CRANBROOK COLLEGE

CLASS TEST EXTENSION 1 TEST 11-5-07
Trigonometric Functions.

Name____________________Class________

Instructions: Show all necessary working throughout the test on A4 paper.

Begin a new page as specified. **Time allowed: 45 minutes**

[Begin a new page] **SKB**

1. (a) Simplify to a single termed expression: $\dfrac{2\sin(\frac{\pi}{2}-2\theta)-1}{\sec\theta}$ [4]

   (b) Find the general solution of: $\sqrt{3}\sin x-\cos x=\sqrt{3}$ [4]

   (c) Find $\int \cos^2 x+\dfrac{1}{\cos^2 x}\,dx$ [3]

   (d) Find the exact volume generated when the area bounded by the curve $y=\sin 2x$, the x-axis and lines $x=0$ and $x=\dfrac{\pi}{6}$, is rotated about the x-axis. [4]

[Begin a new page] **JAH**

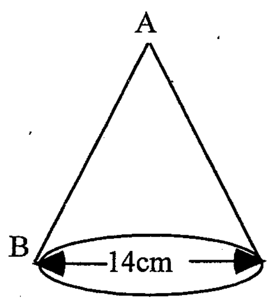
2. (a) Express in degrees : **[1]+[1]**	(b) Express in radians : **[1]+[1]**
(i) $\dfrac{5\pi}{6}$	(i) 240^0
(ii) 2.4 (to nearest minute)	(ii) $38^0\,41'$ (to 2 d.p.)

3.

A cardboard cone of diameter 14cm is cut along the edge AB to form a sector with angle 120^0.

Find : (a) the length of AB. **[4m]**

(b) the area of this sector in cm^2 correct to 2 significant figures ? **[3m]**



[Begin a new page] **CJL**

4. For each of the following functions state the

(a) period **[3m]**
(b) range **[3m]**
(c) amplitude. **[3m]**

Hence sketch each function **on a separate graph** over the domain specified :

(i) $y = 5 \sin 2x$ for $-\pi \le x \le \pi$ **[1m]**

(ii) $y = \tan x$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ **[1m]**

(iii) $y = 4 \sin (\frac{\pi}{2} - x) - 2$ for $0 \le x \le 2\pi$ **[2m]**

[Begin a new page] **BMM**

5. Evaluate the following integrals:

(a) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin^3 x \, dx$ using the substitution $u = \cos x$ **[3m]**

(b) $\int_{0}^{1} \frac{x^2}{\sqrt{1-x^2}} dx$ using the substitution $x = \sin\theta$ **[4m]**

EXT 1 TRIG TEST 11-5-07

1(a) $\dfrac{2\sin\left(\frac{\pi}{2}-2\theta\right)-1}{\sec\theta}$

$= \dfrac{2\cos 2\theta - 1}{\sec\theta}$ ✓

$= \dfrac{2(2\cos^2\theta - 1) - 1}{\sec\theta}$ ✓

$= (4\cos^2\theta - 3)\cos\theta$ ✓

$= 4\cos^3\theta - 3\cos\theta$ ✓

$= \cos 3\theta$

(b) $\sqrt{3}\sin x - \cos x = \sqrt{3}$

$\therefore \sqrt{(\sqrt{3})^2+(-1)^2}\left(\frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x\right) = \sqrt{3}$ ✓

$\therefore \sin(x-\alpha) = \frac{\sqrt{3}}{2}$

where $\cos\alpha = \frac{\sqrt{3}}{2}$

$\sin\alpha = \frac{1}{2}$ ✓

$\therefore \tan\alpha = \frac{1}{\sqrt{3}} \quad \therefore \alpha = \frac{\pi}{6}$

$\therefore \sin\left(x - \frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ ✓

$= \sin\frac{\pi}{3}$

$\therefore x - \frac{\pi}{6} = \pi n + (-1)^n \cdot \frac{\pi}{3}$ ✓

$\therefore x = \pi n + (-1)^n \cdot \frac{\pi}{3} + \frac{\pi}{6}$

where n is any integer

OR Let $\sqrt{3}\sin x - \cos x = \sqrt{3}$

be of the form $R\sin(x-\alpha) = \sqrt{3}$

where $R = \sqrt{(\sqrt{3})^2+(-1)^2} = 2$

$\alpha = \tan^{-1}\frac{1}{\sqrt{3}} = \frac{\pi}{6}$

$\therefore 2\sin\left(x - \frac{\pi}{6}\right) = \sqrt{3}$

$\therefore \sin\left(x - \frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} = \sin\frac{\pi}{3}$ etc.

