CATHOLIC SECONDARY SCHOOLS' ASSOCIATION OF NEW SOUTH WALES

YEAR TWELVE FINAL TESTS 1998

MATHEMATICS

2/3 UNIT

Morning session

Wednesday 12th August 1998.

Time Allowed — Three Hours
(Plus 5 minutes reading time)

EXAMINERS

T. Attard
B. Cosgrove
M. Donaghy
A. Kollias
J. Mann
R. Pantua
E. Rainert
C. Reichel
P. Rockett
J. Wheatley

DIRECTIONS TO CANDIDATES :

* **ALL** questions may be attempted.
* **ALL** questions are of equal value.
* All necessary working should be shown in every question.
* Full marks may not be awarded for careless or badly arranged work.
* Approved slide rules or calculators may be used.
* Table of standard integrals is provided.
* The answers to the ten questions in this paper are to be returned in separate writing sheets clearly marked **QUESTION 1, QUESTION 2** etc. on the top of the sheet.
* If required, additional writing sheets may be obtained from the examinations supervisor upon request.

Students are advised that this is a Trial Examination only and cannot in any way guarantee the content or the format of the Higher School Certificate Examination. However, the committees responsible for the preparation of these 'Trial Examinations' do hope that they will provide a positive contribution to your preparation for the final examinations.

QUESTION 1 *(Use a <u>separate</u> writing sheet)* **Marks**

(a) Simplify $(3x-6)-(5-4x)$ **2**

(b) Find the value of $\dfrac{23{\cdot}1}{56{\cdot}3 \times \sqrt{25{\cdot}04}}$ correct to 3 significant figures. **2**

(c) Differentiate $\tan\dfrac{x}{2}$ with respect to x. **1**

(d) Two dice, with numbers 0 to 5 on their faces are thrown. What is the probability that they both show 4 on their uppermost face? **2**

(e) **2**

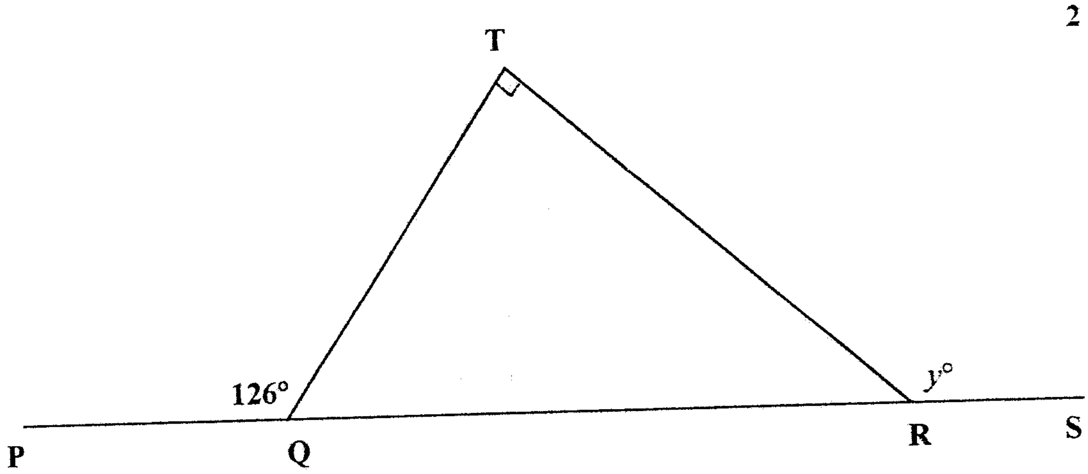


FIGURE NOT TO SCALE

In the diagram $\angle PQT = 126°$ and $\angle QTR = 90°$. Find the value of y.

