Centre Number: | 1 | 2 | 5 |

Student Number: | | | | | | | | |

CRANBROOK SCHOOL

2015
Examination
Assessment Task 2

Mathematics Extension 1

Reading time	5 minutes
Writing time	2 hours
Total Marks	63
Task weighting	30%

General Instructions

- Write using blue or black pen
- Diagrams drawn using pencil
- A Board-approved calculator may be used
- A table of standard integrals can be found on page 12 of this paper
- Use the Multiple-Choice Answer Sheet provided
- All relevant working should be shown for each question

Additional Materials Needed

- Multiple Choice Answer Sheet
- 3 writing booklets
- 1 sheet of working out paper

Structure & Suggested Time Spent

Section I
Multiple Choice Questions
- Answer Q1 – 7 on the multiple choice answer sheet
- Allow 15 minutes for this section

Section II
Extended response Questions 8-10
- Attempt all questions in this section in a separate writing booklet
- Allow about 105 minutes for this section

This paper must not be removed from the examination room

Disclaimer
The content and format of this paper does not necessarily reflect the content and format of the HSC examination paper.

Section I

7 Marks

Allow approximately 15 minutes for this section

Use the multiple choice answer sheet for Questions 1-7.

1. If the angle between the lines $y=3-2x$ and $y=5x+7$ is α, the value of $\tan\alpha$ is:

A. $-\frac{9}{7}$

B. $-\frac{7}{9}$

C. $\frac{3}{11}$

D. $\frac{7}{9}$

2. Suppose that α is obtuse and $\sin\alpha=\frac{\sqrt{6}}{9}$. The exact value of $\sin 2\alpha$ is:

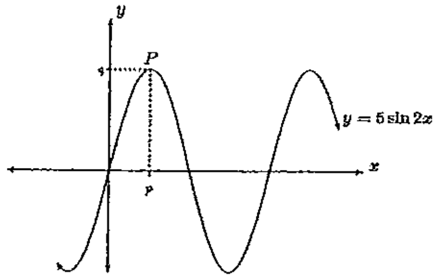
A. $\frac{2\sqrt{6}}{9}$

B. $\frac{-10\sqrt{2}}{27}$

C. $\frac{10\sqrt{2}}{27}$

D. $\frac{-10\sqrt{3}}{81}$

3.



The diagram above shows the wave $y = 5\sin 2x$ with a maximum turning point at (p, q).

The coordinates of the turning point are:

A. $\left(\frac{\pi}{4}, 2\right)$

B. $\left(\frac{\pi}{2}, 2\right)$

C. $\left(\frac{\pi}{2}, 5\right)$

D. $\left(\frac{\pi}{4}, 5\right)$

4. The polynomial $P(x)$ is monic and of degree 5. It has a single zero at $x = 4$ and a double zero at $x = -3$. Its other two zeroes are not real.

Which of the following equations best represents $P(x)$?

A. $x^5 + bx^4 + cx^3 + dx^2 + ex + f$

B. $(x-4)(x+3)^2(x^2+bx+c), \text{ where } b^2 - 4c < 0$

C. $(x-4)(x+3)^2(x-b)(x-c)$

D. $(x-4)(x+3)^2(x^2+bx+c), \text{ where } b^2 - 4c \geq 0$

5. Which of the following is an expression for $\int \sin^2 3x\,dx$?

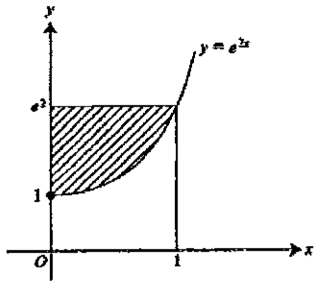
A. $x - \frac{1}{12}\sin 6x + C$

B. $x + \frac{1}{6}\sin 6x + C$

C. $\frac{x}{2} - \frac{1}{12}\sin 6x + C$

D. $\frac{x}{2} + \frac{1}{12}\sin 6x + C$

6. In order to find the area of the shaded region in the diagram below, four different students proposed the following calculations:



Student 1: $\int_0^1 e^{2x}\,dx$

Student 3: $\int_1^{e^2} e^{2y}\,dy$

Student 2: $e^2 - \int_0^1 e^{2x}\,dx$

Student 4: $\int_1^{e^2} \frac{\log_e y}{2}\,dy$

Which of the students is correct?

A. Student 2 only

B. Students 2 and 3 only

C. Students 2 and 4 only

D. Students 1 and 4 only

7. Line TA is a tangent to the circle at A and TB is a secant meeting the circle at B and C.

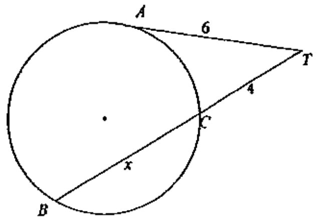


Diagram Not To Scale

Given that TA= 6, CB= x and TC = 4, what is the value of x?

A. -9

B. -5

C. 9

D. 5

END OF SECTION I

Section II

56 Marks

Allow approximately 105 minutes for this section

Answer question 3 in separate booklets.

