CRANBROOK
SCHOOL

Term 1, 2007

Year 11 Extension 1 Mathematics Mini Examination

Tuesday March 27, 2007

Time Allowed: $1\frac{1}{2}$ hours, plus 5 minutes reading time

Total Marks: 60

There are 5 questions, all of equal value

Submit your work in five 4 Page Booklets.

All necessary working should be shown in every question.

Full marks may not be awarded if work is careless or badly arranged.

Board of Studies approved calculators may be used.

Question 1 **Start a new booklet** **Marked by CJL**

a) Write $3.2\dot{6}\dot{7}$ as a mixed numeral showing all steps in your working. 2

b) Simplify $8^{2x+1} \div 16^{3x}$ 2

c) Factorise

i. $12y^2 + 16y - 3$ 1

ii. $343x^3 - y^3$ 1

d) Show that $\dfrac{2}{5-\sqrt{3}} + \dfrac{2}{5+\sqrt{3}}$ 2

e) Simplify $\dfrac{3}{x^2-4} - \dfrac{2}{x^2-3x+2}$ 2

f) If $x + \dfrac{1}{x} = 4$, find the value of $x^2 + \dfrac{1}{x^2}$ 2

Question 2 **Start a new booklet** **Marked by CJL**

a) Solve $|2x-3| = x-7$ 3

b) Solve the inequality $\dfrac{2x+1}{x-3} \geq 4$ 3

c) Solve these simultaneous equations $\begin{array}{l} 2x = 3y+1 \\ xy + x = y = 23 \end{array}$ 3

d) Solve $|x+1| < 4$ 2

e) Solve $2^{5x-1} = 16$ 1

Question 3 **Start a new booklet** **Marked by HRK**

a) Find the obtuse angle between the lines $2x + y = 4$ and $x - y = 2$, to the nearest degree. 3

b) Given $A(-1, 2)$ and $B(3, 5)$, find the coordinates of the point C which divides the interval AB externally in the ratio $3 : 1$. 2

c) Find the equation of the straight line passing through the origin and the point of intersection of the lines $5x - 2y + 3 = 0$ and $3x + 7y - 1 = 0$, WITHOUT SOLVING the equations. 3

d) Find the perpendicular distance of the point $(-2, 6)$ from the straight line which passes through the point $(1, 4)$ and which makes an angle of $45°$ with the positive direction of the x-axis. 4

Question 4 **Start a new booklet** **Marked by HRK**

a) Sketch on separate diagrams, showing essential features:

i $y = |x| + 2$ 1

ii $xy = 4$ 1

iii $(x - 2)^2 + y^2 = 9$ 1

iv $y = \dfrac{1}{x+2}$ 1

b) State the domain and range of the following functions

i $y = x^2 + 2$ 1

ii $y = \sqrt{x+2}$ 2

c) If $f(x) = \begin{cases} x^2 & \text{for } x \le -1 \\ x+1 & \text{for } x > -1 \end{cases}$ find $f(-2) - f(0) + f(-1)$ 2

d) Sketch the region $y \ge \sqrt{9 - x^2}$, $x > -3$, $y < |x - 3|$ 3

Question 5 **Start a new booklet** **Marked by HRK**

a) Consider the function $f(x) = \dfrac{x^2 - 9}{x - 3}$

i Find $\lim\limits_{x \to 3} \dfrac{x^2 - 9}{x - 3}$ 1

ii State the point where $f(x)$ is discontinuous 1

iii Sketch $f(x) = \dfrac{x^2 - 9}{x - 3}$ 1

b) Consider the function $f(x) = \dfrac{x^2}{x^2 - 1}$

i State the domain of the function 1

ii Find $\lim\limits_{x \to \infty} \dfrac{x^2}{x^2 - 1}$ 1

iii Is the function odd or even? Justify your answer. 1

iv Find any x or y intercepts. 1

v Sketch the curve, showing all asymptotes clearly 2

c) Differentiate $f(x) = x^2 + x - 2$ from first principles and hence find the gradient of the tangent to the curve at the point where is crosses the y – axis. 3