(f) Solve for x: **3**

$$\frac{x-2}{2}+\frac{x+1}{5}=2$$

QUESTION 3 *(Use a separate writing sheet)* **Marks**

(a) Differentiate with respect to x: **6**

(i) $(4-3x)^6$

(ii) $x^2 e^{2x}$

(iii) $\dfrac{\sin 2x}{x}$

(b) The decimal $0\cdot\dot{3}\dot{4}$ (i.e. $0\cdot 34343434\ldots$) can be considered as the sum of a geometric sequence. **3**

(i) What is the value of the first term, a, and the common ratio, r ?

(ii) Hence or otherwise express $0\cdot\dot{3}\dot{4}$ as a fraction.

(c) Find the equation of the normal to the curve $y = \ln x$ at the point where $x = 1$. **3**

QUESTION 2 *(Use a separate writing sheet)* **Marks**

(a) Simplify $\sqrt{27} - \sqrt{3} + \sqrt{18}$ **2**

(b) Solve $|x - 1| = 2x - 1$ **3**

(c) **7**

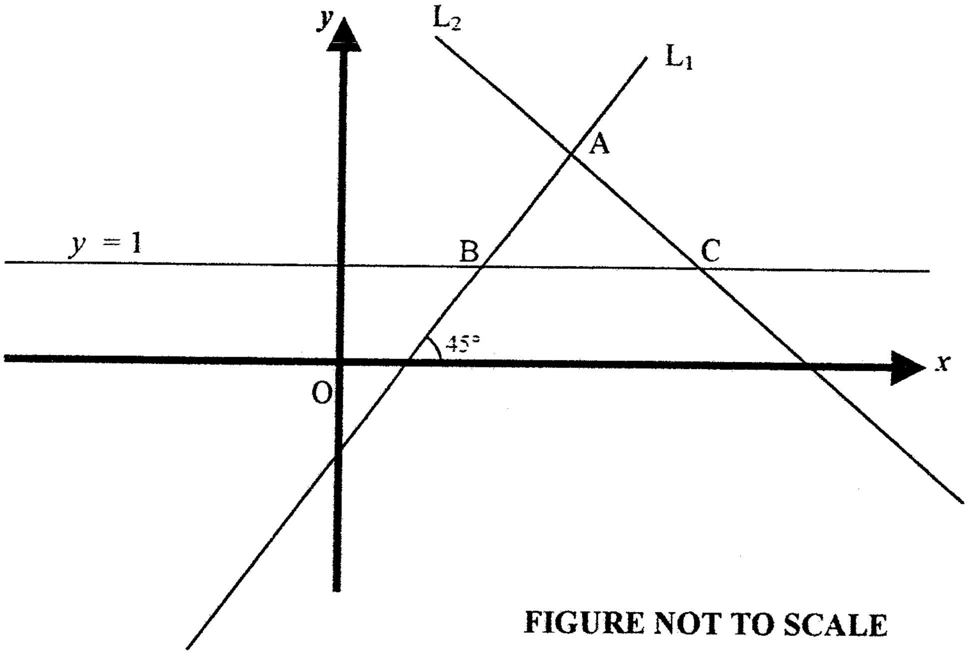


FIGURE NOT TO SCALE

The diagram shows the line $y = 1$. Line L_1 passes through point A(4,3) and makes an angle of 45° with the x axis. L_2 also passes through point A(4,3).

Copy this diagram onto your answer sheet.

(i) Explain how you know the gradient of L_1 is 1.

(ii) Show that the equation of line L_1 is $x - y - 1 = 0$.

(iii) Line L_1 intersects the line $y = 1$ at point B. Find the coordinates of point B.

(iv) Line L_2 is drawn perpendicular to L_1 and passes through point A. Find the equation of L_2.

(v) L_2 intersects $y = 1$ at C. Show that ΔABC is isosceles.

