+ SOLUTIONS

Centre Number | 1 | 2 | 5 |

Student Number

CRANBROOK SCHOOL

HSC Examination
Assessment Task 3

Mathematics (2 unit)

Reading time	5 minutes
Writing time	2 hours
Total Marks	70
Task weighting	30%

General Instructions

- Write using blue or black pen
- Diagrams drawn using pencil
- A Board-approved calculator may be used
- A table of standard integrals can be found on page 16 of this paper
- Use the Formula Sheet
- Use the Multiple-Choice Answer Sheet provided
- All relevant working should be shown for each question

Additional Materials Needed

- Multiple Choice Answer Sheet
- 4 writing booklets

Structure & Suggested Time Spent

Section I
Multiple Choice Questions
- Answer Q1 – 10 on the multiple choice answer sheet
- Allow 17 minutes for this section

Section II
Extended response Questions
- Attempt all questions in this section in a separate writing booklet
- Allow about 103 minutes for this section

This paper must not be removed from the examination room

Disclaimer
The content and format of this paper does not necessarily reflect the content and format of the HSC examination paper.

Section I

10 Marks

Allow about 10 minutes for this section

Use the multiple choice answer sheet for Questions 1-10.

Question 1

What is the exact value of $\tan\frac{5\pi}{6}$?

(A) $\sqrt{3}$

(B) $-\sqrt{3}$

(C) $\frac{1}{\sqrt{3}}$

(D) $-\frac{1}{\sqrt{3}}$

Question 2

The quadratic equation $x^2-5x+3=0$ has roots α and β. What is the value of $\frac{1}{\alpha}+\frac{1}{\beta}$?

(A) $\frac{3}{5}$

(B) $-\frac{3}{5}$

(C) $\frac{5}{3}$

(D) $-\frac{5}{3}$

Question 3

The parabola given by $(y-3)^2 = -8x$ has a focus with which coordinates?

(A) $(-2,3)$

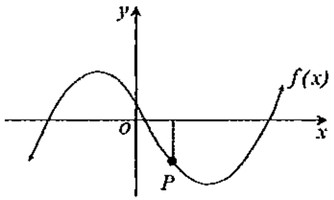
(B) $(3,-2)$

(C) $(2,3)$

(D) $(3,2)$

Question 4

The function $f(x)$ is shown in the diagram below.



At the point P, which of the following are true?

(A) $f'(x) > 0$ and $f''(x) > 0$

(B) $f'(x) > 0$ and $f''(x) < 0$

(C) $f'(x) < 0$ and $f''(x) > 0$

(D) $f'(x) < 0$ and $f''(x) < 0$

Question 5

Evaluate $\log_2 10$ correct to 2 decimal places.

(A) 3.32

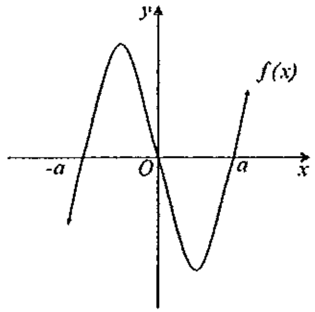
(B) 1.00

(C) 0.30

(D) 2.30

Question 6

The graph of the function $f(x)$ is shown in the diagram below.



If $f(x)$ is an odd function then $\int_{-a}^{a} f(x)\,dx$ is equal to which of the following?

(A) $\int_{-a}^{0} f(x)\,dx + \left|\int_{0}^{a} f(x)\,dx\right|$

(B) $2\left|\int_{0}^{a} f(x)\,dx\right|$

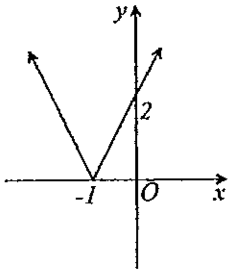
(C) 0

(D) $2\int_{-a}^{0} f(x)\,dx$

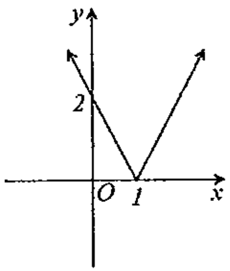
Question 7

Which of the following represents the graph of $f(x)=2|x+1|$?

