**CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NEW SOUTH WALES**

2007
**TRIAL HIGHER SCHOOL CERTIFICATE
EXAMINATION**

Mathematics

Morning Session
Monday 13 August 2007

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided on the back page
- All necessary working should be shown in every question
- Write your Centre Number and Student Number at the top of this page

Total marks – 120

- Attempt Questions 1-10
- All questions are of equal value

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Total marks – 120
Attempt Questions 1-10
All questions are of equal value.

Answer each question in a SEPARATE writing booklet.

Marks

Question 1 (12 marks) Use a SEPARATE writing booklet.

(a)   The radius of Uranus is approximately 25 559 000m. Write the number in scientific notation, correct to two significant figures.   **2**

(b)   Given that $x = \sqrt{5} + 2$ evaluate $x + \dfrac{1}{x}$. Leave your answer in exact form.   **2**

(c)   Express $\dfrac{(3a-1)}{4} - \dfrac{(a-2)}{3}$ as a single fraction in its simplest form.   **2**

(d)   Find a primitive of $5 + \cos x$.   **2**

(e)   Find the values of x for which $|x-2| \leq 3$.   **2**

(f)   Solve the pair of simultaneous equations.   **2**

$$3x + y = 6$$
$$6x - 2y = -8$$

Question 2 (12 marks) Use a SEPARATE writing booklet.

(a)   In the diagram, *ABCD* is a quadrilateral. The coordinates of *A*, *B*, *C* and *D* are $(3, 4)$, $(6, -1)$, $(0, -5)$ and $(x, 0)$ respectively.


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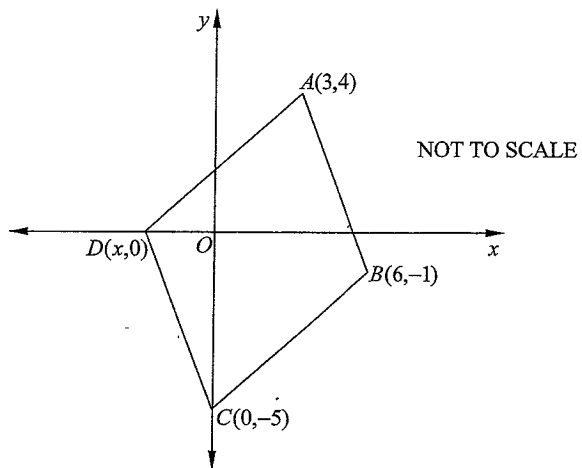
(i)   Calculate the length of *BC*. Leave your answer in exact form.   1

(ii)   Show that the gradient of *AB* is $-\dfrac{5}{3}$.   1

(iii)   Find the *x*-coordinate of *D* such that *AB* is parallel to *DC*.   1

(iv)   Show that the equation of the line *AD* is $2x - 3y + 6 = 0$.   2

(v)   Find the perpendicular distance from *C* to *AD*. Leave your answer in exact form.   2

(vi)   Hence, or otherwise, find the area of the quadrilateral *ABCD*.   1

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Question 2 (continued)

(b)

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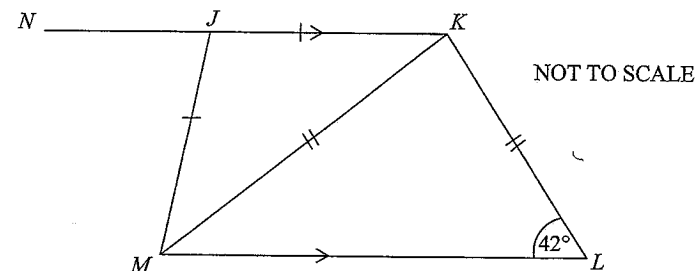
The diagram shows a quadrilateral *JKLM*, in which *JK* is parallel to *ML*, $JM = JK$, $KM = KL$ and $\angle KLM = 42°$.

N is a point on *JK* produced.

(i)   Copy or trace this diagram into your writing booklet.

(ii)   Find the size of $\angle LKM$. Give reasons for your answer.   2

(iii)   Hence, or otherwise, find the size of $\angle MJN$. Give reasons for your answer.   2

End of Question 2

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Question 3 (12 marks) Use a SEPARATE writing booklet.

(a) Differentiate with respect to x:

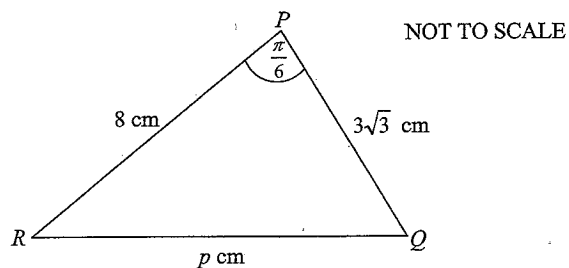   (i) $y = \left(e^x + 3\right)^4$.   **2**

   (ii) $f(x) = \dfrac{x^2}{\tan x}$.   **2**

(b) Find:

   (i) $\displaystyle\int \dfrac{x^2}{x^3 - 5}\, dx$.   **2**

   (ii) $\displaystyle\int_0^2 e^{-x}\, dx$. Leave your answer in exact form.   **2**

