CRANBROOK SCHOOL

Year 9 MATHEMATICS (Accelerated)

ie. YEAR 10 MATHS

Term 3 1999 **Time: 2 Hours / BES**

NAME :__

Show all working and pay particular attention to setting out.
Marks may not be awarded for careless or untidy work, or where working is not shown.
Each question is worth 10 marks.

1.a. Factorise and simplify

i. $4x^2 - 1$

ii. $3x^4 + 81x$

iii. $3x^2 + 5x - 8 = 0$

b. Solve simultaneously

$$2x - 5y = 22$$
$$4x + 3y = 5$$

2. Simplify

(i). $3\sqrt{6}(\sqrt{3}-2\sqrt{2})$

(ii). $\sqrt{4m^3} \div 4$

(iii). $\sqrt{8}+7\sqrt{32}-\sqrt{48}+2\sqrt{75}$

(iv). Write $\frac{\sqrt{7}+3}{7+\sqrt{5}}$ with a rational denominator.

3. (i). Given the points A(-2,4) B(1,-5) C(3,2) D(6,-8) E(-2,5) find the exact distance AD.

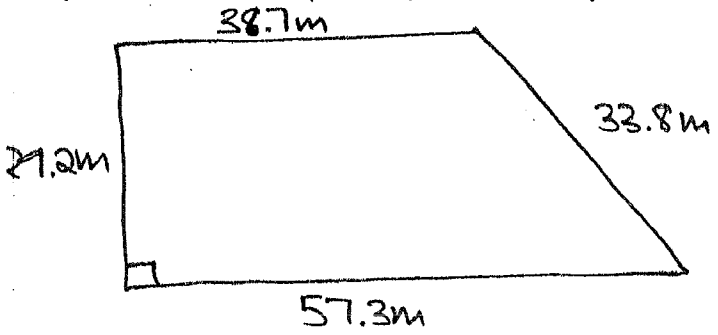
(ii). find the gradient of the line AC.

(iii). Show that the equation of the line AC is $2x+5y-110=0$.

(iv). find the equation of the line through B perpendicular to AC.

(v). does the point E lie on the line AC (show working).

3.(a). The field shown below is to be turfed. How much will this cost if turf is sold for $13.60 per m^2.



(b). A rectangular pyramid has a height of 12m and its base measures 10m by 8m. Find its volume correct to 2 significant figures.

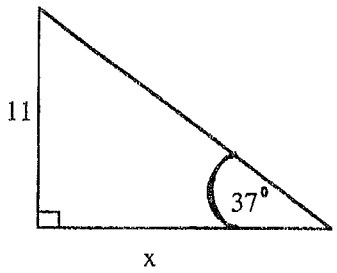
c. Water is being pumped into an in-ground swimming pool which is in the shape of a hemisphere. The pool has a radius of 12 metres. Water is flowing into the pool at a rate of 90,000 litres per hour.

i. What is the volume of the pool in cubic metres?

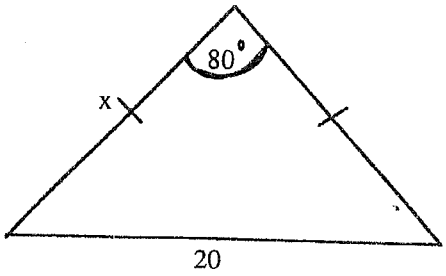
ii. What is the capacity of the pool in litres?

iii. How long will it take to fill the pool to the nearest minute?

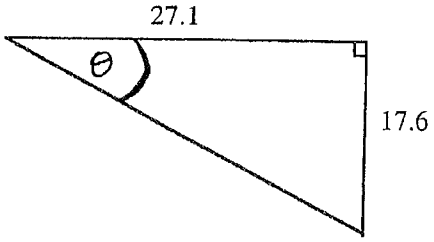
4.a. Find the value of x in the following diagrams correct to 2 decimal places.
i.



ii.

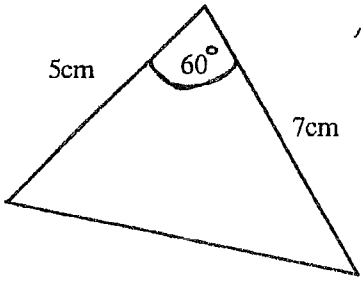


b. Find the value of θ in the diagram to the nearest minute.



b. A triangle has sides 5cm, 7cm and 12cm in length. Find the size of the smallest angle correct to the nearest minute.