Question 8 **Begin a new booklet** **19 Marks**

a. Solve $\frac{3}{x-3}-\frac{1}{x}\leq 0$. 3

b. Write as a Cartesian equation the parametric expressions $x=4t, y=t^2-1$. 1

c. Determine $\lim\limits_{x\to 0}\frac{\sin 3x}{5x}$. 1

d. Evaluate $\int_1^{\sqrt{3}}\frac{x^2}{\sqrt{4-x^2}}dx$ using the substitution $x=2\cos u$. 4

e. Consider the curve whose equation is $f(x)=\frac{x}{9-x^2}$.

i. Find the domain of the function. 1

ii. Show the function is an odd function. 1

iii. Show that the function is increasing throughout its domain. 2

iv. Sketch the curve. 2

Question 8 continues on next page

f.

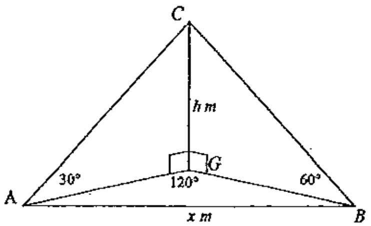


Diagram Not To Scale

In the diagram above Celia is standing at her window C, which is h metres above a point G on the ground. Two boys, Alex and Ben, are standing in the garden below the window at the points A and B respectively. The interval AB subtends an angle of $120°$ at G. From Andrew, the angle of elevation of Celia is $30°$. From Ben, the angle of elevation of Celia is $60°$. The distance between the boys is x metres.

i. Show that the distance AG is $h\sqrt{3}$ metres and find a similar expression for the distance BG. **2**

ii. Show that $3x^2 = 13h^2$. **2**

End Question 8

Question 9 **START A NEW BOOKLET** **22 marks**

a. The point $P(2ap, ap^2)$ is a point on the parabola $x^2 = 4ay$.

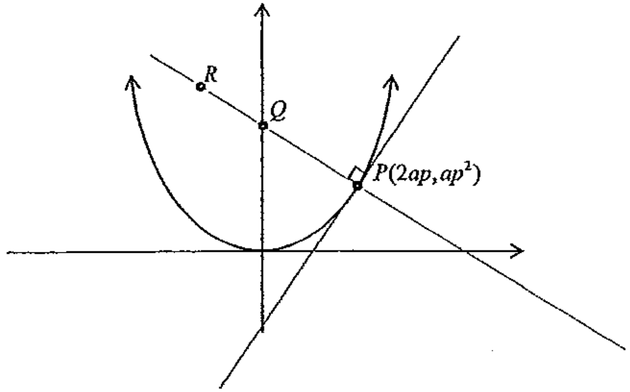


Diagram Not To Scale

i. Show that the equation of the normal to the curve at P is $x + py = 2ap + ap^3$. **2**

ii. Find the coordinates of the point Q where the normal meets the y axis. **1**

iii. Determine the coordinates of the point R which divides PQ externally in the ratio 2:1. **2**

iv. Find the Cartesian equation of the locus of R and describe the locus in general terms. **2**

Question 9 continues on next page

b. Consider the polynomial $P(x)=4x^3+4x^2-3x-3$

i. Express $P(x)$ as a product of its factors. **2**

ii. Hence or otherwise, solve the equation $4\sin^3\theta+4\sin^2\theta-3\sin\theta=3$ for $0\le\theta\le\pi$. **2**

c. Use the substitution $t=\tan\frac{\theta}{2}$ to find the general solution to the equation $3\sin\theta-2\cos\theta=2$. Give your answer <u>to the nearest degree.</u> **4**

d. Using the principle of Mathematical Induction, show that **3**

$$\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+\ldots+\frac{n}{2^n}=2-\frac{n+2}{2^n}, when\ n\ge 1.$$

e. i. Express $3\sin\theta+\sqrt{2}\cos\theta$ in the form $R\sin(\theta+\alpha)$ where R > 0 and $0<\alpha<\frac{\pi}{2}$. **3**

ii. Hence or otherwise, find the minimum value of the expression $3\sin\theta+\sqrt{2}\cos\theta$. **1**

End of Question 9

Question 10 **START A NEW BOOKLET** **15 marks**

a.

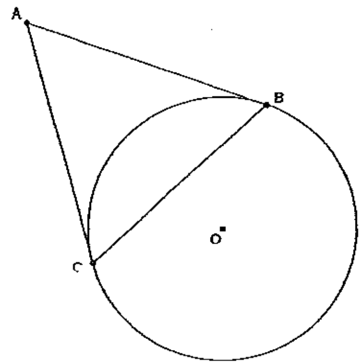


Diagram Not To Scale

AB and AC are tangents of circle with centre O.

i. Neatly copy this diagram into your booklet. **1**

ii. Show that $\angle COB=2\angle ABC$. **2**

iii. Hence or otherwise, prove that ABOC is a cyclic quadrilateral. **2**

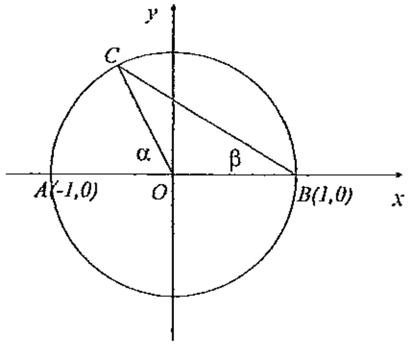
b.

i. Use the facotr theorem to find one factor of $f(x)=x(x+1)-a(a+1)$, where a is a constant. **1**

ii. By long division or otherwise, find the other factor of $f(x)$. **2**

Question 10 continues on next page

c.