QUESTION 4 (*Use a separate writing sheet*) **Marks**

(a) **4**

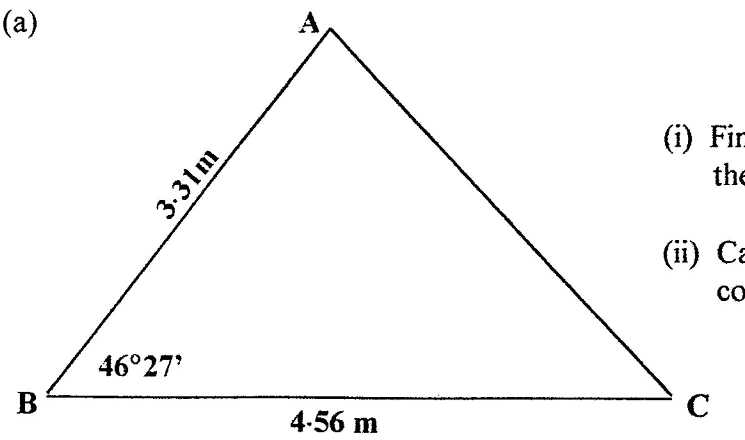


FIGURE NOT TO SCALE

(i) Find the length of AC correct to the nearest cm.

(ii) Calculate the area of ΔABC correct to the nearest m^2.

(b) The gradient function of a curve is given by $\frac{dy}{dx} = 3x^2 - 6x - 9$. The curve passes through the point $(1, -2)$. **8**

(i) Find the equation of the curve.

(ii) Find the coordinates of the stationary points and determine their nature.

(iii) Find the coordinates of the point of inflection.

(iv) Sketch the curve and show its stationary points, point of inflection and y intercept. Also indicate how the curve behaves for large positive and negative values of x.

QUESTION 5 *(Use a* ***separate*** *writing sheet)* **Marks**

(a) **3**

x	1	2	3	4	5
$x \ln x$	0	1·386	3·296	5·545	8·047

The table shows the values of $x \ln x$ for five values of x.

Find an approximate value for $\int_1^5 x \ln x \, dx$ using Simpson's rule with the 5 function values in the table. Express your answer correct to 2 decimal places.

(b) A box has 4 Geography books and 3 Mathematics books in it. Two books are selected at random from the box. **5**

(i) Draw a tree diagram to show all the possible outcomes.

(ii) Find the probability that:
(α) the two books are Mathematics books.
(β) at least one of the books is a Geography book.

(c) **4**

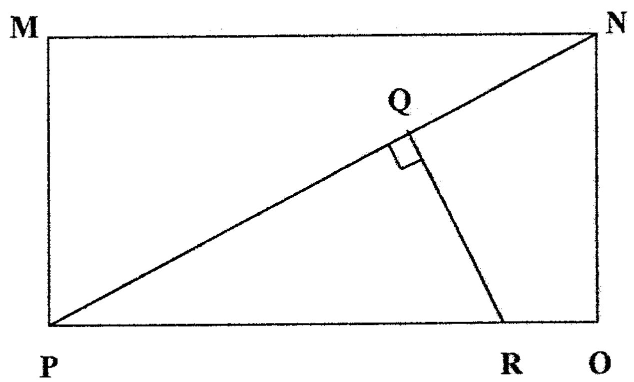


FIGURE NOT TO SCALE

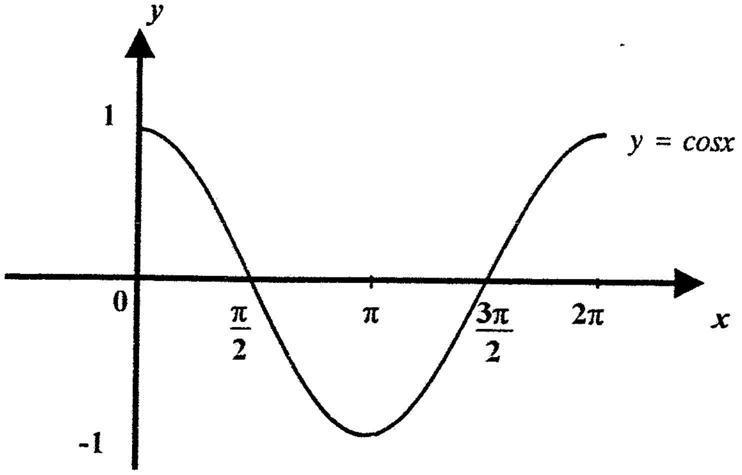
PN is a diagonal of the rectangle MNOP. R is the point on PO and $\angle PQR = 90°$

(i) Prove that ΔPQR is similar to ΔNMP.