(as above)

OR Let $\sqrt{3}\sin x - \cos x = \sqrt{3}$

$\therefore \cos x - \sqrt{3}\sin x = -\sqrt{3}$

$\therefore R\cos(x+\alpha) = -\sqrt{3}$ ✓

where $R = \sqrt{1^2+(-\sqrt{3})^2} = 2$

$\cos\alpha = \frac{1}{2}, \ \sin\alpha = \frac{\sqrt{3}}{2} \quad \therefore \tan\alpha = \sqrt{3}$

$\therefore \alpha = \frac{\pi}{3}$ ✓

$\therefore 2\cos\left(x + \frac{\pi}{3}\right) = -\sqrt{3}$

$\therefore \cos\left(x + \frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2} = \cos\left(\frac{5\pi}{6}\right)$ ✓

$\therefore x + \frac{\pi}{3} = 2\pi n \pm \frac{5\pi}{6}$

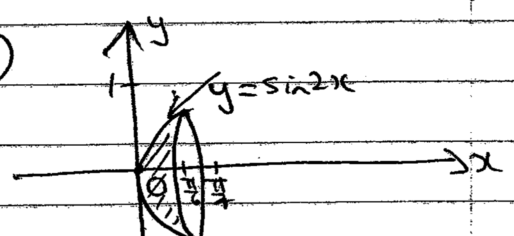
$\therefore x = 2\pi n \pm \frac{5\pi}{6} - \frac{\pi}{3}$ where n is any integer. ✓

(c) Let $I = \int \cos^2 x + \frac{1}{\cos^2 x}\, dx$ ✓

$= \int \frac{1}{2}\left[1 + \cos 2x\right] + \sec^2 x \, dx$

$= \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right] + \tan x + c$ ✓ ✓

(d)



$\text{Volume} = \pi \int_0^{\frac{\pi}{6}} \sin^2 2x \, dx$ ✓

$= \pi \int_0^{\frac{\pi}{6}} \frac{1}{2}\left[1 - \cos 4x\right] dx$ ✓

$= \frac{\pi}{2}\left[x - \frac{\sin 4x}{4}\right]_0^{\frac{\pi}{6}}$

$= \frac{\pi}{2}\left[\left(\frac{\pi}{6} - \frac{\sin\frac{2\pi}{3}}{4}\right) - 0\right]$ ✓

$= \frac{\pi}{2}\left[\frac{\pi}{6} - \frac{\sqrt{3}}{8}\right]$

$= \frac{\pi}{2}\left[\frac{4\pi - 3\sqrt{3}}{24}\right]$

$= \frac{\pi}{48}\left(4\pi - 3\sqrt{3}\right) \text{ units}^3$ ✓

2 (a) (i) $\frac{5\pi}{6} = 150°$ ✓

(ii) $2.4 = 2.4 \times \frac{180°}{\pi}$

$= 137.5098708\ldots°$

$= 137° 31'$ ✓ (to nearest minute)

(b) (i) $240° = 240° \times \frac{\pi}{180°}$

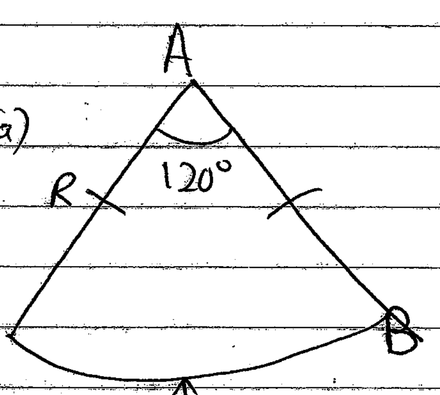
$= \frac{4\pi}{3}$ ✓

(ii) $38° 41' = 38\frac{41}{60}°$

$= 38\frac{41}{60} \times \frac{\pi}{180°}$

$= 0.68$ (2dp) ✓

3 (a)