In the diagram above, the points A(-1,0), B (1,0) and C all lie on the circle with the centre O and radius of 1. Let $\angle COA = \alpha$ and let $\angle CBO = \beta$.

i. Given the line BC has gradient m, find its equation. **1**

ii. Show that the x coordinates of B and C are the solutions to the equation. **2**

$$(1+m^2)x^2 - 2m^2x + m^2 - 1 = 0.$$

iii. Using this equation, find the coordinates of C in terms of m. **2**

iv. Hence deduce that $\tan 2\beta = \dfrac{2\tan\beta}{1-\tan^2\beta}$. **2**

End of Question 10

End of Section II
END OF PAPER

STANDARD INTEGRALS

$\int x^n \, dx$	$= \dfrac{1}{n+1}x^{n+1} + C,$	$n \neq -1;\quad x \neq 0$ if $n < 0$
$\int \dfrac{1}{x} \, dx$	$= \ln x + C,$	$x > 0$
$\int e^{ax} \, dx$	$= \dfrac{1}{a}e^{ax} + C,$	$a \neq 0$
$\int \cos ax \, dx$	$= \dfrac{1}{a}\sin ax + C,$	$a \neq 0$
$\int \sin ax \, dx$	$= -\dfrac{1}{a}\cos ax + C,$	$a \neq 0$
$\int \sec^2 ax \, dx$	$= \dfrac{1}{a}\tan ax + C,$	$a \neq 0$
$\int \sec ax \tan ax \, dx$	$= \dfrac{1}{a}\sec ax + C,$	$a \neq 0$
$\int \dfrac{1}{a^2+x^2} \, dx$	$= \dfrac{1}{a}\tan^{-1}\dfrac{x}{a} + C,$	$a \neq 0$
$\int \dfrac{1}{\sqrt{a^2-x^2}} \, dx$	$= \sin^{-1}\dfrac{x}{a} + C,$	$a > 0, -a < x < a$
$\int \dfrac{1}{\sqrt{x^2-a^2}} \, dx$	$= \ln\left(x + \sqrt{x^2-a^2}\right) + C,$	$x > a > 0$
$\int \dfrac{1}{\sqrt{x^2+a^2}} \, dx$	$= \ln\left(x + \sqrt{x^2+a^2}\right) + C$	

NOTE: $\ln x = \log_e x,\ x > 0$

Question 8

$x \neq 3, 0$ earned a mark

a) $\dfrac{3}{x-3} - \dfrac{1}{x} \leq 0$

$\dfrac{3}{x-3} \leq \dfrac{1}{x}$

$3x^2(x-3) \leq (x-3)^2 x$

$3x^2(x-3) - x(x-3)^2 \leq 0$

$x(x-3)\left[3x - x + 3\right] \leq 0$

$x(x-3)(2x+3) \leq 0$

OR use common denominator

$\dfrac{3x - (x-3)}{x(x-3)} \leq 0$

$\dfrac{(2x+3)}{x(x-3)} \leq 0$

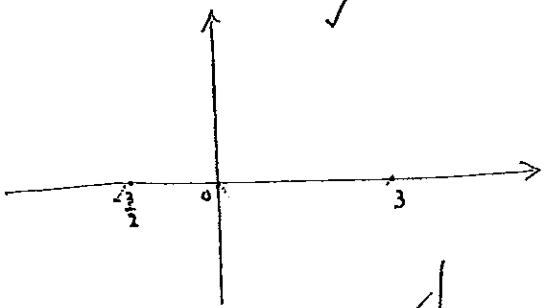
now if $\frac{a}{b} \leq 0$ so too $a \times b \leq 0$

$\therefore (2x+3)x(x-3) \leq 0$

now sketch as below

$y = x(x-3)(2x+3)$

When $x = 1$, $y = -10$



$\therefore \quad x \leq -\frac{3}{2}, \quad 0 < x < 3$

This earned the mark ☺

LUCKY FOR those who went on to rearrange and made arithmetic errors!! ☹

b) $x = 4t, \quad y = t^2 - 1$

$t = \dfrac{x}{4}$

$\therefore y = \left(\dfrac{x}{4}\right)^2 - 1$

$, \ y = \dfrac{x^2}{16} - 1$

c) $\lim\limits_{x \to 0} \dfrac{\sin 3x}{5x} \times \dfrac{3}{3}$

$= \dfrac{3}{5} \lim\limits_{x \to 0} \dfrac{\sin 3x}{3x}$

$= \dfrac{3}{5}$

d) $\displaystyle\int_1^{\sqrt{3}} \frac{x^2}{\sqrt{4 - x^2}}$

$x = 2\cos u$

$\dfrac{dx}{du} = -2\sin u$

$x = \sqrt{3}, \ u = \dfrac{\pi}{6}$ (1)