END OF EXAMINATION

11 EXT 1 MINI 2007

1a) $3.2\dot{6}\dot{7} = 3.2676767\ldots$

$100x = 326.76767\ldots$

$x = 3.267676\ldots$

$99x = 323.5$ ✓

$x = \frac{323.5}{99}$

$x = \frac{3235}{990}$

$x = 3\frac{53}{198}$ ✓

b) $8^{2x+1} \div 16^{3x}$

$= 2^{3(2x+1)} \div 2^{4(3x)}$ ✓

$= 2^{6x+3} \div 2^{12x}$

$= 2^{3-6x}$ ✓

c) i. $12y^2 + 16y - 3$

$= (6y-1)(2y+3)$ ✓

ii. $343x^3 - y^3$

$(7x-y)(49x^2 + 7xy + y^2)$ ✓

d) $\frac{2}{5-\sqrt{3}} + \frac{2}{5+\sqrt{3}}$

$= \frac{2(5+\sqrt{3}) + 2(5-\sqrt{3})}{(5-\sqrt{3})(5+\sqrt{3})}$ ✓

$= \frac{10 + 2\sqrt{3} + 10 - 2\sqrt{3}}{25-3}$

$= \frac{20}{22}$

$= \frac{10}{11}$ which is rational ✓

e) $\frac{3}{x^2-4} - \frac{2}{x^2-3x+2}$

$= \frac{3}{(x-2)(x+2)} - \frac{2}{(x-2)(x-1)}$ ✓

$= \frac{3(x-1) - 2(x+2)}{(x-2)(x+2)(x-1)}$

$= \frac{3x - 3 - 2x - 4}{(x-2)(x+2)(x-1)}$

$= \frac{x-7}{(x-2)(x+2)(x-1)}$ ✓

f) $x + \frac{1}{x} = 4$

Now $\left(x + \frac{1}{x}\right)^2 = x^2 + 2 + \frac{1}{x^2}$ ✓

$\therefore x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$

$= 4^2 - 2$

$= 14$ ✓

2a) $|2x-3| = x-7$

pos case

$2x - 3 = x - 7$

$x = -4$ ✓

neg case

$-(2x-3) = x-7$

$-2x + 3 = x - 7$

$10 = 3x$

$x = \frac{10}{3}$ ✓

check:

LHS $= |-8-3|$

$= 11$

RHS $= -4-7$

$= -11$

✗

LHS $= \left|\frac{20}{3} - 3\right|$

$= 3\frac{2}{3}$

RHS $= \frac{10}{3} - 7$

$= -3\frac{2}{3}$

✗

$\therefore$ No solutions. ✓

b) $\frac{2x+1}{(x-3)} \geq 4$

$(2x+1)(x-3) \geq 4(x-3)^2$ ✓

$(2x+1)(x-3) - 4(x-3)^2 \geq 0$

$(x-3)\left[(2x+1) - 4(x-3)\right] \geq 0$

$(x-3)\left[2x + 1 - 4x + 12\right] \geq 0$

$(x-3)(13-2x) \geq 0$ ✓

aside: $x = 3$, $x = 6\frac{1}{2}$

$\therefore 3 \leq x \leq 6\frac{1}{2}$

Note: $x \neq 3$

$\therefore 3 < x \leq 6\frac{1}{2}$ ✓

c) $2x = 3y + 1$ — ①

$xy + x - y = 23$ — ②

from ① $x = \frac{3y+1}{2}$

sub in ②

$\left(\frac{3y+1}{2}\right)y + \left(\frac{3y+1}{2}\right) - y = 2$

$(3y+1)y + (3y+1) - 2y = 46$

$3y^2 + y + 3y + 1 - 2y - 46 = 0$

$3y^2 + 2y - 45 = 0$ ✓

$y = \frac{-2 \pm \sqrt{4 - 4(3)(-45)}}{6}$

$y = \frac{-2 \pm \sqrt{544}}{6}$ $\sqrt{34}$ $\sqrt{16}$

$y = \frac{-2 \pm 4\sqrt{34}}{6}$

$y = \frac{-1 \pm 2\sqrt{34}}{3}$ ✓

$x = \frac{3\left(\frac{-1 \pm 2\sqrt{34}}{3}\right) + 1}{2}$

$x = \frac{(-1 \pm 2\sqrt{34}) + 1}{2}$

$x = \pm\frac{2\sqrt{34}}{2}$

$x = \pm\sqrt{34}$ ✓

d) $|x+1| < 4$

pos case

$x + 1 < 4$

$x < 3$

neg case

$-x - 1 < 4$ ✓

$-x < 5$

$x > -5$

$\therefore -5 < x < 3$ ✓

e) $2^{5x-1} = 16$

$2^{5x-1} = 2^4$

$\therefore 5x - 1 = 4$

$5x = 5$

$x = 1$ ✓

11 EXT 1 MINI 2007 COMMENTS

1a) R.T.Q. many students gave the answer as $\frac{647}{198}$ which is an IMPROPER fraction, instead of $3\frac{53}{198}$ which is a MIXED NUMERAL