(c)

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Find the value of p in exact form.   **2**

(d) Given that $\sin\theta = \dfrac{3}{5}$ and $\tan\theta < 0$, find the exact value of $\cos\theta$.   **2**

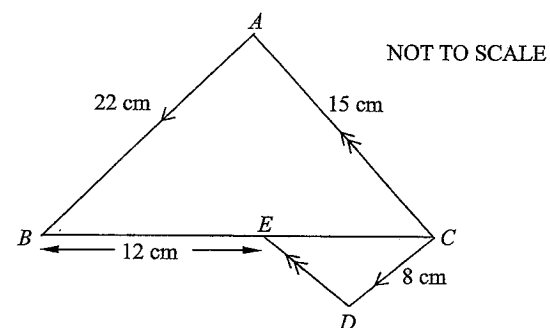**Question 4** (12 marks) Use a SEPARATE writing booklet.

(a) An infinite geometric series has a limiting sum of 24.   **2**

   If the first term is 15, find the common ratio.

(b) In the diagram, CD is parallel to AB and DE is parallel to CA.

   $AC = 15$ cm, $AB = 22$ cm, $CD = 8$ cm and $BE = 12$ cm.

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   (i)   Prove triangle ABC is similar to triangle DCE.   **2**

   (ii)   Hence find the length of BC.   **2**

(c) Find the equation of the normal to $y = e^{\cos x}$ at the point where $x = \dfrac{\pi}{2}$.   **2**

(d) Solve $\sqrt{3}\tan\theta + 1 = 0$ for $0 \le \theta \le 2\pi$.   **2**

(e) α and β are the roots of $3x^2 + 9x + 1 = 0$.

   Find the value of $4\alpha\beta^2 + 4\alpha^2\beta$.   **2**

Question 5 (12 marks) Use a SEPARATE writing booklet.

(a)   Consider the function $f(x)=x^3-x^2-8x-3$.

    (i)   Find the coordinates of the stationary points of the curve $y=f(x)$ and determine their nature.   **3**

    (ii)   Sketch the curve, clearly labelling any stationary points and the y-intercept.   **2**

    (iii)   For what values of x is the curve decreasing?   **1**

(b)   Two urns contain green and red marbles. Urn A contains 6 green and 3 red marbles while Urn B contains 4 green and 7 red marbles. A person selects either of the urns at random and chooses two marbles without replacement from their selected urn.

    (i)   Determine the probability that the person chooses 2 green marbles from Urn B?   **1**

    (ii)   Determine the probability that the person selected at least one green marble from Urn B?   **2**

(c)
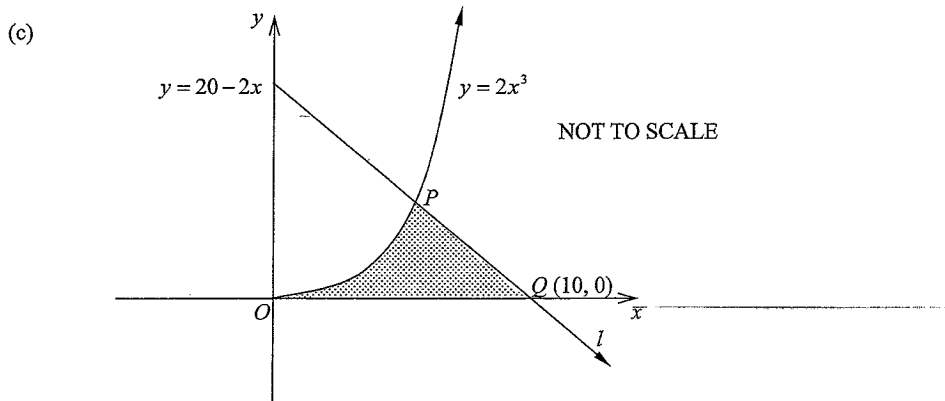
The diagram shows the cubic $y=2x^3$ and the line l with equation $y=20-2x$ for $x\geq 0$ intersecting at the point P, in the first quadrant. Q is the x-intercept of the line l with coordinates $(10, 0)$.

    (i)   Show that the co-ordinates of the point P are $(2, 16)$.   **1**

    (ii)   By considering the sum of two areas, find the shaded area.   **2**

Question 6 (12 marks) Use a SEPARATE writing booklet.

(a)   Evaluate $\displaystyle\sum_{w=0}^{4} w^{\frac{1}{2}}$. Leave your answer in exact form.   **2**

(b)   With the drought ever worsening, James and Theodore design a counting generator that can simulate the number of rain drops per minute that fall over a river during a storm.

The rain drops falling per minute forms the series

$$1+1+3+9+23+\ldots$$

with the nth term given by the formula $R_n=1-2n+2^n$.

    (i)   Verify that 115 is a term of this series.   **1**

    (ii)   Find the total amount of rain drops which fall over the river in the first twenty five minutes.   **2**

    (iii)   If the surface area of the river is 250 m² find the average rainfall per cm² over the first twenty five minutes.   **1**

Question 6 (continued)

(c) (i) Copy and complete the following table into your writing   **1**
booklet, correct to 3 decimal places where necessary, for the
curve $y = x^2 \log_e x$.

x	2	2.5	3	3.5	4
y					

(ii) Use Simpson's rule with 5 function values to find an   **2**

approximation to $\int_{2}^{4} x^2 \log_e x \ dx$.