5.a. Find the area of the following triangle. Use your knowledge of exact trigonometric values to write your answer exactly.



b. David is standing on top of a tree. He sees a bush on the ground, due north of the tree at an angle of depression of 37°. He sees a chip packet on the ground, due south of the tree at an angle of depression of 62°. The bush is 39 metres away (in a straight line) from David. (The base of the tree, the bush and the chip packet are in a straight line)

i. Draw a diagram to show this information.

ii. What is the distance from David to the chip packet to the nearest centimetre?

iii. What area can David see from the tree. Think about this and give reasons.

6. (a). On a single number plane shade the region
$y > 2x+1$, $x \geq -1$, $y < 2$.

(b). On separate diagrams, sketch, showing the important features, the graphs of: $x^2+y^2=16$.

(c). ∴ For the parabola with equation $y = x^2-7x+10$ find:

(i). the equation of the axis of symmetry.

(ii). the vertex.

(iii). the y-intercept

(iv). the x-intercepts

(v). sketch the curve showing the above information.

7.(a). The scores in a class test out of 10 were as follows:

7	8	6	3	9	10	5	6	9	2
6	5	4	9	1	3	5	7	6	8
2	5	4	6	7	6	6	7	9	3

For this set of scores, showing all working and without using the statistics mode on your calculator:

(i). construct a frequency distribution table

(ii). draw a cumulative frequency polygon.

(iii). evaluate the mean.

(b). The golf scores attained by Year 9 students are shown in the following table.

scores (class)	frequency f	class centre x
70-76	2	
77-83	5	
84-90	8	
91-97	12	
98-104	6	
105-111	3	

Complete the table.

Using this information find:

(i). the mean

(ii). the standard deviation.

8.(a). Graeme sat for two mathematics tests this term. In the first he scored 80 and in the second he scored 70. The results for his class are listed below. Given this information explain clearly which is Graeme's better test result.

	Mean	Standard Deviation	Mark
1st test:	72	12	80
2nd test:	62	8	70

(b). A coin is tossed twice and then a four sided die labelled with numbers 1 to 4 is thrown.

(i). Draw a tree diagram and hence list the possible outcomes.

(ii). What is the probability that the result is a head then a tail then a three?

(iii). What is the probability that the result is a head and a tail and an even number?

Cranbrook School
Year 10 Term 3 Exam Solutions 1999

1.(a).(i). $4x^2-1=(2x+1)(2x-1)$

(ii). $3x^4+81x=3x(x^3+27)=3x(x+3)(x^2-3x+9)$

(iii). $3x^2+5x-8=(3x+8)(x-1)$

(b). $2x-5y=22 \quad 4x+3y=5$

Solving simultaneously:

$2(22+5y)+3y=5$

$44+13y=5$

$13y=-39$

$y=-3$

$x=3\frac{1}{2}$.

2.(i). $3\sqrt{6}(\sqrt{3}-2\sqrt{2})=3\sqrt{18}-6\sqrt{12}=9\sqrt{2}-12\sqrt{3}$

(ii). $\frac{\sqrt{4m^3}}{4}=\frac{\sqrt{m^3}}{2}$.

(iii). $\sqrt{8}+7\sqrt{32}-\sqrt{48}+2\sqrt{75}$

$=2\sqrt{2}+28\sqrt{2}-4\sqrt{3}+10\sqrt{3}$

$=30\sqrt{2}+6\sqrt{3}$

(iv). $\frac{\sqrt{7}+3}{7+\sqrt{5}}=\frac{(\sqrt{7}+3)(7-\sqrt{5})}{(7+\sqrt{5})(7-\sqrt{5})}=\frac{7\sqrt{7}-\sqrt{35}+21-3\sqrt{5}}{44}$

3.(i). $AD=\sqrt{(6--2)^2+(-8-4)^2}=\sqrt{208}=4\sqrt{13}$ units

(ii). $m_{AC}=\frac{y_C-y_A}{x_C-x_A}=\frac{2-4}{3--2}=-\frac{2}{5}$

(iii). Equation of line AC:

$$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1} \Rightarrow \frac{y-4}{x--2}=\frac{2-4}{3--2}$$

$$5y+2x-16=0$$

(iv). gradient of line is $\frac{5}{2}$.