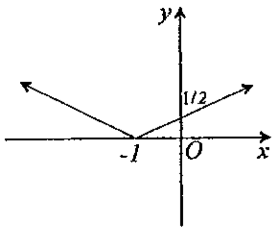
(A)



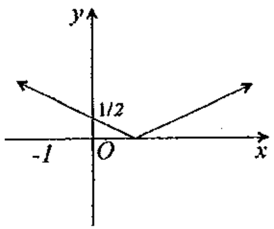
(B)



(C)



(D)



Question 8

What is the value of θ given $\sin\theta=-\frac{\sqrt{3}}{2}$ in the domain $0\le\theta\le 2\pi$?

(A) $\frac{\pi}{3}$ and $\frac{2\pi}{3}$

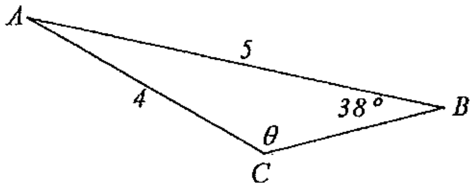
(B) $\frac{\pi}{6}$ and $\frac{5\pi}{6}$

(C) $\frac{4\pi}{3}$ and $\frac{5\pi}{3}$

(D) $\frac{7\pi}{6}$ and $\frac{11\pi}{6}$

Question 9

In ΔABC, $AC=4$ units, $AB=5$ units, $\angle ABC=38^\circ$ and $\angle ACB=\theta^\circ$ where θ is obtuse.



What is the value of θ correct to the nearest degree?

(A) 50°

(B) 23°

(C) 130°

(D) 157°

Question 10

Given $y = 4a^{3x} + b$, express x in terms of a and b.

(A) $x = \frac{1}{12}\log_a(y-b)$

(B) $x = \frac{1}{3}\log_a\left(\frac{y-b}{4}\right)$

(C) $x = \frac{1}{12}\log_a\left(\frac{y}{b}\right)$

(D) $x = \frac{1}{3}\log_a\left(\frac{4y}{b}\right)$

END OF SECTION I

Section II

15 Marks

Allow about 110 minutes for this section

Answer question 11-14 in separate booklets.

Question 11 **Begin a new booklet** **15 Marks**

(a) Differentiate the following functions, simplifying where possible:

(i) $\log_e(4x^2-3x)$ **1**

(ii) $\frac{e^{6x}}{x+1}$ **2**

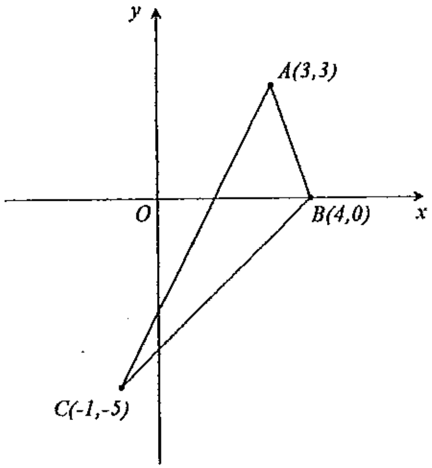
(b) Find $\int e^{-3x+2}\,dx$ **1**

(c) Find the exact value of $\int_0^2 \frac{x}{x^2+6}\,dx$ **3**

(d) Evaluate $\sum_{k=5}^{30} 3k-2$ **2**

Question 11 continues on page 9

(e) In the diagram the points A, B and C have the coordinates $(3,3)$, $(4,0)$ and $(-1,-5)$ respectively.



(i) Calculate the length of the interval AC. **1**

(ii) Show that the equation of the line through A and C is $0=2x-y-3$. **1**

(iii) Calculate the perpendicular distance from point B to the line through A and C. **2**

(iv) Point $D(0,2)$ is a point on the number plane such that the quadrilateral $ABCD$ is a kite.
Find the exact area of the kite $ABCD$. **2**

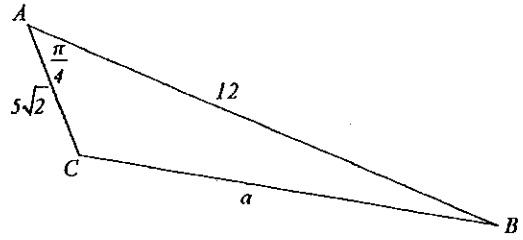
End of Question 11

Question 12 **Begin a new booklet** **15 Marks**

(a) Solve $|2x-1|=3$. **2**

(b) Factorise $9x^2-21x+10$ **2**

(c) In ΔABC, $AC=5\sqrt{2}$ units, $AB=12$ units, $BC=a$ units and $\angle CAB=\dfrac{\pi}{4}$ radians as shown below.



