CRANBROOK SCHOOL

YEAR 11 2 UNIT MATHEMATICS

HSC ASSESSMENT TEST

13[TH] November **TERM 4 2003** **HRK**

Locus and the Parabola **Time:45mins**

All necessary working should be shown in every question.
Full marks may not be awarded if work is careless or badly arranged.
Approved silent calculators may be used.
Begin each question on a new page.

1. (15marks) (Begin a new page)
Marked by HRK

(a)
The point $P(x, y)$ moves such that its distance from the point $R(-1, 0)$ is always twice its distance from the point $S(2, 0)$.
Show that the locus of P is a circle and find the centre and radius.

(b)
The point $P(x, y)$ moves such that its distance from a fixed point $(-2, -2)$ is equal to its distance from the line $y = 0$.
Draw a clear sketch and using the distance formula prove that the locus of P is a parabola.

(c) (i) Find the equation of the tangent to the parabola $x^2 = -12y$ at the point $(-6, -3)$

(ii) This tangent meets the directrix at T.
Find the coordinates of T.

2. (15marks) (Begin a new page) Marked by MJB

(a) For the parabola $(x + 2)^2 = 8(y - 4)$

i. Draw a neat sketch of the curve clearly indicating the:
- α. co-ordinates of the vertex
- β. co-ordinates of the focus
- γ. equation of the directrix
- δ. axis of symmetry

ii. Write down the equation of the line through the focus parallel to the x-axis. Hence find the co-ordinates of the points where this line cuts the parabola.

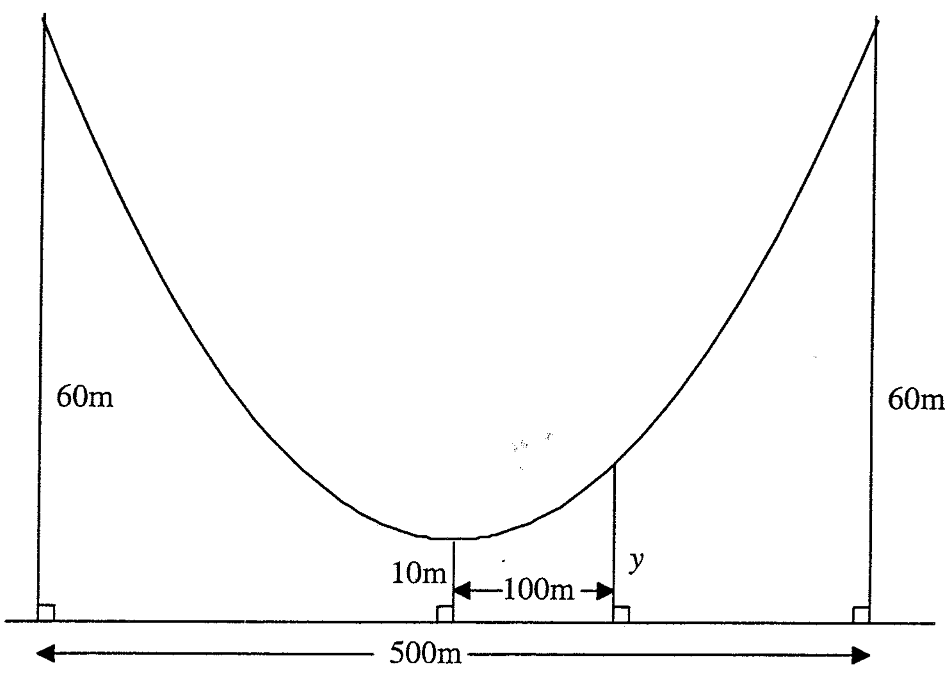
(b) A parabola has equation $y^2 - 4y + 7 = x$

i. Express this equation in the form $(y - y_1)^2 = 4a(x - x_1)$

ii. Draw a neat sketch of the parabola indicating the:
- α. co-ordinates of the focus;
- β. equation of the directrix
- γ. co-ordinates of the point of intersection of the parabola with the x - axis.

(c) i. Find the equation of a parabola with vertex $(0, 10)$ and passing through the point $(250, 60)$

ii. The diagram represents a parabolic suspension bridge of span 500m. Find the value of y.



