CRANBROOK SCHOOL

Year 10 5.2/5.3 & 9 Acc Mathematics

Term 1 Exam, 2013

+ SOLUTIONS

Total Marks: 80

Time allowed: 100 minutes

All questions must be answered in a separate writing booklet
Show all working to gain full marks

Section A (20 marks) **Marked by KH1**

START A NEW BOOKLET

1. Factorise the following:

a. $x^3 - x$ 2

b. $9(x+y)^2 - 121$ 2

c. $10x + 15y - 4xz - 6yz$ 2

d. $-4x^2 + 12x + 7$ 2

e. $4x^3 - 12x^2 - x + 3$ 3

2. Fully simplify the following:

a. $\dfrac{2x-2y}{5y-5x}$ 2

b. $\dfrac{x+1}{2} + \dfrac{x+2}{3}$ 2

c. $\dfrac{1}{x-y} + \dfrac{2x-y}{x^2-y^2}$ 2

d. $\dfrac{x^4-y^4}{x^2-2xy+y^2} \div \dfrac{x^2+y^2}{x-y}$ 3

Section B (20 marks) — Marked by HRK

START A NEW BOOKLET

1. Solve these quadratic equations:

   a. $p(p+6)=0$ — 1

   b. $(3x+5)(2x-1)=0$ — 1

   c. $(x-3)(x+4)=8$ — 2

2. Factorise and solve the following:

   a. $7m-m^2=0$ — 2

   b. $y^2-3y-70=0$ — 2

   c. $5t^2-11t+2=0$ — 2

3. Solve the equation $x^2+6x+7=0$ by completing the square.
   Leave your answer in surd form. — 3

4. a. State the quadratic formula — 1

   b. Solve the equation $a-\frac{1}{2a}=5$, giving roots correct to 3 decimal places. — 3

5. Using $v=x^2$, solve $x^4-5x^2+4=0$. — 3

Section C (20 marks) — Marked by TMS

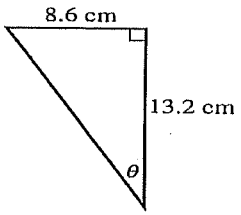
START A NEW BOOKLET

1. If $\cos\theta=0.55$, find the value of θ to the nearest degree.

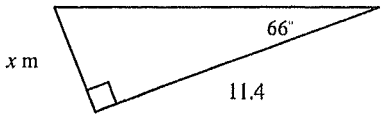
2. If $\tan\theta=0.55$, find $\sin\theta$ correct to 3 decimal places.

3. Evaluate $\frac{\cos 42^\circ}{\tan 8^\circ+\sin 53^\circ}$ correct to 2 significant figures.

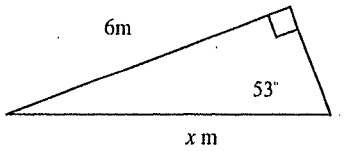
4. Find the value of θ correct to the nearest degree.



5. Find the length x in the following triangle, correct to two decimal places?

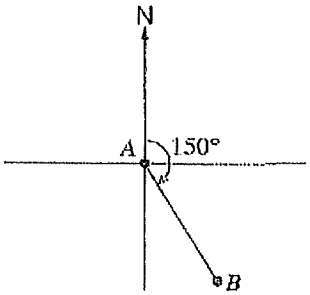


6. Find the value of x correct to 1 decimal place.



7. A plane flies on a bearing of 150° from *A* to *B*. What is the bearing of A from B?

1



NOT TO SCALE

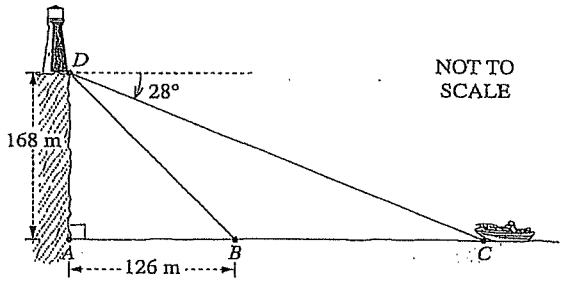
8. The base of a lighthouse, D, is at the top of a cliff 168 metres above sea level.
The angle of depression from D to a boat at C is 28°.
The boat heads towards the base of the cliff, A, and stops at B.

a. What is the angle of depression from D to B, correct to the nearest minute

1

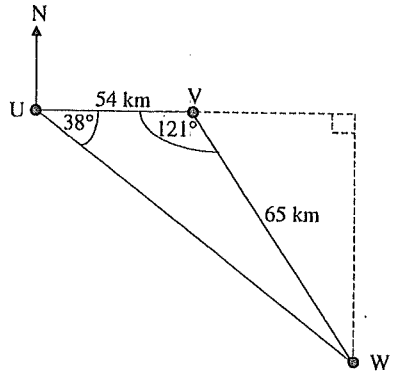
b. How far did the boat travel from C to B, correct to the nearest metre?