$x = 1, \ u = \dfrac{\pi}{3}$

$\therefore I = \displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{6}} \frac{4\cos^2 u}{\sqrt{4 - 4\cos^2 u}}\, du \,.\, -2\sin u$

$4(1 - \cos^2 u)$

(Recall $\cos^2\theta + \sin^2\theta = 1$, $\therefore 1 - \cos^2 u = \sin$

$= \displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{6}} \frac{4\cos^2 u}{\cancel{2\sin u}}\, du \,.\, \cancel{-2\sin u}$

FROM $\cos^2\theta - \sin^2\theta = \cos 2$

$\cos^2 u = \frac{1}{2}(\cos 2u + 1)$

$= -4\displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{6}} \cos^2 u \, du$ (1)

$= -4\displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{6}} \frac{1}{2}(\cos 2u + 1)\, du$

$= -2\displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{6}} \cos 2u + 1 \, du$ (1)

$$= -2\int_{\frac{\pi}{3}} \cos 2u + 1 \; du$$

$$= -2\left[\frac{\sin 2u}{2} + u\right]_{\frac{\pi}{3}}^{\frac{\pi}{6}}$$

$$= -2\left[\left(\frac{\cancel{\sin \frac{\pi}{3}}}{2} + \frac{\pi}{6}\right) - \left(\frac{\cancel{\sin \frac{2\pi}{3}}}{2} + \frac{\pi}{3}\right)\right]$$

$$= -2\left[-\frac{\pi}{6}\right]$$

$$= +\frac{\pi}{3} \text{ units}^2 \quad (1)$$

c) $f(x) = \dfrac{x}{9-x^2}$

i. $x \neq \pm 3$

ii. If $f(-x) = -f(x)$ then the function is odd.

$$f(-x) = \frac{-x}{9-(-x)^2}$$
$$= \frac{-x}{9-x^2}$$
$$= -\left(\frac{x}{9-x^2}\right)$$
$$= -f(x) \quad \therefore \text{ odd.}$$

*Working MUST BE shown here to earn the mark.

iii) $$f'(x) = \frac{\ldots}{(9-x^2)^2}$$
$$= \frac{9-x^2 + 2x^2}{(9-x^2)^2}$$
$$= \frac{9+x^2}{(9-x^2)^2}$$

Both the numerator & denominator are always +ve in the domain $x \neq \pm 3$

$\therefore f'(x) > 0 \quad \therefore$ always increasing.

NOTE: (i)-(iii) were no[t] … these 3 pieces of informa[tion] should be used.
also: SKETCHES and axes should be labelled

iv)

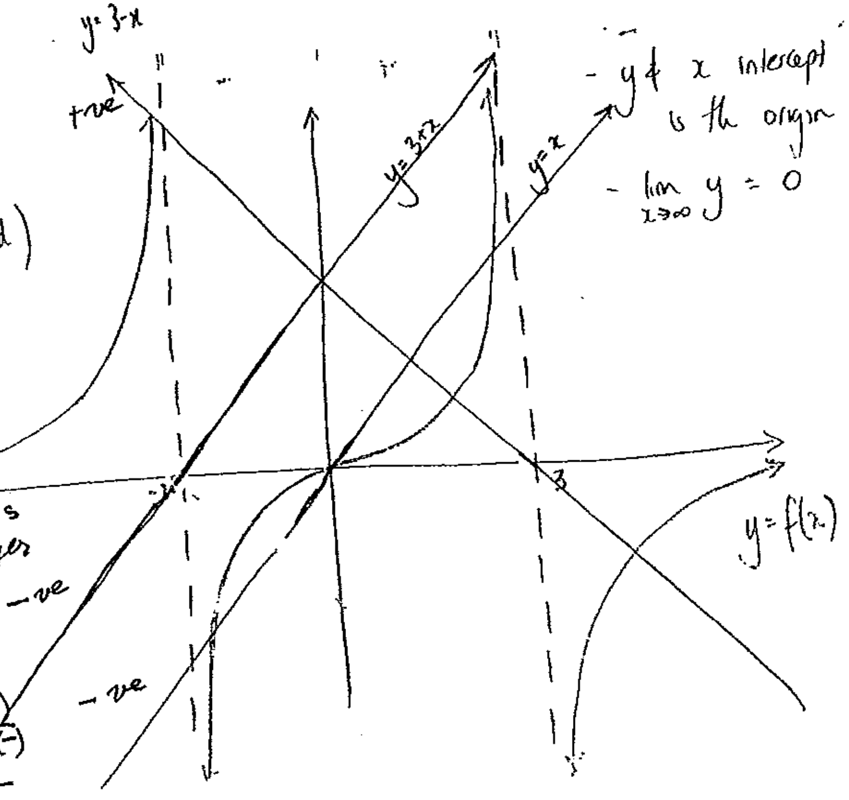
① Correct found info. (domain & odd)

② Correct regions & y intercept.