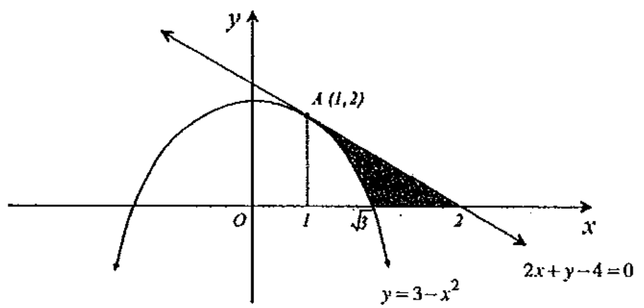
Find the exact value of a. **2**

(d) Solve $2\cos^2\theta-1=0$ for $0\le\theta\le2\pi$ **3**

(e) Find the equation of the normal to the curve $y=e^{2x+1}$ at the point where $x=0$.
Leave your answer in gradient-intercept form. **3**

Question 12 continues on page 11

(f) The diagram shows the graph of $y=3-x^2$. A tangent is drawn to the parabola at the point $(1,2)$ and has the equation $2x+y-4=0$.

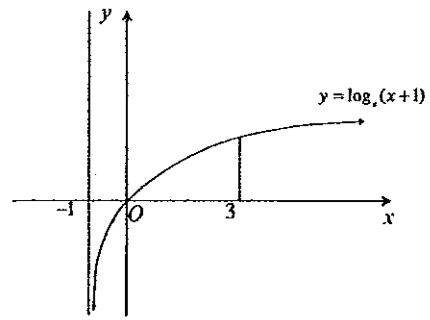


Calculate the exact shaded area bound by the parabola, the tangent, and the x-axis. **3**

End of Question 12

Question 13 **Begin a new booklet** **15 Marks**

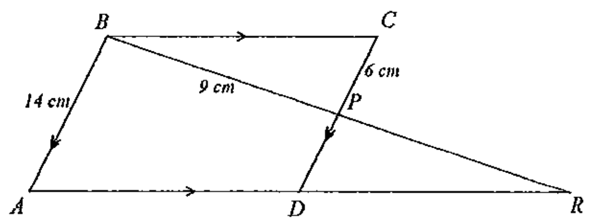
(a) Consider the function $y=\log_e(x+1)$.



(i) Using the trapezoidal rule with four function values, approximate the area under the curve $y=\log_e(x+1)$ between $x=0$ and $x=3$ correct to two decimal places. **3**

(ii) Without evaluating the exact value, determine if the estimated value will be greater than or less than the exact value. Justify your response. **1**

(b) In the diagram $ABCD$ is a parallelogram. P is a point on DC. BP and AD are produced to R. BC is parallel to AR, and AB is parallel to CD. $BP=9\,cm$, $AB=14\,cm$ and $PC=6\,cm$.



(i) Prove $\Delta CBP \,|||\, \Delta ARB$ **2**

(ii) Hence find the length of RP giving reasons. **2**

Question 13 continues on page 13

(c) Rob is planning to purchase an investment property for \$725 000. He currently has a 20% deposit and can afford to pay regular monthly instalments of \$4000.
Westpac have approved a loan for the balance, charging reducible interest monthly at 7.68% p.a.

(i) How much does Rob borrow from Westpac? **1**

(ii) Find an expression for the balance owing after three months. **2**

(iii) Show that the balance owing after n months is given by **1**

$$A_n = 580000(1.0064)^n - 625000(1.0064^n - 1)$$

(iv) How many whole months will it take for Rob to pay off his loan? **2**

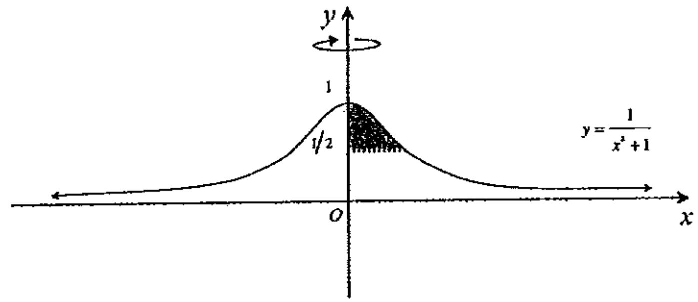
(v) How much interest does he pay over the life of the loan? **1**

End of Question 13

Question 14 **Begin a new booklet** **15 Marks**

(a) For what values of k does $3x^2 - 4kx + k = 0$ have real and distinct roots? **2**

(b) The region bound by the curve $y = \dfrac{1}{x^2+1}$ and the y-axis between $y = \dfrac{1}{2}$ and $y = 1$ is rotated about the y-axis to form a solid.