2



NOT TO SCALE

9. The location of three towns, Ubiri (U), Vanati (V), and Wallarah (W) is shown in the diagram. Vanati is due east of Ubiri.



NOT TO SCALE

a. What is the bearing of W from V?

1

b. How far south of V is W?

2

10. A plane flew 45.8 km due north from *X* to *Y*, then turned and flew due west to *Z*.
The bearing of *Z* from *X* is 285°.

a. Draw a fully labelled diagram clearly showing all the information above.

1

b. Find the distance *XZ*.

2

Section D (20 marks) — Marked by RABS

START A NEW BOOKLET

1. Find the value of x in the following, showing all working.
Full marks may not be awarded if you use a calculator.

a. $\cos x = \sin 31^\circ$ **1**

b. $\cos 2x^\circ = \sin 3x^\circ$ **2**

2. Solve the equations below, correct to the nearest minute.

a. $\sin\theta = 4\cos\theta$ **2**

b. $\dfrac{6}{\sin\theta} = \dfrac{\sqrt{2}}{\cos\theta}$ **2**

3. Prove $\dfrac{\sin(90-\theta)\tan\theta}{\sin\theta} = 1$ **2**
Ensure that you show all steps of working.

4. Find the exact value of each expression, showing all working.
Full marks may not be awarded if you use a calculator.
Ensure that your answer is a single fraction with a rational denominator.

a. $\tan^2 30^\circ$ **2**

b. $\sin 45^\circ + \cos 30^\circ$ **2**

c. $\tan^2 60^\circ - \cos^2 60^\circ$ **2**

5. The height h (in metres) of a lump of concrete, t seconds after it has been thrown vertically upwards is given by:

$$h = 30t - 5t^2$$

a. After what time is the concrete at a height of 40m? **3**

b. At what time does the concrete return to earth? **2**

PLEASE TURN OVER – THIS PAGE IS DOUBLE SIDED

Start here. SECTION A

(Q1) $x^3 - x = x(x^2 - 1)$ ✓

$= x(x-1)(x+1)$ ✓

MANY STUDENTS DID NOT RECOGNISE $(x^2-1) = (x-1)(x+1)$ ⇓ DIFF. OF 2 SQUARES

DONE OK SOMETIMES $x(x^2-x)$

(Q2) $9(x+y)^2 - 121 = (3(x+y) - 11)(3(x+y) + 11)$ ✓

$(3x+3y-11)(3x+3y+11)$ ✓

OVERALL POORLY ATTEMPTED. STUDENTS ALSO FORGOT TO EXPAND BRACKETS & TIDY RESULTS

(Q3) $10x + 15y - 4xz - 6yz$

$= \underbrace{10x - 4xz}_{\text{rearrange}} + 15y - 6yz$ grouping in pairs

$= 2x(5 - 2z) + 3y(5 - 2z)$ Take out HCF ✓

$= (5 - 2z)(2x + 3y)$ ✓

MOST STUDENTS COMPLETED THIS WELL!!

(Q4) $-4x^2 + 12x + 7$

$= -(4x^2 - 12x - 7)$ ← MANY FORGOT TO FACTOR ✓

$= -(2x+1)(2x-7)$ NEGATIVE 1 (-1) ✓

OK!

(Q5) $4x^3 - 12x^2 - x + 3$

$= 4x^2(x-3) - 1(x-3)$

$= (4x^2-1)(x-3)$ ← 3 mark question neg. extra step!!

$= (2x-1)(2x+1)(x-3)$

MANY STUDENTS DID NOT RECOGNISE $(4x^2-1)$ AS A DIFF. OF TWO SQUARES.