OR can use BOXES but takes longer

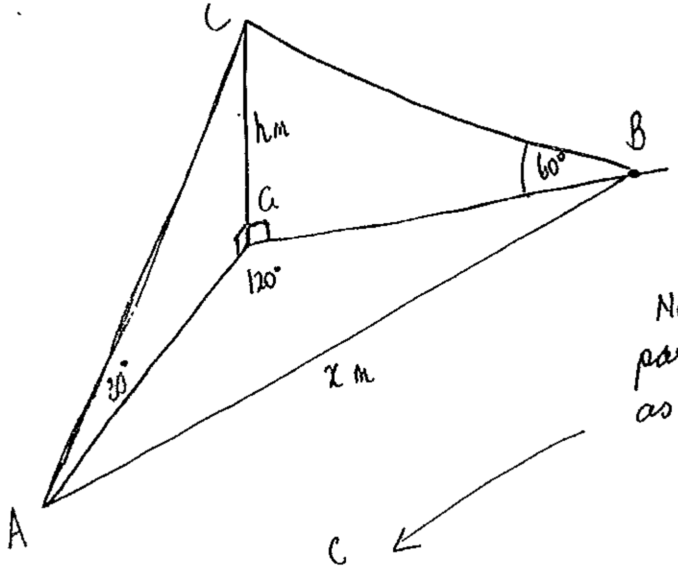
$x \quad -4$
$y \quad +$

$\Rightarrow \dfrac{(x)}{(3-x)(3+x)} \Rightarrow \dfrac{(-)}{(+)(-)} = +$



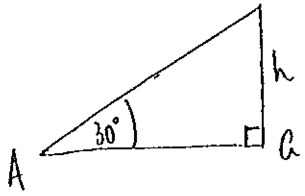
- y & x intercept is the origin
- $\lim_{x\to\infty} y = 0$

f)



NOTE: Redrawing parts of the diagram as shown helps

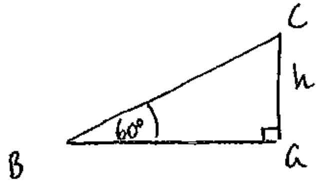
i)



$$\frac{AG}{h} = \cot 30^\circ$$

$$\frac{AG}{h} = \sqrt{3} \quad (1)$$

$$AG = \sqrt{3}h$$

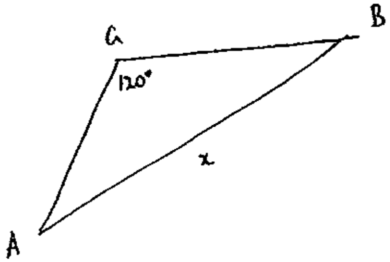


$$\frac{BG}{h} = \cot 60^\circ$$

$$= \frac{1}{\sqrt{3}} \quad (1)$$

$$BG = \frac{h}{\sqrt{3}}$$

ii)



Using the cosine rule

$$x^2 = BG^2 + AG^2 - 2 \times BG \times AG \times \cos 120^\circ \quad (1)$$

$$= \frac{h^2}{3} + 3h^2 + \cancel{2} \times \frac{h}{\sqrt{3}} \times \sqrt{3}h \times \frac{1}{\cancel{2}}$$

$$x^2 = \frac{h^2}{3} + 4h^2 \quad (1) \quad \therefore 3x^2 = 13h^2$$

uestion 9

$$x^2 = 4ay$$

$$y = \frac{x^2}{4a}$$

$$\frac{dy}{dx} = \frac{2x}{4a}$$

ere $x = 2ap$

$$\frac{dy}{dx} = \frac{2ap}{2a}$$

$\therefore m_{norm} = -\frac{1}{p}$. $(2ap, ap^2)$ ✓

Mostly well done.

$$y - ap^2 = -\frac{1}{p}(x - 2ap)$$

$$py - ap^3 = -x + 2ap$$

$x + py = 2ap + ap^3$ as req. ✓

Must ensure answer is same as "Show" statement

When norm meets y axis $x = 0$

(common error was to make $y=0$).

$$0 + py = 2ap + ap^3$$

$$y = 2a + ap^2$$

$\therefore Q(0, 2a + ap^2)$ ✓

iii) Needed to use P + Q. in that order

$P(2ap, ap^2)$ $Q(0, 2a + ap^2)$.

$-2 : 1$

External so negative number in ratio!

$x = \frac{2ap + 0}{-1}$ $y = \frac{ap^2 - 4a - 2ap^2}{-1}$

If using formula, learn it correctly!!