Find the exact volume of the solid. **3**

(c) Consider the geometric series

$$1 + \frac{1}{(1-\sqrt{5})} + \frac{1}{(1-\sqrt{5})^2} + \ldots$$

(i) Explain why the geometric series has a limiting sum. **1**

(ii) Find the exact value of the limiting sum. Write your answer with a rational denominator. **2**

(d) The equation of a parabola is given by $x^2 + 4x - 12y + 16 = 0$.

(i) Find the coordinates of the vertex and the focus. **2**

(ii) Give the equation of the directrix. **1**

Question 14 continues on page 15

(e) The energy consumption of a laser is given by the equation

$$E=(M-2)A+3$$

E is the energy consumption measured in kilojoules per second and $E>0$.
M is the intensity of the light measured in Lux and $M>0$.
A is the cross section of the beam in square millimetres and $A>0$.

The light intensity is dependent on the area of the beam by the given formula

$$M=\frac{1}{1-\log_e A}$$

(i) Given that the lab operators can only control the light intensity. Show that: **1**

$$E=(M-2)e^{1-\frac{1}{M}}+3$$

(ii) Hence find the light intensity the lab operators should use in order to have the lowest possible energy consumption. **3**

END OF SECTION II

END OF EXAM

STANDARD INTEGRALS

$\int x^n\,dx$	$=\frac{1}{n+1}x^{n+1}+C,$	$n\neq -1;\quad x\neq 0$ if $n<0$
$\int \frac{1}{x}\,dx$	$=\ln x+C,$	$x>0$
$\int e^{ax}\,dx$	$=\frac{1}{a}e^{ax}+C,$	$a\neq 0$
$\int \cos ax\,dx$	$=\frac{1}{a}\sin ax+C,$	$a\neq 0$
$\int \sin ax\,dx$	$=-\frac{1}{a}\cos ax+C,$	$a\neq 0$
$\int \sec^2 ax\,dx$	$=\frac{1}{a}\tan ax+C,$	$a\neq 0$
$\int \sec ax\tan ax\,dx$	$=\frac{1}{a}\sec ax+C,$	$a\neq 0$
$\int \frac{1}{a^2+x^2}\,dx$	$=\frac{1}{a}\tan^{-1}\frac{x}{a}+C,$	$a\neq 0$
$\int \frac{1}{\sqrt{a^2-x^2}}\,dx$	$=\sin^{-1}\frac{x}{a}+C,$	$a>0,\ -a<x<a$
$\int \frac{1}{\sqrt{x^2-a^2}}\,dx$	$=\ln\left(x+\sqrt{x^2-a^2}\right)+C,$	$x>a>0$
$\int \frac{1}{\sqrt{x^2+a^2}}\,dx$	$=\ln\left(x+\sqrt{x^2+a^2}\right)+C$	

NOTE: $\ln x=\log_e x,\ x>0$

MATHEMATICS (2UNIT) - HALF YEARLY EXAM SOLUTIONS 2015

SECTION 1 - MULTIPLE CHOICE

Q1) $\tan\frac{5\pi}{6} = -\tan\frac{\pi}{6}$

$= -\frac{1}{\sqrt{3}}$ (D)

Q2) $x^2-5x+3=0$

$a=1 \quad b=-5 \quad c=3$

$\alpha+\beta = -\frac{b}{a} \qquad \alpha\beta = \frac{c}{a}$

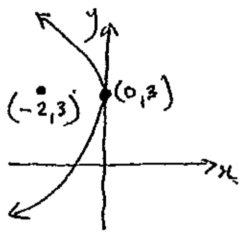
$= 5 \qquad\qquad = 3$

$\frac{1}{\alpha}+\frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$

$= \frac{5}{3}$ (C)

Q3) $(y-3)^2 = -8x$

Vertex $(0,3)$ $\quad a=2$



focus $(-2,3)$ (A)

Q4) decreasing $\rightarrow f'(x)<0$

concave up $\rightarrow f''(x)>0$

(C)

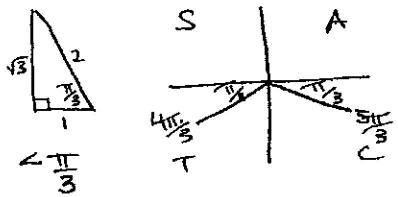
Q5) $\log_2 10 = \frac{\log 10}{\log 2}$