(Q6) $\frac{2x-2y}{5y-5x}$

$= \frac{2(x-y)}{-5(-y+x)}$ } STUDENTS DID NOT FACTOR −5 IE. $2(x-y)$ ⇎ $5(y-x)$ NOT THE SAME ✓

$= \frac{2(x-y)}{-5(x-y)}$

$= \frac{2}{-5}$

$(x-y) \neq (y-x)$ ∴ CANNOT CANCEL

(Q7) $\frac{x+1}{2} + \frac{x+2}{3}$

$= \frac{3(x+1)}{6} + \frac{2(x+2)}{6}$ OK!

$= \frac{3x+3+2x+4}{6}$

$= \frac{5x+7}{6}$

Additional writing space

Start here.

Q8) $\frac{1}{x-y} + \frac{2x-y}{x^2-y^2}$ ← MANY STUDENTS DID NOT FACTORISE

$= \frac{1}{x-y} + \frac{2x-y}{(x-y)(x+y)}$

↑ COMMON ↑

$= \frac{x+y}{(x-y)(x+y)} + \frac{2x-y}{(x-y)(x+y)}$ ✓

$= \frac{3x+y-y}{(x-y)(x+y)}$

$= \frac{3x}{(x-y)(x+y)}$ ✓

COMPLETED OK! $(x-y)(x+y) = x^2-y^2$ WAS AN UNECESSARY STEP

$\frac{x^4-y^4}{x^2-2xy+y^2} \div \frac{x^2-y^2}{x-y}$ ← flip

↑ change sign

$\frac{x^4-y^4}{x^2-2xy+y^2} \times \frac{x-y}{x^2+y^2}$ ← THIS IS NOT x^2-y^2 ∴ CANNOT FACTORISE AS $(x-y)(x+y)$ ✓

$\frac{(x^2-y^2)(x^2+y^2)}{(x-y)(x-y)} \times \frac{x-y}{x^2+y^2}$

↑ PERFECT SQ.

$\frac{x^2-y^2}{x-y}$

$= \frac{(x-y)(x+y)}{x-y}$ ← MANY STUDENTS DID NOT SEE THIS AS DIFF. OF TWO SQUARES!

$= x+y$ ✓

OVERALL, VERY POORLY ATTEMPTED. OFTEN THIS MISTAKE WAS MADE

$\frac{x^{2^2}-y^{2^2}}{\dots} \times x^{2^2}+y^{2^2}$ ←

Office Use Only – Do NOT write anything, or make any marks below this line.

Start here.

Office Use Only – Do NOT write anything, or make any marks below this line.

SECTION B

1 a) $p(p+6)=0$

$p=0 \quad p=-6$

Q1(a)(b) are ALREADY FACTORISED FOR YOU! JUST SOLVE!!!

JUSTIFICATION FOR this method is $AB=0$ $\therefore$ IT MUST BE EQUAL TO ZERO

b) $(3x+5)(2b-1)=0$

① $3x+5=0 \quad 2b-1=0$

$x=-\frac{5}{3}, \quad b=\frac{1}{2}$

c) MUST Be EXPANDED

$x^2+x-12=8$

$x^2+x-20=0$ ←

② $x^2+x-20=0$

$(x+5)(x-4)=0$

$x=-5, 4$

2 a) $7m-m^2=0$

② $m(7-m)=0$

$m=0 \quad 7-m=0$

$m=7$

BASIC FACTORISING NEEDS TO HAPPEN :)

TRINOMIAL FACTORISATION NEEDS MORE PRACTICE

AND R I Q

Read the Question

MARKS WERE NOT GIVEN UNLESS THE REQUESTED FACTORISING WAS DONE.