Line: $y--5=\frac{5}{2}(x-1) \Rightarrow 2y+10=5x-5$

(v). AC: $2x+5y-16=0$ E(-2,5).

Substitute the point E into the line AC.

LHS = 2(-2) + 5(5) - 16

= 5

RHS = 0

≠ LHS.

E doesn't lie on the line AC.

3.(a). Area = (29.2) × ( ) + $\frac{1}{2}$ × (29.2) × (18.6)

= 1401.6 m^2

Cost = $19061.76

(b).(i). $V = \frac{1}{3} \times$ base × height

$= \frac{1}{3} \times (10 \times 8) \times 12$

$= 320\ m^3$

(c).(i). $V = \frac{\frac{4}{3}\pi r^3}{2} = \frac{\frac{4}{3}\pi(12)^3}{2} = \frac{2304\pi}{2} = 1152\pi\ m^3$

(ii). V = 3619.114737...

∴ capacity = 3,619,114.7 L

(iii). Time = $\frac{\text{capacity}}{\text{Rate}} = \frac{3619114.7}{90{,}000} = 40.21238556...$ hours

= 2412.74... mins

= 40 hrs 13m (nearest min).

4.(a).(i). $\tan(37°) = \frac{11}{x}$

$x = 14.60$ (2dp).

(ii). Sine rule:

$\frac{20}{\sin 80} = \frac{x}{\sin 50}$

$x = 15.56$ (2dp).

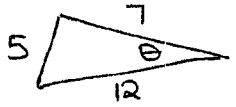
(b). $\tan\theta = \frac{17.6}{27.1}$

$\theta = 33°00'$

(c). Cosine rule:

$\cos\theta = \frac{9^2 + 12^2 - 5^2}{2 \times 9 \times 12}$

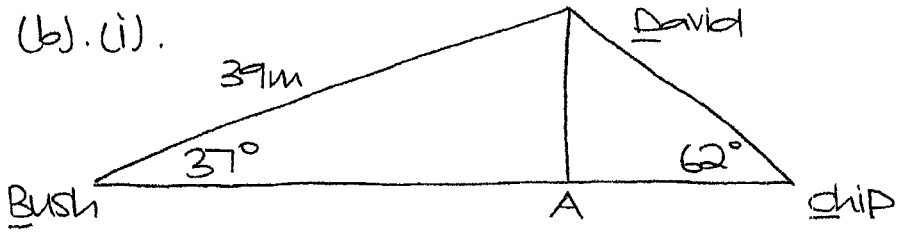
$\theta = 22°11'$



smallest angle is opposite to smallest side.

5.(a). $A = \frac{1}{2} \times 5 \times 7 \times \sin 60 = \frac{35\sqrt{3}}{4}$ cm²

(b).(i).

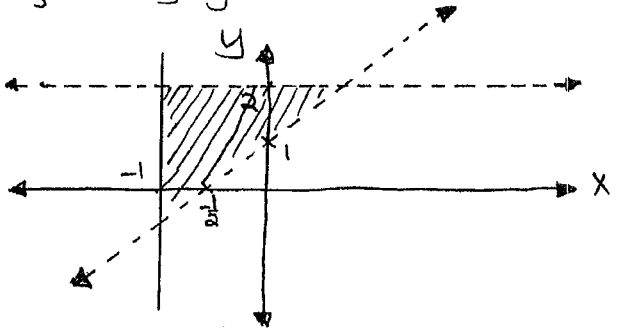


(ii). $\sin 62 = \frac{23.47}{CD}$

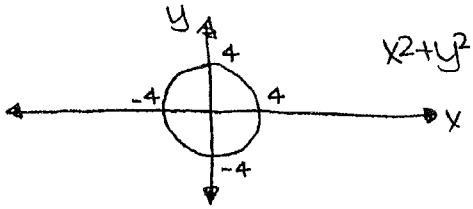
$CD = 26.58m = 2658$ cm (nearest cm).