$= 3.32$ (A)

Q6) (C)

Q7) (A)

Q8) $\sin\theta = -\frac{\sqrt{3}}{2}$



$\angle\frac{\pi}{3}$

$\theta = \frac{4\pi}{3}, \frac{5\pi}{3}$ (C)

Q9) $\frac{\sin\theta}{5} = \frac{\sin 38}{4}$

$\theta = 50^\circ$ but θ is obtuse

$\therefore \theta = 130^\circ$ (C)

Q10) $y = 4a^{3x}+b$

$a^{3x} = \frac{y-b}{4}$

$3x = \log_a\left(\frac{y-b}{4}\right)$

$x = \frac{1}{3}\log_a\left(\frac{y-b}{4}\right)$ (B)

QUESTION 11 - Marked by CJC.

a) (i) $\frac{d}{dx}\log_e(4x^2-3x) = \frac{8x-3}{4x^2-3x}$ (1)

(ii) $\frac{e^{6x}}{x+1}$ $\qquad u=e^{6x} \qquad v=x+1$

$u'=6e^{6x} \qquad v'=1$

$\frac{d}{dx} = \frac{vu'-uv'}{v^2}$

$= \frac{6e^{6x}(x+1)-e^{6x}}{(x+1)^2}$ (1) correct application of quotient rule

$= \frac{6xe^{6x}+6e^{6x}-e^{6x}}{(x+1)^2}$

$= \frac{6xe^{6x}+5e^{6x}}{(x+1)^2}$ (1) simplified answer.

(b) $\int e^{-3x+2}\,dx = -\frac{e^{-3x+2}}{3}+c$ (1) must have $+c$

(c) $\int_0^2 \frac{x}{x^2+6}\,dx = \frac{1}{2}\int_0^2\frac{2x}{x^2+6}\,dx$ (1) factor $\frac{1}{2}$

$= \frac{1}{2}\left[\ln(x^2+6)\right]_0^2$ (1) integration

$= \frac{1}{2}\left[\ln 10 - \ln 6\right]$

$= \frac{1}{2}\ln\frac{5}{3}$ (1) substitution / simplification

(d) $\sum_{k=5}^{30} 3k-2 = 13 + 16 + 18 + \ldots + 88$

$30-5+1=26$

$a = 13$

$l = 88$

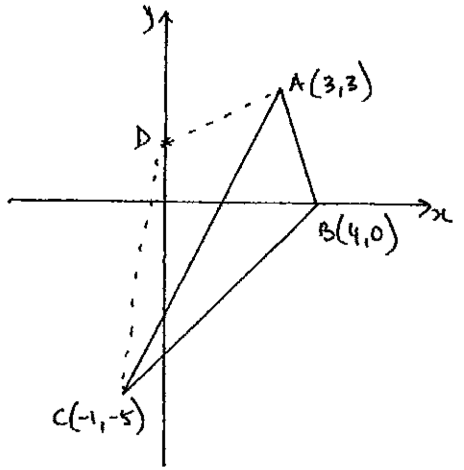
$n = 26$

$S_n = \frac{n}{2}(a+l)$ ① correct formula

$S_{26} = 13(13+88)$

$= 1313$ ① correct answer

(e)



(i) $A(3,3)$ $C(-1,-5)$

$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$d_{AC} = \sqrt{(3+1)^2 + (3+5)^2}$

$= \sqrt{16+64}$

$= \sqrt{80}$

$= 4\sqrt{5}$ units ①

(ii) $m = \frac{y_2-y_1}{x_2-x_1}$

$m_{AC} = \frac{-5-3}{-1-3}$

$= \frac{-8}{-4}$

$= 2$

① $m_{AC} = 2$

$m = 2$ $(3,3)$

$y - y_1 = m(x - x_1)$

$y - 3 = 2(x-3)$

$y = 2x - 6 + 3$

$0 = 2x - y - 3$ ①

(iii) $\perp d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$

$0 = 2x - y - 3$ $(4,0)$

$\perp d = \frac{|2(4) - 1(0) - 3|}{\sqrt{2^2+1^2}}$ ① correct substitution into formula.

$= \frac{|8-3|}{\sqrt{5}}$

$= \frac{5}{\sqrt{5}}$

$= \sqrt{5}$ units. ① correct evaluation.

(iv) $A = \frac{1}{2}xy$

$A = \frac{1}{2} \times 2\sqrt{5} \times 4\sqrt{5}$ ① progress towards area with formula and distances.