$= -2ap$ ✓ $y = 4a + ap^2$ ✓

$\therefore R(-2ap, 4a + ap^2)$

iv) $x = -2ap$ $y = ap^2 + 4a$

$p = \frac{x}{-2a}$ $y = a\left(\frac{x}{-2a}\right)^2 + 4a$

$$y - 4a = \frac{ax^2}{4a^2}$$

$x^2 = 4a(y - 4a)$ ✓

Parabola with vertex $(0, 4a)$. ✓

(Must include the word parabola + other details)

b) $P(x) = 4x^3 + 4x^2 - 3x - 3$

$P(-1) = -4 + 4 + 3 - 3$

$= 0$. $\therefore (x+1)$ is a factor) ✓

$$\begin{array}{r} 4x^2 - 3 \\ x+1 \overline{) 4x^3+4x^2-3x-3} \\ -(4x^3+4x^2) \\ \hline -3x-3 \\ -(-3x-3) \\ \hline 0 \end{array}$$

$\therefore P(x) = (x+1)(4x^2-3)$

or

$P(x) = (x+1)(2x-\sqrt{3})(2x+\sqrt{3})$

Do not write $(P(x) = (x+1)(x-\frac{\sqrt{3}}{2})(x+\frac{\sqrt{3}}{2}))$

i) $\therefore 4\sin^3 x + 4\sin^2 x - 3\sin x - 3 = 0$ $\quad 0 \le x \le \pi$

must be in radians

$(\sin x - 1)(4\sin^2 x - 3) = 0.$

$\therefore \sin x = -1$ $\qquad \sin^2 x = \frac{3}{4}$

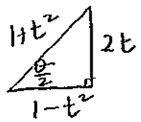
$x = \frac{3\pi}{2}$ $\qquad \sin x = \pm\frac{\sqrt{3}}{2}$

not in the domain. $\qquad x = \frac{\pi}{3}, \frac{2\pi}{3}$

(must consider domain)

$\therefore x = \frac{\pi}{3}, \frac{2\pi}{3}$

c) $t = \tan\frac{\theta}{2}$



$3\sin\theta - 2\cos\theta = 2$

$3\left(\frac{2t}{1+t^2}\right) - 2\left(\frac{1-t^2}{1+t^2}\right) = 2$

$6t - 2 + 2t^2 = 2 + 2t^2$ $\qquad t = \frac{2}{3}$

test for $\theta = 180°$.

$3\sin(180) - 2\cos(180) = 2$

$3(0) - 2(-1) = 2$

$\therefore 2 = 2$

$\therefore \frac{\theta}{2} = 33°41' + 180n$

$\theta = 2(180n + 33°41')$

$= 360n + 66°82'$

d). $\frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \dots + \frac{n}{2^n} = 2 - \frac{n+2}{2^n} \quad n \ge 1$

Step 1

Show true for $n=1$

LHS $= \frac{1}{2}$ $\qquad$ RHS $= 2 - \frac{3}{2} = \frac{1}{2}$

$\therefore$ LHS = RHS $\qquad \therefore$ true for $n=1$

Step 2

Assume true for $n=k$

$\frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \dots + \frac{k}{2^k} = 2 - \frac{k+2}{2^k}$

Step 3

Prove true for $n=k+1$

eg $\frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \dots + \frac{k}{2^k} + \frac{k+1}{2^{k+1}} = 2 - \frac{(k+1)+2}{2^{k+1}}$

LHS $= 2 - \frac{k+2}{2^k} + \frac{k+1}{2^{k+1}}$

$= 2 + \frac{-2(k+2)}{2^{k+1}} + \frac{k+1}{2^{k+1}}$

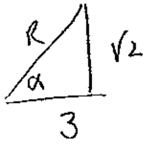
$= 2 + \frac{-k-3}{2^{k+1}}$

$= 2 - \frac{(k+1)+2}{2^{k+1}}$ = RHS.

$\therefore$ True by Mathematical Induction.

). $3\sin\theta + \sqrt{2}\cos\theta$. $\quad R>0 \quad 0<\alpha<\frac{\pi}{2}$

Must be in radians!