Eva Lee

Yr 11 2u maths HSC assessment Test

a) $PR = 2PS$

$\sqrt{(x+1)^2+(y-0)^2} = 2\sqrt{(x-2)^2+(y)^2}$

$(x+1)^2 + y^2 = 4\left[(x-2)^2+y^2\right]$

$x^2+2x+1+y^2 = 4x^2-16x+16+4y^2$ ✓

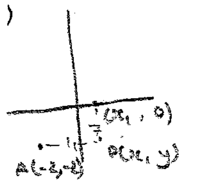
$0 = 3x^2-18x+15+3y^2$

$0 = x^2-6x+5+y^2$

$4 = (x-3)^2+y^2$ ✓

$C = (3,0) \quad R = 2$ ✓

)



$\sqrt{(x_1-x_1)^2+(y-0)^2} = \sqrt{(x+2)^2+(y+2)^2}$ ✓

$y^2 = x^2+4x+4+y^2+4y+4$

$= x^2+4x+4y+8$ ✓

i) $x^2 = -12y$

$\frac{x^2}{-12} = y$

$\frac{dy}{dx} = \frac{x}{-6}$ ✓

at $x = -6$

$\frac{dy}{dx} = 1$

$y-3 = x+6$ ✓

$y = x+3$

$x^2 = -4ay$

$-4a = -12$

$a = 3$ ✓

at $a = 3$

directrix is $y = 3$

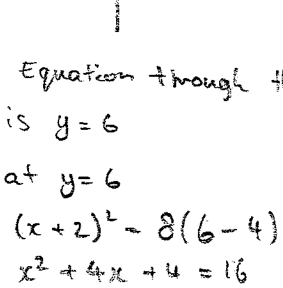
at $y = 3$

$3 = x+3$

$x = 0$ ✓

∴ $(0,3)$

2a



✓

$4a = 8$

$a = 2$

∴ $S(-2,6)$ ✓

directrix $y = 0$ ✓

axis of sym $x = -2$

Equation through the focus parrallel to the x-axis is $y = 6$

at $y = 6$

$(x+2)^2 = 8(6-4)$ ✓

$x^2+4x+4 = 16$ ✓

$x^2+4x-12 = 0$

$(x+6)(x-2) = 0$

∴ at $x = -6$

$(-6+2)^2 = 8(y-4)$ ✓

$16 = 8(y-4)$

$2 = y-4$

$6 = y$ ✓

∴ $(-6,6)$ ✓

at $x = 2$

$(2+2)^2 = 8(y-4)$ ✓

$16 = 8(y-4)$ ✓

$2 = y-4$

$6 = y$ ✓

∴ $(2,6)$

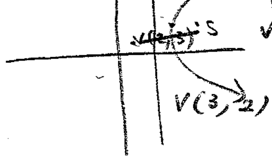
hence the line cuts at $(-6,6)$ & $(2,6)$

bi. $y^2-4y = x-7$ ✓

$(y-2)^2 = x-7+4$ ✓

$(y-2)^2 = (x-3)$

ii.



✗

$4a = 1$

$a = \frac{1}{4}$ ✓

~~S(2¼,3)~~ $S(3\frac{1}{4}, 2)$

directrix ~~x = 1¾~~ $x = 2\frac{3}{4}$

at $y = 0$

$(-2)^2 = x-3$

$4 = x-3$

$7 = x$ ✓

∴ point of intersect $(7,0)$

ci) $(x-h)^2 = 4a(y-k)$

$(x-0)^2 = 4a(y-10)$

$x^2 = 4a(y-10)$ ✓

given it pass through $(250,60)$

$(250)^2 = 4a(60-10)$ ✓

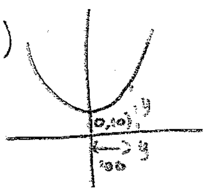
$62\,500 = 200a$

$a = 312.5$ ✓

∴ $x^2 = 4(312.5)(y-10)$

$x^2 = 1250(y-10)$ ✓

ii)



$x^2 = 1250(y-10)$

$100^2 = 1250(y-10)$ ✓

$\frac{10000}{1250} = y-10$

$8 = y-10$

$y = 18$ ✓