(d) A set of three concentric circles with centre O and radii 3.5 cm, 9 cm
and 12.5 cm is drawn below. The outer arc length of AB is 22.5 cm.

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22.5 cm
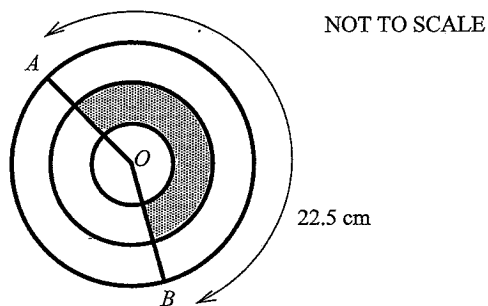
(i) Calculate the size of the reflex angle AOB. Give your answer in   **1**
radians.

(ii) Calculate the area of the shaded region.   **2**

End of Question 6

Question 7 (12 marks) Use a SEPARATE writing booklet.

(a) The population P of Sydney rose from 1 million at the beginning of 1980
to 1.7 million at the beginning of 1990.

Assuming natural growth, $P = 10^6 \times e^{kt}$ where t is the time in years since
the beginning of 1980 and k is a constant.

(i) Show that $\dfrac{dP}{dt} = kP$ satisfies the equation $P = 10^6 \times e^{kt}$.   **1**

(ii) Find the value of k, correct to three decimal places.   **2**

(iii) Find the population of Sydney at the beginning of 2007, correct to   **2**
three significant figures.

(iv) The population of Sydney's sister city Guangzhou in China at the   **2**
beginning of 2007 was 8.5 million.

In what year will the population of Sydney reach the population of
its sister city Guangzhou?

(b) Another sister city of Sydney is San Francisco. Sydney City Council
decides to build an art gallery in San Francisco to allow local Sydney
artists to exhibit their work.

The loan required to build the art gallery is $\$P$ with interest charged at an
introductory rate of 6% p.a. for the first three months. The loan is to be
initially repaid in equal monthly repayments of $4 000 over 3 years and
interest is charged monthly before each repayment.

Let $\$A_n$ be the amount owing by Sydney City Council at the end of the
nth repayment

(i) Find an expression for A_1.   **1**

(ii) Show that $A_3 = P(1.005)^3 - 4000(1 + 1.005 + 1.005^2)$.   **2**

At the end of three months interest rates rise to 9% p.a. and the loan is to be
repaid in total in equal monthly repayments of $4 800 for the next 2.75
years.

(iii) If the loan's interest rate is fixed at 9% p.a. for the remainder of the   **2**
loan, find the value of $\$P$.

Question 8 (12 marks) Use a SEPARATE writing booklet.

(a)   Evaluate $\displaystyle \lim_{x \to 0} \frac{\sin x}{x}$.   1

(b)   The displacement of a particle is given by:

$$x = t^2 - 4\log_e(t-1) + 5, \qquad t > 1$$

where x is in metres and t is in seconds.

(i)   Find the exact displacement of the particle when $t = 4$.   1 ·

(ii)   Find an expression for v and hence find when the particle comes to rest.   2

(iii)   Show that the acceleration remains positive for $t > 1$.   2

(iv)   Find the distance travelled by the particle between the times the particle comes to rest and $t = 4$.   2

(c)   The shaded region bounded by the graph $y = \dfrac{1}{1-x}$ for $x < 1$, the line $y = 2$ and the y-axis is rotated about the y-axis to form a solid of revolution.

$y = \dfrac{1}{1-x}$

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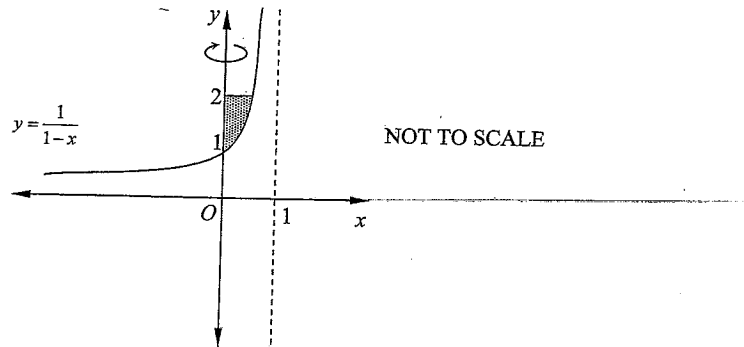
(i)   Show that the volume of the solid is given by: 2

$$V_y = \pi \int_1^2 1 - \frac{2}{y} + \frac{1}{y^2}\, dy.$$

(ii)   Find the volume V_y of the solid formed.   2

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Question 9 (12 marks) Use a SEPARATE writing booklet

(a)   Consider the function $g(x) = \dfrac{2}{x^2 - 1}$.

