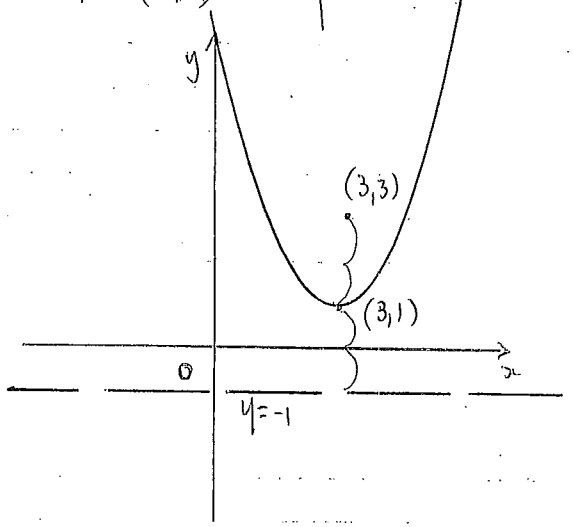
ii)



① for $x>0$

① for $x \leq 0$

d) Vertex $(3,1)$ $y=-1$



i) $a=2$ ①

ii) $(3,3)$ ①

iii) $(x-h)^2 = 4a(y-j)$

$(x-3)^2 = 8(y-1)$ ①

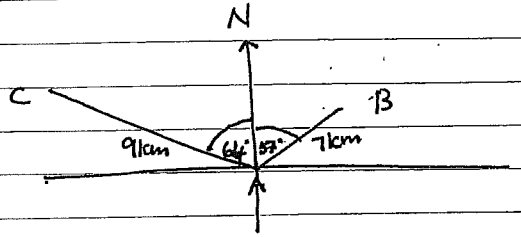
iv) $\therefore x=3$ ①

INCORRECT ROUNDING

Question 7 Solutions

a)

i)



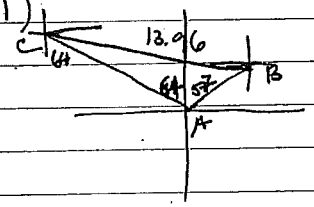
allowed with horizontal line so long as angles BAC and ① lengths were correct.

ii) $BC^2 = 7^2 + 9^2 - 2\times7\times9\times\cos 121°$ ①

$= 198.62\ldots$

$BC = 14.0$ km (1dp) ①

iii)



$\frac{\sin C}{7} = \frac{\sin 123}{13.96}$

$C = 25°27'$ ①

$90 - 25°27' - 64 = 0°33'$

$\therefore$ Bearing $= 90 + 0°33'$

$= 90°33'$

$= 091°T.$ ①

This question was poorly done.
Many people found ∠C and minused it from 90° for the angle

b) $x^6 + 7x^3 - 8 = 0$ let $u = x^3$ ①

$u^2 + 7u - 8 = 0$

$(u+8)(u-1) = 0$ ①

$u = 1$ $\quad u = -8$

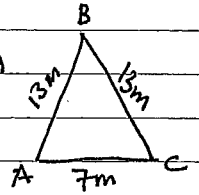
$x^3 = 1$ $\quad x^3 = -8$

$x = 1$ $\quad x = -2$

① ②

Common issue was misconception that you can't cube root a negative number

c)



i) $\cos\beta = \frac{13^2 + 13^2 - 7^2}{2\times13\times13}$ ①

$\beta = 31°14'$

$\beta = 31°$ ①

ii) $A = \frac{1}{2}ab\sin C$ ②

$= \frac{1}{2}\times13\times13\times\sin 31°$

$= 43.8$ m (1dp) ①

Needed to use unrounded angle to get correct answer.

d) $(x-3)^2 + (y+2)^2 = 121$