(ii) Given MP = 5 cm, MN = 10 cm and QR = 2 cm, find the length of PQ.

QUESTION 6 *(Use a separate writing sheet)* **Marks**

(a) **2**



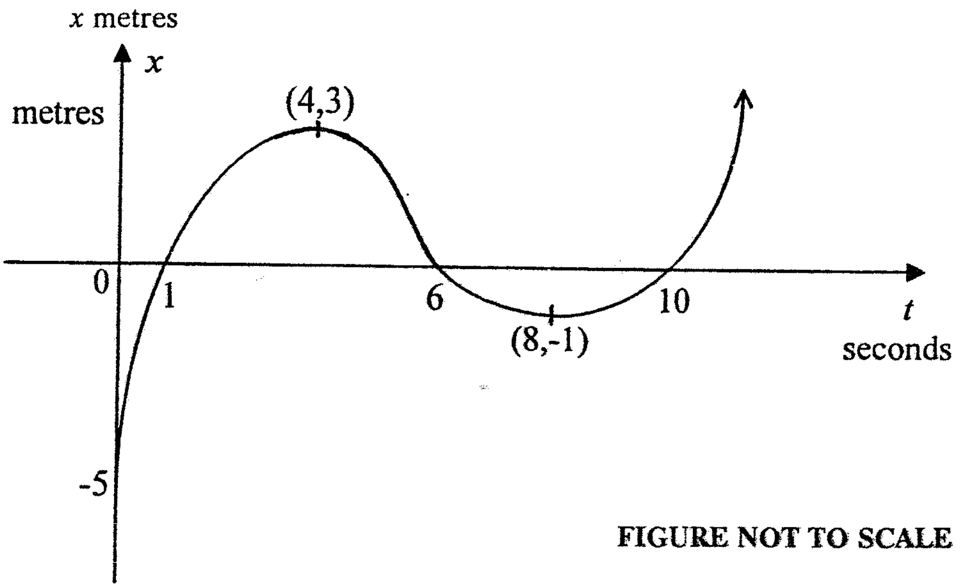
The diagram shows the graph of $y = \cos x$ in the domain $0 \le x \le 2\pi$.

(i) Copy this graph onto your answer sheet.

(ii) On the same graph, draw the graph of $y = \sin 2x$ in the domain $0 \le x \le 2\pi$.

(iii) How many solutions are there to the equation $\sin 2x = \cos x$ for $0 \le x \le 2\pi$?

(b) **5**



The graphs shows the displacement, x metres from the origin, at any time t seconds, of a particle moving in a straight line.

(i) Where was the particle initially?

(ii) When was the particle at the origin?

(iii) When was the particle at rest?

(iv) Estimate the time when the acceleration was zero.

(v) How far did the particle travel during the first 10 seconds?

Marks

(c) Bruce borrowed \$60 000 to buy a small business. He was charged $\frac{1}{2}$ % per month interest on the balance owing and he repaid the loan plus interest in equal monthly repayments over 5 years. **5**

(i) Show that Bruce owed \$(60 300 – M) immediately after making his first monthly repayment of \$M.

(ii) Show that Bruce owed $\$[60\,000(1{\cdot}005)^3 - M(1{\cdot}005^2 + 1.005 + 1)]$ immediately after he made three monthly repayments.

(iii) Calculate the value of his monthly repayment, \$M.