✓

arc length $= 2\pi r$ (where $r = 7$) ✓

$\therefore R \times \frac{2\pi}{3} = 2\pi \times 7$

$\therefore R = \frac{14\pi}{2\pi/3}$ ✓

$\therefore R = 21$

$\therefore$ length of AB $= 21$ cm ✓

(b) $A_{sector} = \frac{1}{2} R^2 \theta$

$= \frac{1}{2} \times 21^2 \times \frac{2\pi}{3}$ ✓

$= 461.8141201\ldots$ ✓

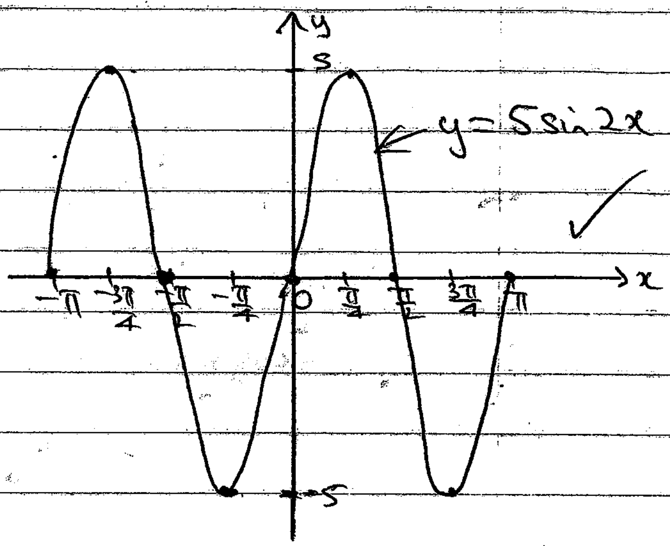
$= 460 \text{ cm}^2$ (2 s.f.) ✓

4 (i) $y = 5\sin 2x \quad -\pi \le x \le \pi$

(a) period $= \frac{2\pi}{2} = \pi$ ✓

(b) range: $-5 \le y \le 5$ ✓

(c) amplitude is 5 units ✓



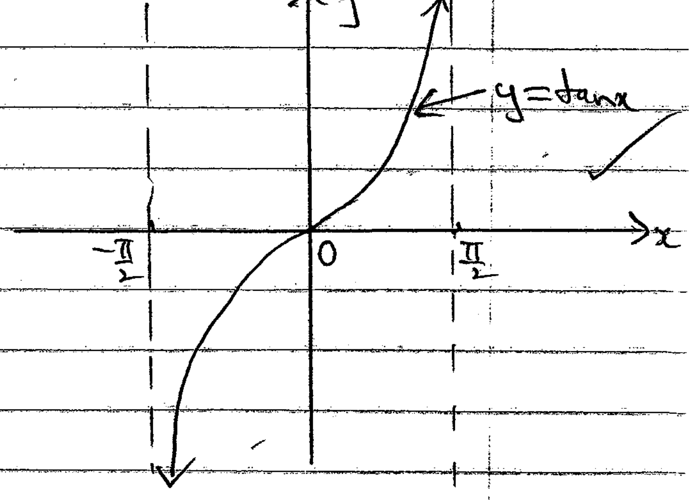
✓

(ii) $y = \tan x \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$

(a) period $= \pi$ ✓

(b) range: all real y ✓

(c) amplitude is infinite ✓



✓

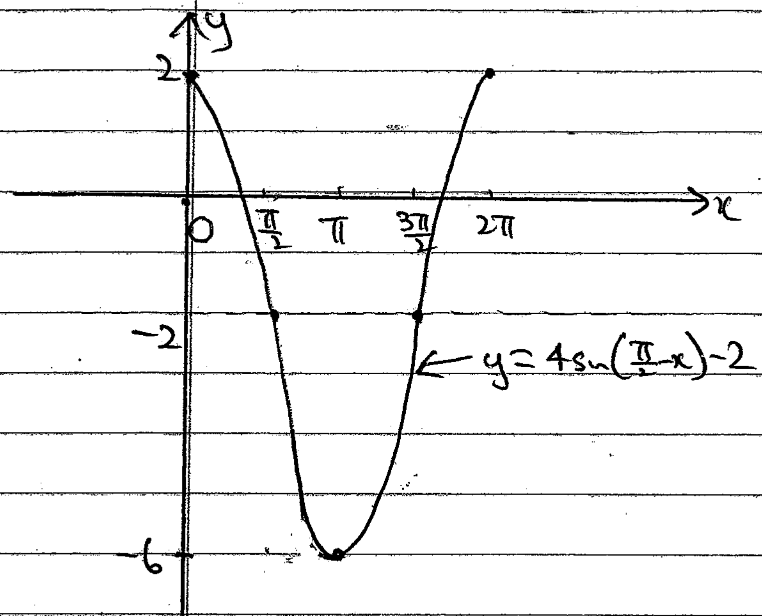
(iii) $y = 4\sin\left(\frac{\pi}{2} - x\right) - 2$

$\therefore y = 4\cos x - 2$

(a) period = 2π

(b) range: $-6 \le y \le 2$

(c) amplitude = 4 units



5 (a) Let $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin^3 x \, dx$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin x\left(1 - \cos^2 x\right) dx$$

let $u = \cos x$ when $x = \frac{\pi}{6}$ $u = \frac{\sqrt{3}}{2}$

$\therefore \frac{du}{dx} = -\sin x$ $x = \frac{\pi}{2}$ $u = 0$

$$\therefore I = \int_{\frac{\sqrt{3}}{2}}^{0} (1 - u^2) \cdot -du$$

$$= \int_{0}^{\frac{\sqrt{3}}{2}} 1 - u^2 \, du$$

$$= \left[u - \frac{u^3}{3}\right]_0^{\frac{\sqrt{3}}{2}}$$

$$= \frac{\sqrt{3}}{2} - \frac{1}{3}\left(\frac{3\sqrt{3}}{8}\right)$$

$$= \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{8}$$

$$= \frac{3\sqrt{3}}{8}$$

(b) Let $I = \int_0^1 \frac{x^2}{\sqrt{1 - x^2}} dx$

let $x = \sin\theta$ when $x = 0$ $\theta = 0$

$\frac{dx}{d\theta} = \cos\theta$ $x = 1$ $\theta = \frac{\pi}{2}$

$$\therefore I = \int_0^{\frac{\pi}{2}} \frac{\sin^2\theta}{\sqrt{1 - \sin^2\theta}} \cdot \cos\theta \, d\theta$$

$$= \int_0^{\frac{\pi}{2}} \sin^2\theta \, d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{2}\left[1 - \cos 2\theta\right] d\theta$$

$$= \frac{1}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{2}\left[\left(\frac{\pi}{2} - 0\right) - 0\right]$$

$$= \frac{\pi}{4}$$