(i)   Show that $g(x)$ is an even function.   1

(ii)   State the domain of $y = g(x)$.   1

(b)   Consider the trigonometric function $y = 1 - 3\cos 2x$.

(i)   State the amplitude of $y = 1 - 3\cos 2x$.   1

(ii)   Draw a neat and accurate graph of $y = 1 - 3\cos 2x$ for $0 \le x \le \pi$.   2

(iii)   On the same diagram accurately draw the graph of $y = x + 1$. Hence determine the number of solutions to the equation $x + 3\cos 2x = 0$ over the domain $0 \le x \le \pi$.   2

(c)   Helen is training to compete in a mini triathlon. The course she practises on consists of three legs which starts at S and finishes at F. The first leg is a straight line swim from S to a point X. The second leg is a bike ride from X to Y along a straight road OY and the final leg is a jog from Y to F around a circular path. The perpendicular distance from S to O is 2km while the distance OY is 4km.

Helen can swim at 6km/h, bike ride at 12km/h and jog at 8km/h.

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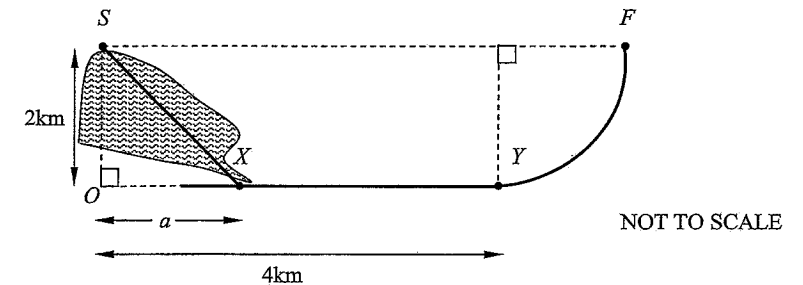
(i)   If the distance $OX = a$ km, show that the time T that it takes Helen to complete the three legs is given by   2

$$T = \frac{4\sqrt{a^2 + 4} - 2a + (8 + 3\pi)}{24} \text{ hours.}$$

(ii)   Find the value of a that will allow Helen to minimise the time taken to complete the three legs of her practise course.   3

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Question 10 (12 marks) Use a SEPARATE writing booklet.

(a)   A rural water dam is to be empted by means of a control valve. The valve operates so that the volume of the water , V litres , remaining in the dam varies with the time , t minutes, according to the equation

$$\frac{dV}{dt} = -bt, \qquad \text{where } b \text{ is a constant.}$$

   (i)   Initially the dam contains 250 000 litres of water.   **2**

   Show that after t minutes $V = 250\,000 - \frac{1}{2}bt^2$.

   (ii)   If $b = 0.431$, at what rate will the dam be emptying when   **2**
   $V = 85\,000$ litres?

(b)   In the diagram below, $ABCD$ is a square of side 15 cm leaning against a wall at an angle θ to the vertical and as well to the ground XY.

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15 cm.
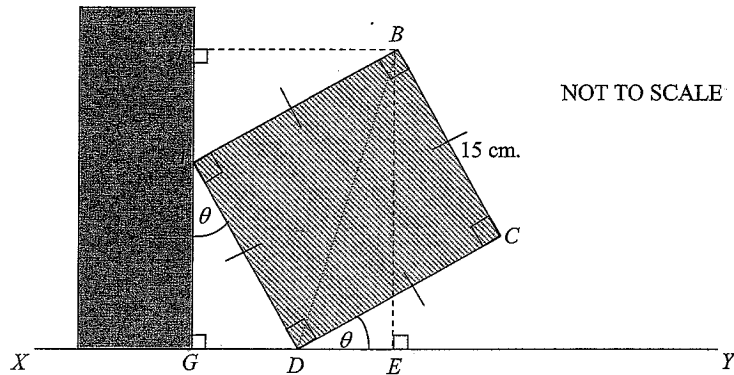
   (i)   Show that BD $= 15\sqrt{2}$ cm.   **1**

   (ii)   Hence by using triangle DBE, prove that the perpendicular   **2**
   distance of B from the line XY is $15\sqrt{2}\sin\left(\frac{\pi}{4}+\theta\right)$.

   (iii)   By using triangles DAG and BFA, find an expression for the length   **3**
   of FG.

   (iv)   Hence, prove that $\sin\theta + \cos\theta = \sqrt{2}\sin\left(\frac{\pi}{4}+\theta\right)$.   **2**

End of Paper

(b) (ii) (2 marks)
Outcomes Assessed: P3, P4
Targeted Performance Band: 3-5

Criteria	Marks
• Identifies $\angle BDC$ and correctly expresses the sine ratio	1
• Deduces the correct result	1

Sample Answer

$\angle BDC = \dfrac{\pi}{4}$ (diagonals of a square bisect the right angle)

$\therefore \sin\left(\dfrac{\pi}{4} + \theta\right) = \dfrac{BE}{15\sqrt{2}} \qquad \therefore BE = 15\sqrt{2} \times \sin\left(\dfrac{\pi}{4} + \theta\right) \dots \text{①}$

$\qquad\qquad\qquad$ as required.