$= 4 \times 5$

$= 20$ units^2 ① correct evaluation.

QUESTION 12 - Marked by CJC.

(a) $|2x-1| = 3$

$2x - 1 = 3$ $\qquad$ $2x-1 = -3$

$2x = 4$ $\qquad$ $2x = -2$

$x = 2$ (1) $\qquad$ $x = -1$ (1)

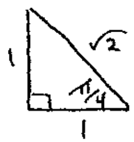
(b) $9x^2 - 21x + 10$

$= (3x-2)(3x-5)$ (2)

(c) $a^2 = b^2 + c^2 - 2bc\cos A$ $\qquad$ (1) correct substitution into cosine rule

$a^2 = 12^2 + (5\sqrt{2})^2 - 2(12)(5\sqrt{2})\cos\frac{\pi}{4}$

$a^2 = 144 + 50 - 120\sqrt{2} \times \frac{1}{\sqrt{2}}$



$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$a^2 = 194 - 120$

$a^2 = 74$

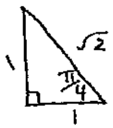
$a = \sqrt{74}$ $\qquad$ (1) correct answer

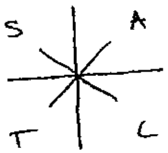
(d) $2\cos^2\theta - 1 = 0$

$2\cos^2\theta = 1$

$\cos^2\theta = \frac{1}{2}$

$\cos\theta = \pm\frac{1}{\sqrt{2}}$ (1)





$\angle\frac{\pi}{4}$ (1) related acute angle

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

(1) all 4 solutions.

(e) $y = e^{2x+1}$

$y' = 2e^{2x+1}$ $\qquad$ $y = e^{2x+1}$

when $x = 0$ $\qquad$ when $x = 0$

$y' = 2e$ $\qquad$ $y = e$ $\qquad$ $(0, e)$

(1) point.

$m_T = 2e$

$m_N = -\frac{1}{2e}$ $\qquad$ (1) $m_N = -\frac{1}{2e}$

$y - y_1 = m(x - x_1)$

$y - e = -\frac{1}{2e}(x - 0)$

$y = -\frac{x}{2e} + e$ $\qquad$ (1) substitution/simplify.

(f) $\text{Area} = \left(\frac{1}{2} \times 1 \times 2\right) - \int_1^{\sqrt{3}} 3 - x^2\,dx$ $\qquad$ (1) correct integral statement

$= 1 - \left[3x - \frac{x^3}{3}\right]_1^{\sqrt{3}}$ $\qquad$ (1) correct integration

$= 1 - \left[\left(3\sqrt{3} - \frac{(\sqrt{3})^3}{3}\right) - \left(3 - \frac{1}{3}\right)\right]$

$= 1 - \left[3\sqrt{3} - \frac{3\sqrt{3}}{3} - \frac{8}{3}\right]$

$= 1 - \left(2\sqrt{3} - \frac{8}{3}\right)$

$= 1 - 2\sqrt{3} + \frac{8}{3}$

$= \frac{11}{3} - 2\sqrt{3}$ units^2 $\qquad$ (1) correct evaluation.

QUESTION 13

(a)(i) $f'(x) \doteqdot \frac{h}{2}\left[(y_0 + y_n) + 2(y_1 + y_2 + y_3 + \dots)\right]$.

$f(x)$	0	1	2	3
$f'(x)$	0	$\ln 2$	$\ln 3$	$\ln 4$

$h = 1$ ① correct values in table and $h=1$

$f'(x) \doteqdot \frac{1}{2}\left[(0 + \ln 4) + 2(\ln 2 + \ln 3)\right]$ ① correct substitution into formula

$\doteqdot 2.48$ (2 d.p) ① correct evaluation

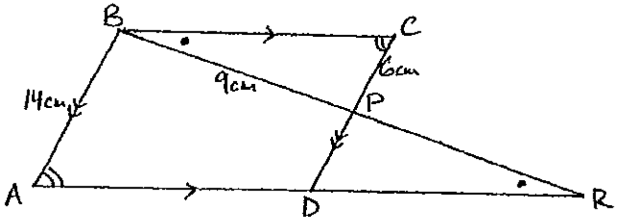
→ On the whole - Trap. Rule was done very poorly. Learn the rule!! It's simple!!

(ii) The estimated value will be less than the exact value as the curve is concave down so the trapezia will lie totally underneath the curve.

① must mention concavity and trapeziums

Many used a combination of diagrams + explanations with words. This seemed to be most effective.