$\tan\alpha = \frac{\sqrt{2}}{3}$

$R^2 = 3^2 + (\sqrt{2})^2$

$R = \sqrt{11}$ ✓

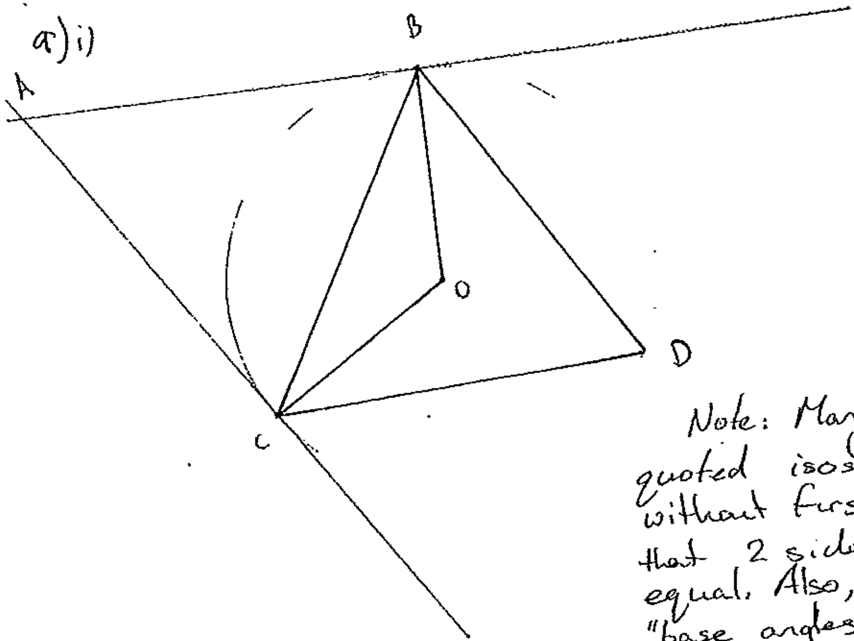
$\alpha = 0.44$ ✓

$\therefore \quad 3\sin\theta + \sqrt{2}\cos\theta = \sqrt{11}\sin(\theta + 0.44)$ ✓

) min value $= -\sqrt{11}$ ✓

Question 10

a) i)



Note: Many students quoted isosceles Δ without first proving that 2 sides were equal. Also, should state "base angles isosceles Δs are equal".

ii) Let D be a point on the major arc BC

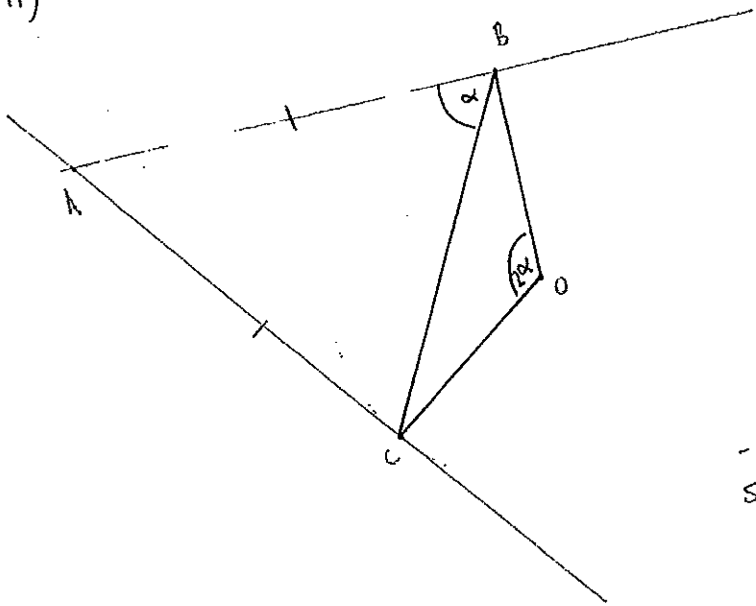
$\angle ABC = \angle BDC$ (∡ between a chord & a tangent is equal to the angle subtended in the opposite segment)

$\angle BOC = 2 \times \angle BDC$ (∡ subtended at the centre is equal to the twice the angle subtended at the circumference)

$\therefore \angle BOC = 2 \times \angle ABC$.

Notes: To get 2 marks students must have a complete solution – all steps shown.
1 Mark – Part solution with at least one circle geometry proof used.

ii)



Note: *Only need to prov[e] one set of opposite angles supplementary.

* Could simply use $\angle ABO = \angle ACO = 9$[0]

∴ Opposite angle[s] supplementary (giving full ne[...]

let $\angle ABC = \alpha$. $\therefore \angle BOC = 2\alpha$ (from previous)

$\angle ACB = \alpha$ (Base ∡'s of isosceles Δ are equal)

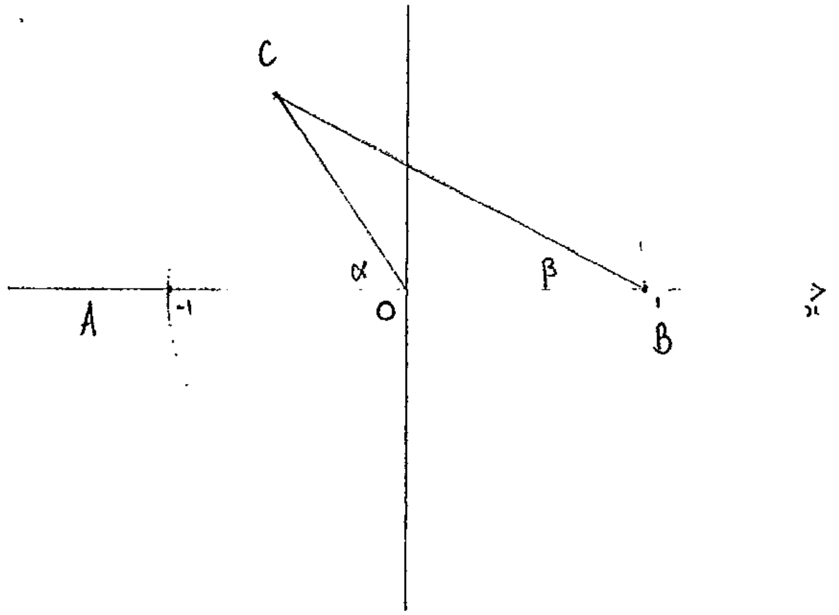
NB $AB = AC$ (tangents from the same external point are equal)

$\angle CAB = 180 - 2\alpha$ (Angle sum of Δ ACB)

$\therefore$ ABOC is cyclic as opposite angles ($\angle BAC$ & $\angle BOC$) are supplementary.