(b) (iii) (3 marks)
Outcomes Assessed: P3, P4
Targeted Performance Band: 3-6

Criteria	Marks
• Correctly shows $AG = 15\cos\theta$	1
• Correctly shows $AF = 15\sin\theta$	1
• Finds the correct expression for FG	1

Sample Answer

In $\quad \triangle AGD, \cos\theta = \dfrac{AG}{15} \Rightarrow AG = 15\cos\theta \quad \dots \text{②}$

$\qquad \triangle AFB, \sin\theta = \dfrac{AF}{15} \Rightarrow AF = 15\sin\theta \quad \dots \text{③}$

②+③

$AG + AF = 15\sin\theta + 15\cos\theta$

$\therefore \ FG = 15(\sin\theta + \cos\theta) \qquad\qquad \dots \text{④}$

(b) (iv) (2 marks)
Outcomes Assessed: P3, P4
Targeted Performance Band: 2-4

Criteria	Marks
• Identifies $BE = FG$ and correctly equates suitable equations	1
• Deduces the correct result	1

Sample Answer
We know $BE = FG$ and from ① and ④

$\therefore 15(\sin\theta + \cos\theta) = 15\sqrt{2}\sin\left(\dfrac{\pi}{4} + \theta\right)$

$\therefore \ \sin\theta + \cos\theta = \sqrt{2}\sin\left(\dfrac{\pi}{4} + \theta\right)$ as required.

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CATHOLIC SECONDARY SCHOOLS ASSOCIATION
2007 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION
MATHEMATICS – SUGGESTED SOLUTIONS

These marking guidelines show the criteria to be applied to responses along with the marks to be awarded in line with the quality of responses. These guidelines are suggested and not prescriptive. This is not intended to be an exhaustive list but rather an indication of the considerations that students could include in their responses.

Question 1 (12 marks)
(a) (2 marks)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 2-3

Criteria	Marks
• Correctly expresses in scientific notation	1
• Correctly rounds to TWO significant figures.	1

Sample Answer

$25\,559\,000 = 2.5559 \times 10^7$

$\qquad\qquad = 2.6 \times 10^7$ m $\quad$ (2 significant figures)

(b) (2 marks)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 2-3

Criteria	Marks
• Correctly write the conjugate and expands to get the denominator OR the numerator correct	1
• Simplifies to give the correct answer	1

Sample Answer

$\sqrt{5} + 2 + \dfrac{1}{\sqrt{5}+2} \times \dfrac{\sqrt{5}-2}{\sqrt{5}-2}$

$= \sqrt{5} + 2 + \left(\dfrac{\sqrt{5}-2}{5-4}\right)$

$= \sqrt{5} + 2 + \sqrt{5} - 2$

$= 2\sqrt{5}$

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(c) (2 marks)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 2-3

Criteria	Marks
• Correctly expands the numerator, having found common denominator	1
• Simplifies	1

Sample Answer

$$\frac{(3a-1)}{4} - \frac{(a-2)}{3}$$

$$= \frac{3(3a-1)}{12} - \frac{4(a-2)}{12}$$

$$= \frac{9a-3-4a+8}{12}$$

$$= \frac{5a+5}{12}$$

$$= \frac{5(a+1)}{12}$$

(d) (2 marks)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 2-3

Criteria	Marks
• Correctly integrates 5	1
• Correctly integrates $\cos x$	1

Sample Answer
$$= 5x + \sin x + c$$

(e) (2 marks)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 2-3

Criteria	Marks
• Gives ONE correct answer	1
• Gives the second correct answer	1

Sample Answer

$$-3 \leq x-2 \leq 3$$
$$-1 \leq x \leq 5$$

(f) (2 marks)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 2-3

Criteria	Marks
• Correctly evaluates x or y	1
• Correctly evaluates both x and y	1

Sample Answer

$$6x + 2y = 12$$
$$6x - 2y = -8$$
adding both equations
$$12x = 4$$
$$\therefore x = \frac{1}{3}$$
$$2y = 10$$
$$\therefore y = 5 \qquad \therefore x = \frac{1}{3}, y = 5$$

Question 2 (12 marks)
(a)(i) (1 mark)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 2-3

Criteria	Mark
• Correctly finds the length of BC.	1

Sample Answer

$$BC = \sqrt{6^2 + 4^2}$$
$$= \sqrt{52}$$
$$= 2\sqrt{13}$$

(a)(ii) (1 mark)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 2-3

Criteria	Mark
• Correctly shows the gradient is $-\frac{5}{3}$	1

Sample Answer
$$m = \frac{4-(-1)}{3-6} = -\frac{5}{3}$$

(a) (iii) (1 mark)
Outcomes Assessed: P3, P4, P5
Targeted Performance Bands: 2-4

Criteria	Mark
• Correctly finds the value of the x co-ordinate of D	1

Sample Answer

$$-\frac{5}{3} = \frac{5}{x}$$
$$5x = -15$$
$$\therefore x = -3$$

(a) (iv) (2 marks)
Outcomes Assessed: P4, P5
Targeted Performance Bands: 3

Criteria	Marks
• Correctly determines the gradient of AD	1
• Correctly substitutes into the equation of a line to show $2x - 3y + 1 = 0$	1