(b)



(i) In ΔCBP and ΔARB

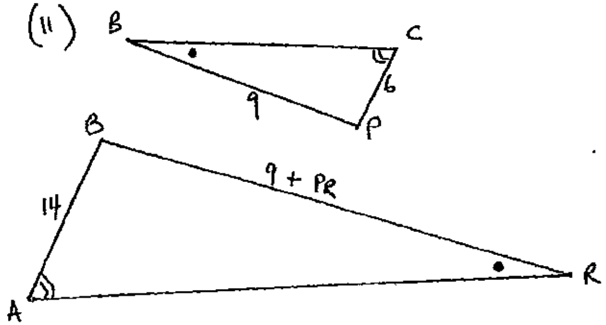
- $\angle CBP = \angle ARB$ (alt $\angle$'s on $\parallel$ lines $=$)
- $\angle PCB = \angle BAR$ (opp $\angle$'s in $\parallel$gram $=$)

① one pair matching angles with reason

$\therefore \Delta CBP \parallel\!\mid \Delta ARB$ (equiangular) ① equiangular

→ Various methods used - appropriate reasons had to be given to achieve full marks.

(ii)



Must give reasons!! No mark awarded if no reason.

$\frac{RP + 9}{9} = \frac{14}{6}$ (corresponding sides in $\parallel\!\mid \Delta$'s $=$)

① correct ratio and reasoning.

$PP + 9 = 21$

$PR = 12$ units ① correct evaluation from given ratio.

c) (i) $0.8 \times 725\ 000 = 580\ 000$

Rob borrows \$580 000 ① correct answer.

(ii) $P = 580\ 000$ $r = 7.68\%$ p.a $= 0.64\%$ p.m $= 0.0064$ $M = 4000$

$A_1 = 580\ 000(1.0064) - 4000$ ① showing added interest - payment pattern.

$A_2 = [580\ 000(1.0064) - 4000](1.0064) - 4000$

$= 580\ 000(1.0064)^2 - 4000(1.0064) - 4000$

$= 580\ 000(1.0064)^2 - 4000(1.0064 + 1)$

$A_3 = \left[580\,000(1.0064)^2 - 4000(1.0064+1)\right](1.0064) - 4000$

$= 580\,000(1.0064)^3 - 4000(1.0064^2 + 1.0064 + 1)$

1 developed series to A_3.

(iii) $A_n = 580\,000(1.0064)^n - 4000\underbrace{(1 + 1.0064 + \ldots + 1.0064^{n-1})}$

$S_n = \dfrac{a(r^n - 1)}{r-1}$ $\quad a = 1$, $r = 1.0064$, $n = n$

$A_n = 580\,000(1.0064)^n - 4000\left[\dfrac{1.0064^n - 1}{0.0064}\right]$ 1 correct sum formula and simplify.

$A_n = 580\,000(1.0064)^n - 625\,000(1.0064^n - 1)$

Had to show this using step.

(iv) When $A_n = 0$

$0 = 580\,000(1.0064)^n - 625\,000(1.0064^n - 1)$

$0 = 580\,000(1.0064)^n - 625\,000(1.0064)^n + 625\,000$

$45(1.0064)^n = 625$ 1 Simplified to this point.

$1.0064^n = \dfrac{625}{45} = \dfrac{125}{9}$

$n\ln(1.0064) = \ln\left(\dfrac{625}{45}\right)$

$n = \dfrac{\ln\left(\frac{625}{45}\right)}{\ln(1.0064)}$

$n = 412.42$ mths

$\therefore$ after $n = 413$ months. 2 correct whole months.

v) $4000 \times 413 = 1652000$

$1652000 - 580000$
$= \$1072000$

$\therefore$ He paid \$1072000 in interest.

1 correct amount.

QUESTION 14

(a) $3x^2 - 4kx + k = 0$

$a = 3 \quad b = -4k \quad c = k$

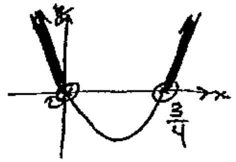
$\Delta = b^2 - 4ac$

$\Delta = 16k^2 - 12k$ 1 Δ

$\Delta > 0$ for real and distinct roots

$16k^2 - 12k > 0$

$4k(4k-3) > 0$



$k < 0$ and $k > \frac{3}{4}$

1 correct inequalities with test or sketch.