(iii). Infinite area ⇒ look straight up.

6.(a). $y > 2x+1, x \geq -1, y < 2$.



(b).



$x^2 + y^2 = 16$

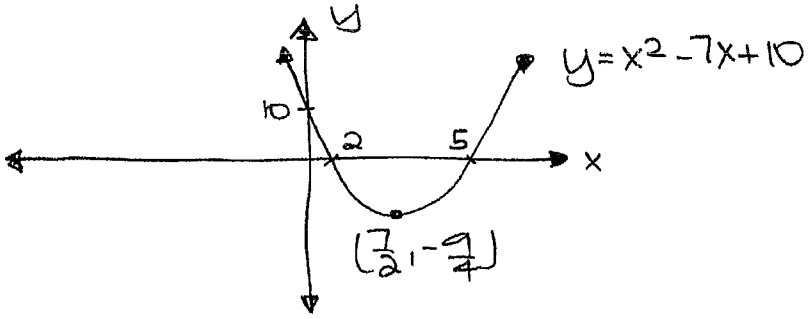
6.(i). $x = -\frac{b}{2a} = \frac{7}{2}$

(ii). Vertex: $\left(\frac{7}{2}, -\frac{9}{4}\right)$.

(iii). y-intercept: $(0, 10)$

(iv). x-intercepts: $(2, 0)$ and $(5, 0)$.

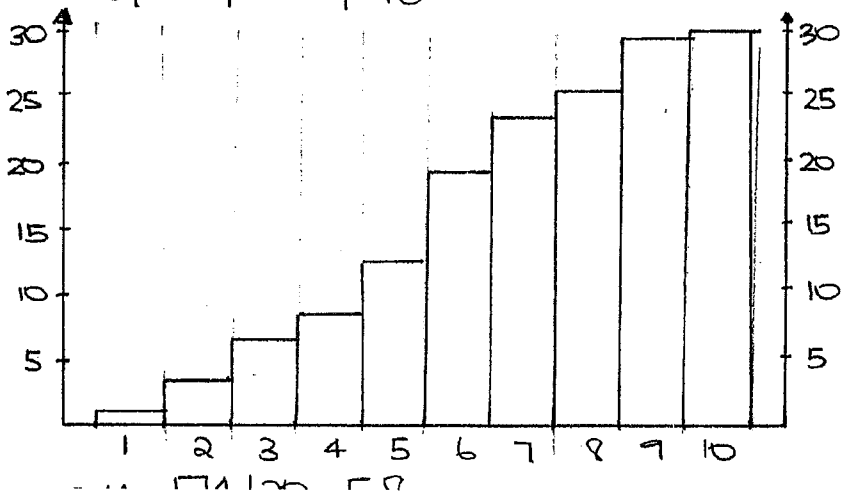
(v).



7.(a).(i).

x	f	cf	fx
1	1	1	1
2	2	3	4
3	3	6	9
4	2	8	8
5	4	12	20
6	7	19	42
7	4	23	28
8	2	25	16
9	4	29	36
10	1	30	10

(ii).



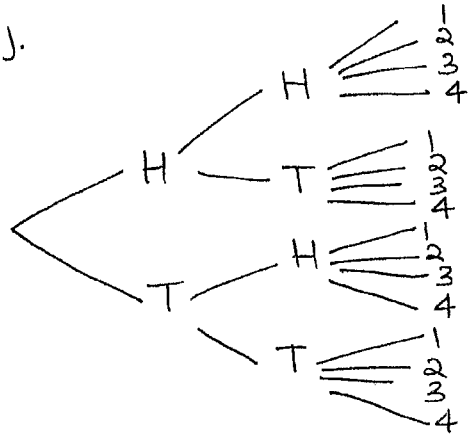
(b). Use a calculator.

(i). mean = $91\frac{2}{3}$

(ii). Std. dev. = 9.04 (2dp).

8.(a). In the first test, Graeme is $\frac{2}{3}$ of a std. dev. above the mean. In the second test, he is 1 std. dev. above the mean. The second test is the better result.

(b).(i).



∴ 16 possible outcomes

(ii). $P = \frac{1}{16}$

(iii). $P = \frac{4}{16} = \frac{1}{4}$