QUESTION 7 (*Use a separate writing sheet*) **Marks**

(a) The quadratic equation $x^2 + (k + 3)x - k = 0$ has real roots. Find all the possible values k can take. **3**

(b) The volume, V, of water in a dam at time t was monitored over a period of time. When monitoring began, the dam was 75% full. During the monitoring period, the volume decreased at an increasing rate due to a long period of drought. **3**

(i) What does this tell us about $\frac{dV}{dt}$ and $\frac{d^2V}{dt^2}$?

(ii) Sketch the graph of V against t.

(c) A particle moves in a straight line such that its distance, x metres, from a fixed point O at any time t seconds is given by: **6**

$$x = 5 + ln\,(1 + 2t)$$

(i) Find expressions for the velocity and acceleration of the particle at any time t seconds.

(ii) When will the particle be 10 metres from O?

(iii) Find the velocity and acceleration of the particle when it is 10 metres from O.

(iv) Show that the particle does not change direction during its motion.

QUESTION 8 *Use a separate writing sheet* **Marks**

(a) During the first week in January a car dealer sold 15 cars. In the second he sold 18, and each week after that he sold 3 more cars than he sold the previous week. **4**

i.e.	Week	1	2	3	4	....
	Sales	15	18	21	24	....

There are 52 weeks in a year.

(i) How many cars did he sell during the last week in December?

(ii) Calculate the total number of cars he sold during the year.

(b) Evaluate $\int_1^3 \frac{x}{x^2+1}\,dx$ **2**

(c) **6**

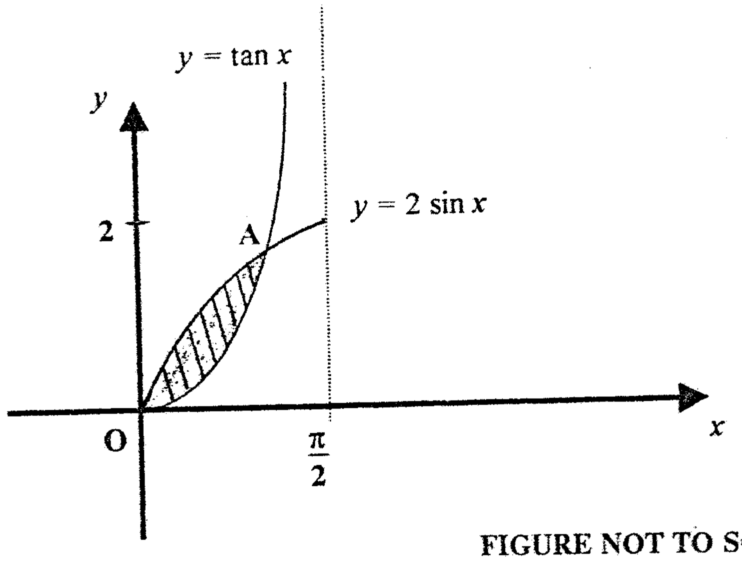


FIGURE NOT TO SCALE

The diagram shows the curves $y = \tan x$ and $y = 2\sin x$ for $0 \le x \le \frac{\pi}{2}$

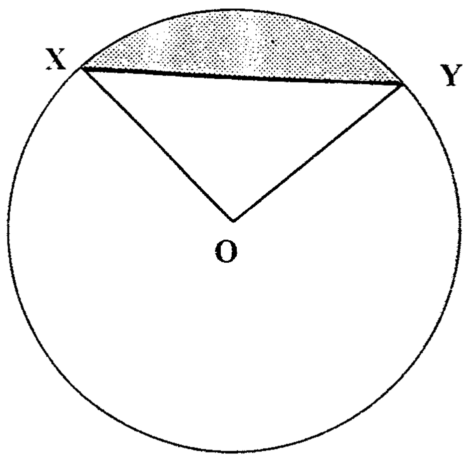
(i) Show that the coordinates of A are $(\frac{\pi}{3}, \sqrt{3})$.