b) $y = \dfrac{1}{x^2+1}$

$x^2 + 1 = \dfrac{1}{y}$

$x^2 = \dfrac{1}{y} - 1$

$V = \pi\int_{\frac{1}{2}}^{1} \dfrac{1}{y} - 1\,dy$ 1 x^2, π

$= \pi\left[\ln y - y\right]_{1/2}^{1}$ 1 integration

$= \pi\left[(\ln 1 - 1) - (\ln\tfrac{1}{2} - \tfrac{1}{2})\right]$

$= \pi\left[-1 - (\ln 1 - \ln 2 - \tfrac{1}{2})\right]$

$= \pi\left[\ln 2 - \tfrac{1}{2}\right]$ 1 substitution/simplification

$= \left(\ln 2 - \tfrac{1}{2}\right)\pi$ units^3

* Use log laws

(c) $1 + \frac{1}{1-\sqrt{5}} + \frac{1}{(1-\sqrt{5})^2} + \cdots$

(i) $r = \frac{1}{1-\sqrt{5}}$

$\doteqdot -0.8$

as $-1 < r < 1$ the series has a limiting sum. ①

(ii) $S_\infty = \frac{a}{1-r}$ $\quad a = 1 \quad r = \frac{1}{1-\sqrt{5}}$

$S_\infty = \dfrac{1}{1 - \left(\frac{1}{1-\sqrt{5}}\right)}$ ① substitute correct values into correct formula

Substitute Carefully!!

$= \dfrac{1}{\frac{1-\sqrt{5}-1}{1-\sqrt{5}}}$

$= \dfrac{1}{-\frac{\sqrt{5}}{1-\sqrt{5}}}$

$= -\dfrac{1-\sqrt{5}}{\sqrt{5}}$

$= \dfrac{-\sqrt{5}(1-\sqrt{5})}{\sqrt{5}\sqrt{5}}$

$= \dfrac{5-\sqrt{5}}{5}$ ① evaluation with rational denominator.

(d) $x^2 + 4x - 12y + 16 = 0$

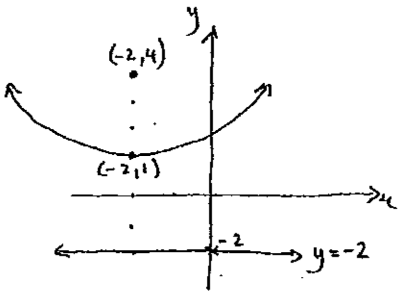
$x^2 + 4x + 4 = 12y - 12$

$(x+2)^2 = 12(y-1)$

(i) vertex $(-2, 1)$ ①

$a = 3 \quad \therefore$ focus $(-2, 4)$ ①

(ii) directrix $y = -2$ ①



(e)

(i) $M = \dfrac{1}{1 - \log_e A}$

$1 - \log_e A = \dfrac{1}{M}$

$\log_e A = 1 - \dfrac{1}{M}$

$A = e^{1-\frac{1}{M}}$

$E = (M-2)A + 3$

$E = (M-2)e^{1-\frac{1}{M}} + 3$

① rearrange/substitute

(ii) $E = \underbrace{(M-2)}_{u}\underbrace{e^{1-\frac{1}{M}}}_{v} + 3$ use Product rule

$u = M-2$ $\quad v = e^{1-\frac{1}{M}}$

$u' = 1$ $\quad v' = \dfrac{e^{1-\frac{1}{M}}}{M^2}$

$E' = e^{1-\frac{1}{M}} + \dfrac{(M-2)e^{1-\frac{1}{M}}}{M^2}$

$E' = \dfrac{M^2 e^{1-\frac{1}{M}} + Me^{1-\frac{1}{M}} - 2e^{1-\frac{1}{M}}}{M^2}$

$E' = \dfrac{e^{1-\frac{1}{M}}(M^2 + M - 2)}{M^2}$

when $E' = 0$ ① substituting $E' = 0$

$0 = \dfrac{e^{1-\frac{1}{M}}(M^2 + M - 2)}{M^2}$

$0 = e^{1-\frac{1}{M}}(M+2)(M-1)$

$e^{1-\frac{1}{M}} \neq 0 \quad M = -2 \quad M = 1 \quad$ but $M > 0$

$\therefore M = 1$ only. ① $M = 1$ only.

test.

$f(x)$	$\frac{1}{2}$	1	2
$f'(x)$	-1.8	0	1.6

$\diagdown \quad — \quad \diagup$

① checking for min.

$\therefore$ Minimum when $M = 1$.

END OF SOLUTIONS