b) Some students thought the base 2 would disappear and tried to solve a non-existant equation
$$2^{6x+3} \div 2^{12x} \neq 6x+3-12x!$$

d) Quickest way is to find a common denominator

f) Most students had NO CLUE!
$$\left(x+\frac{1}{x}\right)^2 \neq x^2+\frac{1}{x^2}$$
There should be a middle term.

2 a) Many students DID NOT check their solutions. Those who did, did not check into the ORIGINAL equation, so did not use $|2x-3| = x-7$

b) done well. Most students remembered to sketch a parabola or test on a number line.

c) Too many students gave up!! Students could not use the substitution method correctly and could not get to the correct quadratic equation.

If they did get the correct quadratic, they gave up because it wouldn't factorise and they didn't use the quadratic formula.

d) Most students got $x<3$, $x>-5$ but didn't combine it to ONE answer $-5<x<3$

e) done well

Q3 SOLUTION COMMENTS

a) $m_1 = -2$
③ $m_2 = 1$

$$\tan\theta = \left|\frac{-2-1}{1-2}\right| \checkmark$$
$$= 3$$
$$\therefore \theta = 72^\circ \checkmark$$
OBTUSE $\angle = 108^\circ$ ✓

Done well, though some did not RTQ (Read the Question!) and find the obtuse angle.
Also some did not RTQ and left answer in degrees and minutes!

b) ② $$(x,y) = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right) \checkmark$$
$$= \left(\frac{3\times3+-1\times-1}{3-1}, \frac{3\times5+-1\times2}{3-1}\right)$$
$$= \left(5, 6\tfrac{1}{2}\right) \checkmark$$

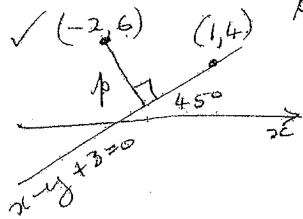
Well done except for some careless substitution

c) ③ $$ax_1+by_1+c+k(ax_2+by_1+c)=0 \checkmark$$
$$3+(-1)k=0$$
$$k=3 \checkmark$$
$$5x-2y+3+3(3x+7y-1)=0$$
$$14x+19y=0 \checkmark$$

Clear division into those who knew the method and got it right and those who didn't. Marks were not given if equations were solved - instructions were very clear

d) FIRST find eq'n of line
④ $m = \tan 45^\circ = 1$ $x_1 = 1$ $y_1 = 4$
$$y-y_1 = m(x-x_1) \checkmark$$
$$y-4 = 1(x-1)$$
$$x-y+3=0 \checkmark$$

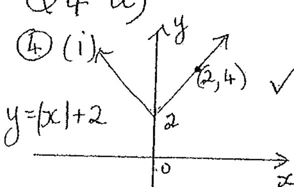
$$p = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}} \checkmark \quad \begin{cases} x_1 = -2 \\ y_1 = 6 \\ a = 1 \\ b = -1 \\ c = 3 \end{cases}$$
$$= \frac{|-2-6+3|}{\sqrt{2}}$$
$$= \frac{5}{\sqrt{2}} \text{ OR } \frac{5\sqrt{2}}{2} \checkmark$$



Well done apart from some careless substitution
Question interpretation generally good though some confused the points
Though not asked for a sketch will often help to clarify such questions.