(ii) Show that $\frac{d}{dx}[(ln \cos x)] = -\tan x$

(iii) Hence find the shaded area in the diagram.

QUESTION 9 ***(Use a separate writing sheet)*** **Marks**

(a) **2**



In the diagram XY is an arc of a circle of radius 10 cm and $\angle XOY = \frac{3\pi}{8}$

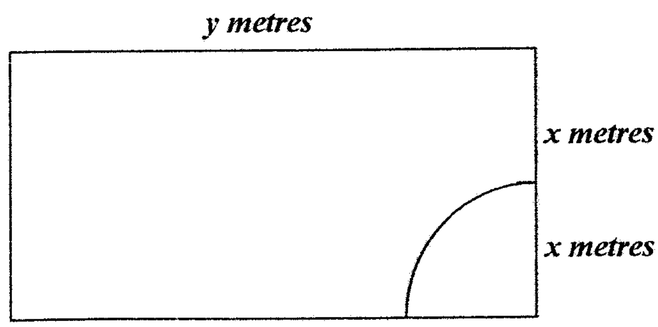
Find the area of the shaded region correct to the nearest cm^2.

(b) An organisation donated \$50 000 to charity during 1990. Each subsequent year after 1990 it donated 75% of its previous year's donation to charity. **4**

(i) What was the amount donated to charity during 1995?

(ii) If this trend continued indefinitely, what is the maximum total amount the organisation would donate to charity?

(c) **6**



The diagram shows a rectangular paddock which has an area of 1 hectare (10 000 m^2). The paddock requires fencing around the perimeter and also along a circular arc in one corner. The radius of the arc is half the width of the paddock.

(i) Show that the total amount of fencing required is given by

$L = 4x + \frac{\pi x}{2} + \frac{10\,000}{x}$ metres, where x is the radius of the circular arc.

(ii) Hence show that when x is approximately 42·4 metres, the paddock will require the least amount of fencing.

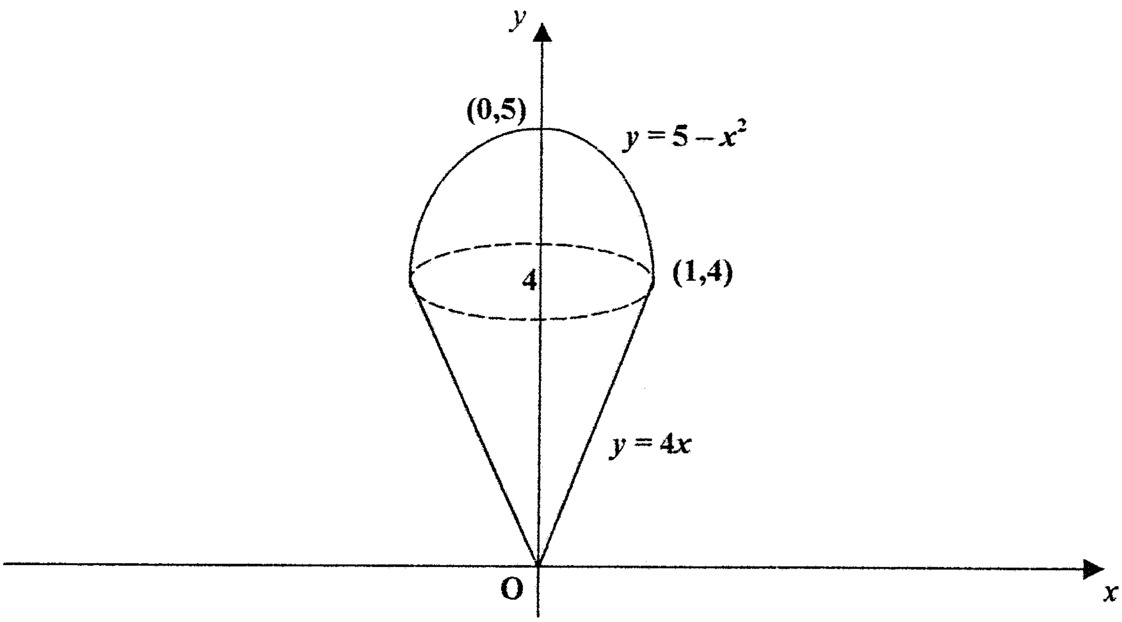
QUESTION 10 (*Use a separate writing sheet*) **Marks**