b) $y^2-3y-70=0$

② $(y-10)(y+7)=0$

$y=10, -7$

c) $5t^2-11t+2=0$

② $(5t-1)(t-2)=0$

$t=\frac{1}{5}, 2$

3 / ③ $x^2+6x+7=0$

$x^2+6x=-7$

$x^2+6x+(+3)^2=-7+9$

$(x+3)^2=2$

$x+3=\pm\sqrt{2}$

$x=-3\pm\sqrt{2}$

Remember $\pm$

a number of students u

$+3\pm\sqrt{2}$:) BASIC EQU SOLU needs atten

4 a) $ax^2+bx+c=0$ has solution

① $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

b) $a-\frac{1}{2a}=5$

③ so $\times 2a$

NOTE: Before using this f Equation must loo THIS

$a\times 2a-\frac{1}{2a}\times 2a=5\times 2a$

$2a^2-1=10a$

$2a^2-10a-1=0$

$a=\frac{-(-10)\pm\sqrt{(-10)^2-4\times 2\times -1}}{2\times 2}$

$=\frac{10\pm\sqrt{100+8}}{4}$

$=-0.098, 5.098$

5 / ③ $V=x^2 \therefore V^$

V^2-5V+

$(V-4)(V-1)=$

$V=4, 1$

ie $x^2=4, 1$

$\therefore x=\pm 2, \pm 1$

NB $x^2=4$

$x=\pm\sqrt{4}$

Section C

$\cos Q = 0.55$

$Q = \cos^{-1} 0.55$

$Q = 56^\circ 37' 58.75''$

$Q = 57^\circ$ ✓

$\tan Q = 0.55$

$Q = \tan^{-1} 0.55$

$Q = 28^\circ 48' 38.86''$

$\sin 28^\circ 48' 38.86'' = 0.4819$

$= 0.482$ ✓

0.79 (2 sig. figs.) ✓ ✓

Note:
Rounding correctly to 2 sig. figs. was an issue for many.

$\tan Q = \frac{8.6}{13.2}$ ✓

$Q = \tan^{-1}\left(\frac{8.6}{13.2}\right)$

$Q = 33^\circ$ ✓

$\tan 66 = \frac{xc}{11.4}$

$1.4 \tan 66 = xc$ ✓

$xc = 25.60$ ✓

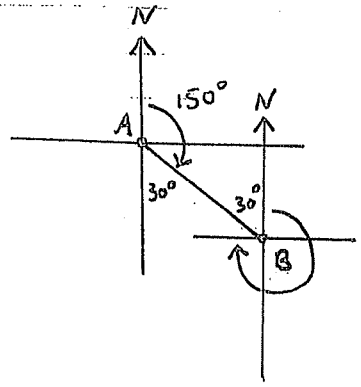
$\sin 53 = \frac{6}{xc}$ ✓

$= \sin 53 = 6$

$xc = \frac{6}{\sin 53}$ ✓

$xc = 7.5$ m

7.



Bearing $= 360^\circ - 30^\circ$

$= 330^\circ$ ✓

8.(a) Let $\angle ABD = Q$

$\therefore \tan Q = \frac{168}{126}$

$Q = \tan^{-1}\left(\frac{168}{126}\right)$

$Q = 53^\circ 08'$ (nearest minute) ✓

Angle of depression $= 53^\circ 08'$

(b) $\tan 28^\circ = \frac{168}{AC}$

$AC \tan 28^\circ = 168$

$AC = \frac{168}{\tan 28^\circ}$

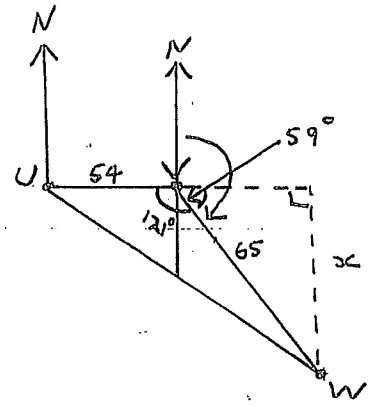
$AC = 315.96$ m ✓

$\therefore$ Distance CB $= 315.96 - 126$

$= 189.96..$

$= 190$ m (to nearest m) ✓

(a)



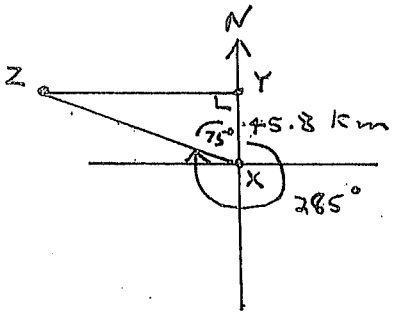
bearing : 90° + 59°
: 149°

) sin 59° : $\frac{x}{65}$

x : 65 sin 59°
x : 55.72 km (2 d.p)

Note:
Many students had poor diagrams.

). (a)



(b) XZ : cos $\quad \cos 75° : \frac{45.8}{XZ}$

$XZ \cos 75° : 45.8$

$XZ : \frac{45.8}{\cos 75°}$

XZ : 176.96 km (2 d.p)

SECTION D.