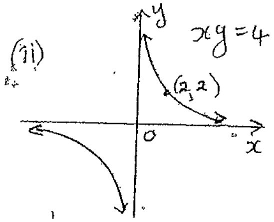
Q4 a)

① (i)



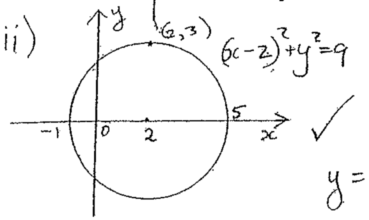
$y=|x|+2$ ✓

(ii)



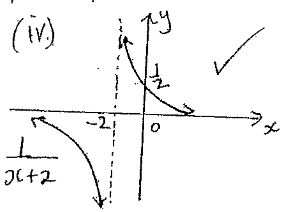
✓

(iii)



✓

(iv)



$y=\frac{1}{x+2}$ ✓

Sketching demonstrated a sound knowledge of basic curves and variations but many lacked attention to detail such as labelling axes, use of a ruler. Some were very sloppy.

b)(i) ① domain : all real x ✓
range : all real $y \geq 2$

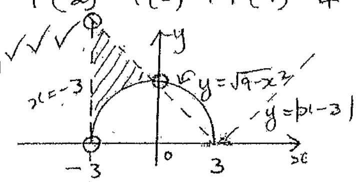
(ii) ② domain : all real $x \geq -2$ ✓
range : all real $y \geq 0$ ✓

Good on the whole but more practice needed with square roots.

c) ② $f(-2) - f(0) + f(-1) = 4$ ✓✓

Quite well done

d) ✓✓✓ ③

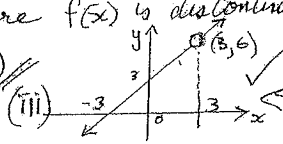


Not well done - again basics were known but detail of boundaries correctly indicated and labelling not good.

5 a) i) $\lim\limits_{x \to 3} \frac{x^2-9}{x-3} = \lim\limits_{x \to 3} \frac{(x-3)(x+3)}{(x-3)}$
$= 6$ ✓ Well done

ii) Point where $f(x)$ is discontinuous
✓ is $(3,6)$ //

(iii)



Many did not give the POINT - a point has 2 coordinates!!

Candidature divided clear + good sketch or no clue!

b) i) all real $x \neq \pm 1$ ✓

ii) $\lim\limits_{x \to \infty} \frac{x^2}{x^2-1} = \lim\limits_{x \to \infty} \frac{\frac{x^2}{x^2}}{\frac{x^2}{x^2} - \frac{1}{x^2}}$

$= \lim\limits_{x \to \infty} \frac{1}{1-\frac{1}{x^2}}$

$= 1$ ✓

Well done by most but notation needs attention

(iii) $f(x) = \frac{x^2}{x^2-1}$

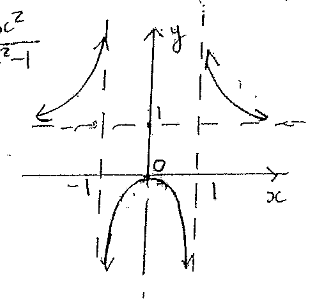
$f(-x) = \frac{(-x)^2}{(-x)^2-1} = \frac{x^2}{x^2-1} = f(x)$ ✓
∴ EVEN

Most knew what they were doing but use of brackets needed more care.

5 b)(iv) When $x = 0$
$f(x) = \frac{0}{0-1} = 0$ } ie passes through $(0,0)$ the origin ✓

Some wasted time here checking $y=0$!!! this has already been found of course.

$f(x) = \frac{x^2}{x^2-1}$



✓✓

Generally good but again candidature divided - those who put all the information together perfectly and those who had not revised or did not make the appropriate connections

c) $f'(x) = \lim\limits_{h \to 0} \frac{f(x+h) - f(x)}{h}$ ✓

$= \lim\limits_{h \to 0} \frac{(x+h)^2 + x + h - 2 - (x^2+x-2)}{h}$

$= \lim\limits_{h \to 0} \frac{x^2+2xh+h^2+x+h-2-x^2-x+2}{h}$

$= \lim\limits_{h \to 0} \frac{2xh+h^2+h}{h}$

$= \lim\limits_{h \to 0} \frac{\cancel{h}(2x+h+1)}{\cancel{h}}$

$= 2x+1$ ✓

A mark was lost for incorrect notation eg leaving out the $\lim\limits_{h \to 0}$ BEFORE taking the limit.

1ST Principles method extremely well done - only a couple took the shortcut which of course did not answer the question and so did not earn the marks!

Curve crosses the ~~x~~ y axis when $x = 0$

When $x = 0$, $f'(x) = 2x+1$

becomes $f'(0) = 2(0)+1$
$= 1$ // ✓

The last part was left out by some who did not use their knowledge of function notation and see the link here.