(a) A number of electrical components are wired together in an alarm so that it will operate if at least one of the components works. The probability that each one of these components will work is 0·6. **3**

(i) If an alarm had three components wired together, find the probability that at least one of the components will work.

(ii) Find the minimum number of components that must be wired together to be 99% certain that the alarm will operate.

(b) The diagram shows a cone and a paraboloid. It represents an icecream cone which is completely full of icecream and which has an additional scoop of icecream on top. **4**



All measurements in cm.
FIGURE NOT TO SCALE

To calculate the volume of icecream, the area bounded by the section of the line $y = 4x$ between (0,0) and (1,4), the part of the parabola between (1,4) and (0,5) and the y axis, was rotated about the y axis.

(i) Determine the total quantity of icecream contained in the cone and the scoop on top.

(ii) How many of these icecreams can be made from a 1 litre container of icecream? (1 000 cm^3 = 1 litre)

QUESTION 10 *(Continued)* **Marks**

(c) As soon as advertising and other promotions of a particular product ended, the sales, S, in thousands of units of the product fell at a rate proportional to the number of sales made, that is $\frac{dS}{dt} = -kS$, where k is a constant and the time, t, is measured in days after the advertising and promotions ended **5**

(i) Show that $S = S_0 e^{-kt}$ satisfies the equation $\frac{dS}{dt} = -kS$ where S_0 is the number of sales, in thousands of units, made at the end of the promotions period.

(ii) 30 days after the promotions ended the number of sales had fallen by 50%. Find the value of k.

(iii) The decision was made to stop making this product as soon as the sales fell to 15% of the level at the end of the promotions. How many days after the promotions ended did that occur?

END OF EXAMINATION

USE RULER TO TEAR OFF HERE

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

(1)(a) $7x - 11$

(b) 0.0820 (to 3 sig. fig.)

(c) $\frac{1}{2}\sec^2\frac{x}{2}$

(d) $\frac{1}{36}$

(e) 144^0

(f) $x = 4$

(2) (a) $2\sqrt{3} + 3\sqrt{2}$

(b) $x = 0$ (invalid); $x = \frac{2}{3}$

(c) (i) $m = \tan 45^0 = 1$

(ii) Proof

(iii) $B(2, 1)$

(iv) $x + y - 7 = 0$

(v) Since $\angle B = \angle C$, therefore ΔABC is isosceles.

(3)(a)(i) $-18(4 - 3x)^5$

(ii) $2xe^{2x}(1 + x)$

(iii) $\frac{2x\cos 2x - \sin 2x}{x^2}$

(b)(i) $a = \frac{17}{50}$, $r = \frac{1}{100}$

(ii) $x = \frac{34}{99}$

(c) $x + y - 1 = 0$

(4) (a) (i) $AC = 331$ cm

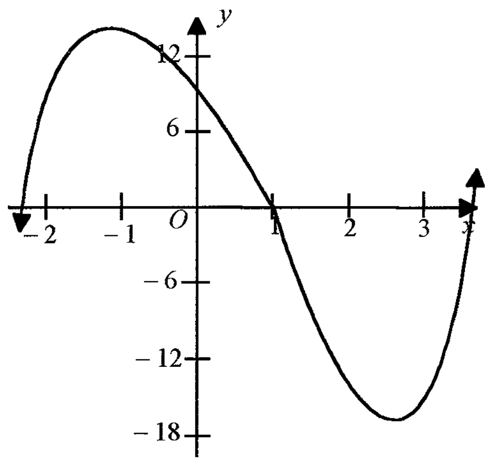
(ii) Area $= 5\ \text{m}^2$

(b) (i) $x^3 - 3x^2 - 9x + 9 = 0$

(ii) $(3, -18)$ rel. min.
$(-1, 14)$ rel. max.