2 Marks: Correct proof
1 Mark: Partly correct proof or correct use of final conclusion for cyclic quad.

i)



i) $m = \dfrac{y-0}{x-1}$

$y = mx - m$ ✓ 1

ii) B & C are the points of intersection between the circle $x^2+y^2=1$ & the line $y = mx - m$

∴ the x coordinate of B & C are the results of solving those equation simultaneously.

(1) Explanation.

$x^2 + (mx-m)^2 = 1$ (1) Substitution

$x^2 + m^2x^2 - 2m^2x + m^2 = 1$ 2 Marks

$(1+m^2)x^2 - 2m^2x + m^2 - 1 = 0$ As required.

(N) 1 Mark awarded for subbing in B and

iii) $(1+m^2)x^2 - 2m^2x + (m^2-1) = 0$

Quadratic Eq^n

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \dfrac{2m^2 \pm \sqrt{4m^4 - 4(1+m^2)(m^2-1)}}{2(1+m^2)}$

$= \dfrac{2m^2 \pm \sqrt{4m^4 - 4(m^4-1)}}{2(m^2+1)}$

$= \dfrac{2m^2 \pm \sqrt{4m^4 - 4m^4 + 4}}{2(m^2+1)}$

$= \dfrac{2m^2 \pm 2}{2(m^2+1)}$

$\Rightarrow x = \dfrac{m^2+1}{m^2+1}$ or $\dfrac{m^2-1}{m^2+1}$ ✓ (1)

$= 1$ ∴ C

B

find y coordinate;

$$y = mx - m$$

$$y = \frac{m(m^2-1)}{m^2+1} - m$$

$$= \frac{m(m^2-1) - m(m^2+1)}{m^2+1}$$

$$= \frac{-2m}{m^2+1} \quad ✓ \quad 2$$

See Alternative Method.

iii) x coordinate of B is 1

let x coordinate of C be α 1

from $(1+m^2)x^2 - 2m^2x + m^2 - 1 = 0$

$$\alpha \times 1 = \frac{m^2-1}{1+m^2} \quad \text{(Product of the roots)}$$

$$\therefore \alpha = \frac{m^2-1}{1+m^2} \quad 1$$

Alternative Method

iv) $M_{OC} = \tan(180 - \alpha)$

$= -\tan\alpha$ (2nd quadrant)

$M = \tan(180 - \beta)$

$= -\tan\beta$ (2nd quadrant)

From diagram

$\angle OCB = \beta$ (Base $\angle$s of isosceles Δ are equal)

$\therefore \alpha = 2\beta$ (external $\angle$ of a Δ is equal to the sum of the interior opposites)

$\therefore M_{OC} = -\tan 2\beta$

c iv) $OC = OB$ (radii of circle)

$\angle OCB = \angle OBC$ (Base $\angle$'s of isosceles Δ)

$\therefore \alpha = 2B$ (Exterior angle theorem)

$\tan\alpha = \dfrac{y-0}{x-0}$ $\dfrac{\text{rise}}{\text{run}}$ from origin

$= \dfrac{\frac{-2m}{m^2+1}}{\frac{m^2-1}{m^2+1}}$

$\tan\alpha = \dfrac{-2m}{m^2-1}$

$\tan\alpha = \dfrac{2m}{1-m^2}$ $m = \tan B$, $\alpha = 2B$

$\tan 2B = \dfrac{2\tan B}{1-\tan^2 B}$

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1 Mark: $\alpha = 2B$ and some use of $\tan\alpha$.

2 Marks: Correct result.

i) $f(x) = x(x+1) - a(a+1)$

$f(a) = a(a+1) - a(a+1)$

$= 0$

$\therefore$ $x-a$ is a factor of $f(x)$. ✓

Method 1:

ii) $f(x) = x^2 + x - a^2 - a$

$f(x) = (x-a)(x-b)$ (As $f(x)$ is a quadratic)

$= x^2 - (a+b)x + ab$

$\therefore$ $-a+b = 1$

$ab = -a^2 - a$

$b = -(a+1)$

$\therefore$ other factor is $[x + (a+1)]$

(*) For B marks needed to state the results as factors, not roots.

(*) 1 mark awarded if reasonable attempt at long division made.

ii) $\alpha + \beta = -\frac{b}{a}$

Easy Way

Method 2

$a + \beta = -1$

$\beta = -1 - a$

$\therefore (x - (-a-1))$ is a factor

$(x + a + 1)$

bii) Long Division Method 3

$$
\begin{array}{r}
x + (1+a) \\
x - a \overline{) x^2 + x - a^2 - a} \\
x^2 - ax \\
\hline
0 \quad x + ax - a^2 - a \\
x + ax - a^2 - a \\
\hline
0
\end{array}
$$

$\Rightarrow (x-a)(x+1+a)$