Sample Answer

Gradient of $AD = \dfrac{4-0}{3+3} = \dfrac{4}{6} = \dfrac{2}{3}$

$$y - 4 = \frac{2}{3}(x-3)$$
$$3y - 12 = 2x - 6$$
$$\therefore 2x - 3y + 6 = 0$$

(a) (v) (2 marks)
Outcomes Assessed: P4, P5
Targeted Performance Bands: 2-4

Criteria	Marks
• Correctly substitutes into the formula	1
• Correctly determines the perpendicular distance in exact form.	1

Sample Answer

$$\perp \text{distance} = \frac{|2(0) - 3(-5) + 6|}{\sqrt{4+9}}$$
$$= \frac{21}{\sqrt{13}}$$

(a)(vi) (1 mark)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 2-3

Criteria	Mark
• Correctly determines the area of the quadrilateral $ABCD$	1

Sample Answer

$$A = bh$$
$$= 2\sqrt{13} \times \frac{21}{\sqrt{13}}$$
$$= 42u^2$$

(b) (ii) (2 marks)
Outcomes Assessed: P3, P4
Targeted Performance Band: 2-4

Criteria	Marks
• Correctly determines the value of $\angle LKM$	1
• Gives correct reasoning for the value of $\angle LKM$	1

Sample Answer

$\angle KML = \angle KLM = 42°$ (base angles of isosceles triangle)
$\angle LKM + 42 + 42 = 180$ (angle sum of triangle equals $180°$)
$\therefore \angle LKM = 96°$

(b) (iii) (2 marks)
Outcomes Assessed: P3, P4
Targeted Performance Band: 2-4

Criteria	Marks
• Correctly determines the value of $\angle MJN$.	1
• Gives correct reasoning for the value of $\angle MJN$.	1

Sample Answer

$\angle JKM = \angle KML$ (alternate angles in parallel lines, $JK \parallel ML$)

$\therefore \angle JKM = 42°$
$\angle JMK = \angle JKM$ (base angles of isosceles triangle)
$\therefore \angle MJN = 42 + 42 = 84°$ (exterior angle of a triangle is equal to the sum of the two interior opposite angles).

Question 3 (12 marks)

(a) (i) (2 marks)
Outcomes Assessed: P7, H5
Targeted Performance Band: 2-3

Criteria	Marks
• Correctly uses the chain rule of differentiation but has ONE mistake in calculation	1
• Correctly finds the answer	1

Sample Answer

$$\frac{dy}{dx} = 4(e^x + 3)^3 \cdot e^x$$
$$= 4e^x(e^x + 3)^3$$

(a) (ii) (2 marks)
Outcomes Assessed: P7, H5
Targeted Performance Band: 2-4

Criteria	Marks
• Correctly uses the quotient rule of differentiation but has ONE mistake in calculation	1
• Correctly finds the answer	1

Sample Answer

$$f'(x) = \frac{2x \tan x - x^2 \sec^2 x}{\tan^2 x}$$

(b) (i) (2 marks)
Outcomes Assessed: H3, H5
Targeted Performance Band: 2-4

Criteria	Marks
• Gives an correct logarithmic primitive but has ONE mistake e.g. $\log_e(x^3 - 5) + c$	1
• Finds the correct answer	1

Sample Answer

$$\frac{1}{3} \int \frac{3x^2}{x^3 - 5}$$
$$= \frac{1}{3} \log_e(x^3 - 5) + c$$

(b) (ii) (2 marks)
Outcomes Assessed: H3, H5
Targeted Performance Band: 3-4

Criteria	Marks
• Finds the primitive $-e^{-x}$ but has an error in calculating the integral	1
• Correctly applies the Newton-Leibnitz formula to obtain the correct answer in exact form	1

Sample Answer

$$\int_0^2 e^{-x} dx = \left[-e^{-x}\right]_0^2 - \left[\frac{1}{e^2} - 1\right] = 1 - \frac{1}{e^2} \text{ or } \frac{e^2 - 1}{e^2}$$

(c) (2 marks)
Outcomes Assessed: P3, H5
Targeted Performance Band: 2-4

Criteria	Marks
• Correctly substitutes into the cosine rule	1
• Correctly evaluates the answer, in exact form.	1

Sample Answer

$$p^2 = 8^2 + (3\sqrt{3})^2 - 2 \times 8 \times 3\sqrt{3} \times \cos\frac{\pi}{6}$$
$$= 64 + 27 - 2 \times 3\sqrt{3} \times 8 \times \frac{\sqrt{3}}{2}$$
$$= 19$$
$$\therefore p = \sqrt{19} \text{ cm}$$

(d) (2 marks)
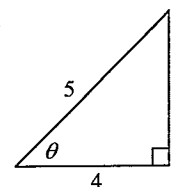*Outcomes Assessed: P4, H5*
Targeted Performance Band: 3-4

Criteria	Marks
• Determines the correct ratio, using right angled triangles	1
• Correctly determines the quadrant in which $\cos\theta$ lies.	1

Sample Answer

By Pythagoras' theorem, $BC = 4$
The angle lies in the second quadrant.
$$\therefore \cos\theta = -\frac{4}{5}$$