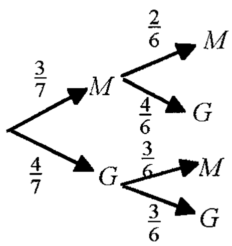
(iii) $(1, -2)$ point of inflexion.

(iv)



(5) (a) 14.12 (to 2 d.p.)

(b) (i)

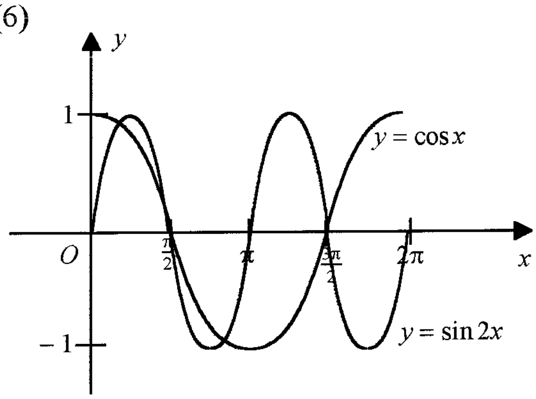


(ii) (α) $\frac{1}{7}$ (β) $\frac{6}{7}$

(c) (i) Proof

(ii) $PQ = 4$ cm

(6)



(iii) 4 solutions

(b) (i) -5 m

(ii) $t = 1,\ 6,\ 10$s

(iii) $t = 4,\ 8$s

(iv) ≈ 6s

(v) 13 m

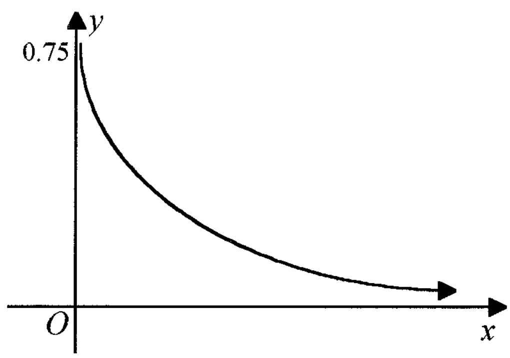
(c) (i), (ii) Proofs

(iii) $1159.97

(7)(a)(i) $k \le -9$ or $k \ge -1$

(b) (i) $\frac{dV}{dt} < 0,\ \frac{d^2V}{dt^2} > 0$

(ii)



(c) (i) $v = \frac{2}{1+2t};\ a = \frac{-4}{(1+2t)^2}$

(ii) $t = \frac{e^5 - 1}{2} \approx 73.7$ s

(iii) $v = 0.1$ m/s; $a = -1.7 \times 10^{-2}$ m/s^2

(iv) For the particle to change directions, $v = 0$ but $\frac{2}{1+2t} \ne 0$, therefore particle cannot change directions.

(8) (a) (i) 168 cars

(ii) 4758 cars

(b) $\frac{1}{2}\ln 5$

(c) (i) Solve simultaneously

(ii) Proof

(iii) $1 - \ln 2$

(9) (a) 13 cm^2 (to the nearest sq. cm)

(b) (i) $15 820.31

(ii) $200 000

(c) (i) Proof (ii) Proof

(10) (a) (i) 0.936 (ii) $n = 6$

(b) (i) $\frac{11\pi}{6}$ cm^3

(ii) 174 ice creams

(c) (i) Proof

(ii) $k = \frac{\ln 0.5}{-30} \approx 0.023$ (to 2 d.p.)

(iii) 83 days.