1a. $\sin\theta = \cos(90° - \theta)$

$\therefore \sin 31° = \cos(90° - 31°)$

$\therefore x = 59°$ ✓

b. COMPLEMENTARY ANGLES AGAIN.

IF $\cos x = \sin y$
THEN $x + y = 90°$

$\therefore 2x + 3x = 90°$
$5x = 90°$ ✓
$x = 18°$ ✓

- MANY ISSUES WITH THIS QUESTION.
- MANY TRIED THE TAN RATIO OR SEPARAT
  ∴ 2x FROM COS.

2a. $\sin\theta = 4\cos\theta$ ÷ cosθ BOTH SID

$\frac{\sin\theta}{\cos\theta} = 4$

$\tan\theta = 4$ ✓

$\theta = \tan^{-1}(4)$,
$= 75°58'$ ✓

- MANY DID NOT IDENTIFY THAT IT WAS A $\frac{\sin}{\cos}$

b. $\frac{6}{\sin\theta} = \frac{\sqrt{2}}{\cos\theta}$ · CROSS MULTIPLY

$6\cos\theta = \sqrt{2}\sin\theta$ ✓

$\div\sqrt{2}$ BOTH SIDES

$\frac{6\cos\theta}{\sqrt{2}} = \sin\theta$

$\div\cos\theta$ BOTH SIDES

$\frac{6}{\sqrt{2}} = \frac{\sin\theta}{\cos\theta}$

$\frac{6}{\sqrt{2}} = \tan\theta$

$\theta = \tan^{-1}\left(\frac{6}{\sqrt{2}}\right) = 76°44'$ ✓

- AGAIN, MANY COULD NOT IDENTIFY WHICH TECHNIQUES REQUIRED TO SOLVE THIS EQ'N.
- TOO MANY ALGEBRAIC ERRORS!

---

3. $RHS = \frac{\sin(90°-\theta)\tan\theta}{\sin\theta}$ · USE $\cos\theta = \sin(90-\theta)$

$= \frac{\cos\theta\tan\theta}{\sin\theta}$ ✓

· USE $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \frac{\cos\theta \times \frac{\sin\theta}{\cos\theta}}{\sin\theta}$ ✓

$= \frac{\sin\theta}{\sin\theta} = 1 = RHS$

- MOST STUDENTS WERE ON THE RIGHT T
- A FEW DID NOT SHOW ENOUGH WORKING GAIN FULL MARKS
- MANY RANDOMLY SUBSTITUTED, SAY, θ INTO THE EQ'N. THIS IS NOT A PRO

4a. $\tan^2 30° = (\tan 30°)^2$

$= \left(\frac{1}{\sqrt{3}}\right)^2$ ✓

$= \frac{1}{3}$ ✓

b. $\sin 45° + \cos 30°$

$= \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}$ ✓

$= \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{2}+\sqrt{3}}{2}$ ✓

c. $\tan^2 60° - \cos^2 60°$

$= (\sqrt{3})^2 - (\frac{1}{2})^2$ ✓

$= 3 - \frac{1}{4}$

$= \frac{11}{4}$ ✓

- FORGETTING EXACT VALUES WAS THE MAIN ISSU
- SOME STUDENTS NEED TO REVISE SURDS FRACTIONS!

5a. $h = 40$

$\therefore 40 = 30t - 5t^2$

$5t^2 - 30t + 40 = 0$ ✓

$5(t^2 - 6t + 8) = 0$

$5(t-2)(t-4) = 0$ ✓

$\therefore t = 2, 4$ seconds ✓

- MANY FAILED TO RECOGNISE A QUADRATIC EQUATION
- OTHERS TRIED HIGHLY UNUSUAL METHODS RATHER THAN COLLECTING TERMS ON ONE SIDE OF THE EQ'N.

b. THE LUMP OF CONCRETE RETURNS TO EARTH AT $h = 0$

$0 = 30t - 5t^2$

$0 = 5t(t-t)$ ✓

$\therefore t = 0$ or 6 seconds

HOWEVER, IT "RETURNS" TO EARTH AT $t = 6$ s. ✓

- A LOT OF STUDENTS RANDOMLY ADDED OR SUBTRACTED THEIR ANSWERS FROM a. WHY?
- MARKS WERE NOT AWARDED IF STUDENTS DID NOT RECOGNISE THAT $t = 0$ DIDN'T ANSWER THE QUESTION.