Question 4 (12 marks)

(a) (2 marks)
Outcomes Assessed: H5
Targeted Performance Band: 3-4

Criteria	Marks
• Writes the limiting sum formula with correct a and S_∞	1
• Gives the correct answer	1

Sample Answer

$$S_\infty = \frac{a}{1-r}$$

$$24 = \frac{15}{1-r}$$

$$24(1-r) = 15$$

$$r = \frac{3}{8}$$

(b) (i) (2 marks)
Outcomes Assessed: P2, H2
Targeted Performance Band: 2-4

Criteria	Marks
• Shows that $\angle ACE = \angle CED$ (alternate $\angle's$, AC $\parallel$ ED) or equivalent	1
• Correctly completes proof using equiangular triangles	1

Sample Answer

In triangles ABC and DCE

$\angle ACB = \angle DEC$	(alternate $\angle's$, AC $\parallel$ ED)
$\angle ABC = \angle DCE$	(alternate $\angle's$, AB $\parallel$ CD)
$\therefore \angle BAC = \angle CDE$	(third angle)

$\therefore \triangle ABC$ is similar to $\triangle DCE$ (equiangular)

(b) (ii) (2 marks)
Outcomes Assessed: P2, H2
Targeted Performance Band: 3-4

Criteria	Marks
• Gives correct proportion statement $\dfrac{EC}{BC} = \dfrac{8}{22}$	1
• Gives the correct answer	1

Sample Answer

$$\frac{CE}{BC} = \frac{DC}{AB}$$

$$\frac{CE}{12+CE} = \frac{8}{22}$$

Let CE be represented by x

$$22x = 8(12+x)$$

$$x = 6\frac{6}{7}$$

Length of $BC = 12 + x$, $\therefore BC = 18\frac{6}{7}$ cm

(c) (2 marks)
Outcomes Assessed: P6, H3, H5
Targeted Performance Band: 2-4

Criteria	Marks
• Correctly finds the gradient of the normal	1
• Correctly substitutes the values for x and y into the point/gradient formula to find the equation of the normal	1

Sample Answer

$$y = e^{\cos x} \therefore \frac{dy}{dx} = -(\sin x)e^{\cos x} \rightarrow \text{At } x = \frac{\pi}{2}, \frac{dy}{dx} = -(\sin \frac{\pi}{2})e^{\cos \frac{\pi}{2}} = -1 \therefore m_T = -1 \therefore m_N = 1$$

At $x = \frac{\pi}{2}$, $y = 1 \therefore (\frac{\pi}{2}, 1)$ is the point on the curve

$\therefore$ equation of normal is given by $y - 1 = 1(x - \frac{\pi}{2})$

(d) (2 marks)
Outcomes Assessed: P4, H5
Targeted Performance Band: 2-4

Criteria	Marks
• Gives ONE correct answer in radians	1
• Gives TWO correct answers in radians.	1

Sample Answer

$$\sqrt{3}\tan\theta+1=0 \qquad \therefore \tan\theta=\frac{-1}{\sqrt{3}}$$

Basic angle is $\dfrac{\pi}{6}$ (First Quadrant).

$$\therefore \theta=\frac{5\pi}{6},\frac{11\pi}{6} \qquad\qquad 0\le\theta\le2\pi$$

(e) (2 marks)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 2-3

Criteria	Marks
• Correctly writes down the sum **and** product of the roots	1
• Gives the correct answer.	1

Sample Answer

$$3x^2+9x+1=0 \ , \ \alpha+\beta=-3 \ \text{ and } \ \alpha\beta=\frac{1}{3}$$

$$4\alpha\beta^2+4\alpha^2\beta=4\alpha\beta(\beta+\alpha)$$
$$=4\times\frac{1}{3}\times(-3)$$
$$=-4$$

Question 5 (12 marks)
(a) (i) (3 marks)
Outcomes Assessed: P7, H6
Targeted Performance Band: 3-5

Criteria	Marks
• Finds the stationary points	1
• Finds the nature of ONE stationary point	1
• Finds the nature of the other stationary point	1

Sample Answer

$$f(x)=x^3-x^2-8x-3 \ \therefore f'(x)=3x^2-2x-8$$

For stationary points $f'(x)=0 \quad \therefore (3x+4)(x-2)=0 \quad \therefore x=-\frac{4}{3} \ \text{ or } \ x=2$

$\therefore$ the stationary points are $\left(-\frac{4}{3},3\frac{14}{27}\right) \ \& \ (2,-7)$

Also for the nature of the stationary points, $f''(x)=6x-2$

At $x=-\frac{4}{3}$, $f''(-\frac{4}{3})=-10<0 \ \therefore \left(-\frac{4}{3},3\frac{14}{27}\right)$ is a MAXIMUM stationary point

At $x=2$, $f''(2)=10>0 \ \therefore (2,-7)$ is a MINIMUM stationary point

(a) (ii) (2 marks)
Outcomes Assessed: P6, H6, H7, H9
Targeted Performance Band: 3-5

Criteria	Marks
• Draws the correct cubic curve	1
• Plots all important points	1

Sample Answer

$$f(x)=x^3-x^2-8x-3$$
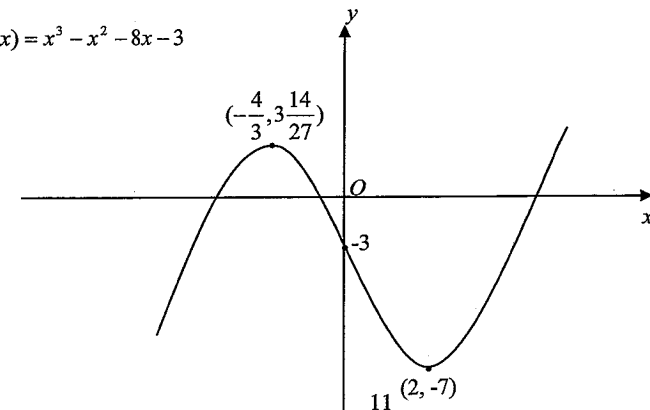
(a) (iii) (1 mark)
Outcomes Assessed: P6, H6, H7
Targeted Performance Band: 2-4

Criteria	Mark
• Correctly solves the equation $f'(x) < 0$ or gives correct answer from graph.	1

Sample Answer

$$\therefore f'(x) = 3x^2 - 2x - 8$$

For the curve to be decreasing, $f'(x) < 0$ $\therefore (3x+4)(x-2)<0$

$$\therefore -\frac{4}{3} < x < 2$$

(b) (i) (1 mark)
Outcomes Assessed: H5
Targeted Performance Band: 3-4

Criteria	Mark
• Gives the correct answer	1

Sample Answer $P(S) = \frac{1}{2} \times \frac{4}{11} \times \frac{3}{10} = \frac{3}{55}$

(b) (ii) (2 marks)
Outcomes Assessed: H5
Targeted Performance Band: 3-4

Criteria	Marks
• Uses the complementary events method (or otherwise)	1
• Gives the correct answer with required working	1

Sample Answer

$$P(S) = 1 - \left[\frac{1}{2} \times \frac{7}{11} \times \frac{6}{10}\right] = 1 - \left(\frac{21}{110}\right) = \frac{89}{110}$$

(c) (i) (1 mark)
Outcomes Assessed: P4
Targeted Performance Bands: 2-3

Criteria	Mark
• Correctly substitutes $(2, 16)$ into both functions	1

Sample Answer

$$y = 2x^3 \Rightarrow 16 = 2(2)^3 \text{ and } y = 20 - 2x \Rightarrow 16 = 20 - 2(2)$$

(c) (ii) (2 marks)
Outcomes Assessed: H8
Targeted Performance Bands: 2-4

Criteria	Marks
• Correctly uses the sum of integrals to find area under curves and their primitives	1
• Correctly finds the area using Leibnitz – Newton Formula	1

Sample Answer

$$A_{POQ} = \int_0^2 2x^3\,dx + \int_2^{10}(20-2x)\,dx = \frac{1}{2}\left[x^4\right]_0^2 + \left[20x - x^2\right]_2^{10}$$

$$\therefore A_{POQ} = \frac{1}{2}(16-0) + \left[\left(20 \times 10 - 10^2\right) - (40-4)\right] = 8 + 64 = 72 \text{ square units}$$

Question 6 (12 marks)
(a) (2 marks)
Outcomes Assessed: P3, P4, H5
Targeted Performance Band: 2-4

Criteria	Marks
• Writes correct series of FIVE terms	1
• Gives the correct answer using series or otherwise	1

Sample Answer

$$\sum_{n=0}^{4} w^{\frac{1}{2}} = 0^{\frac{1}{2}} + 1^{\frac{1}{2}} + 2^{\frac{1}{2}} + 3^{\frac{1}{2}} + 4^{\frac{1}{2}} = 0 + 1 + \sqrt{2} + \sqrt{3} + 2 = 3 + \sqrt{2} + \sqrt{3}$$

(b) (i) (1 mark)
Outcomes Assessed: P4, H5
Targeted Performance Band: 2-4

Criteria	Mark
• Gives the correct answer	1

Sample Answer

$R_6 = 53$

$R_7 = 115$

$\therefore n = 7, R = 115$

(b) (ii) (2 marks)
Outcomes Assessed: P4, H5
Targeted Performance Band: 2-4

Criteria	Marks
• Correctly finds the sum of ONE of the series	1
• Gives the correct answer	1

Sample Answer
$1+1+3+9+23+... = (-1+2)+(-3+4)+(-5+8)+...(-49+33\,554\,432)$

$$AP: S_{25} = \frac{25}{2}(-1+-49) = -625$$

$$GP: S_{25} = \frac{2(2^{25}-1)}{2-1} = 67\,108\,862$$

After 25 mins we have $-625 + 67\,108\,862 = 67\,108\,237$ rain drops

(b) (iii) (1 mark)
Outcomes Assessed: P4, H5
Targeted Performance Band: 2-3

Criteria	Mark
• Gives the correct answer	1

Sample Answer
Raindrops per $cm^2 = 67\,108\,237 \div 2\,500\,000 = 26.8$ drops/cm^2

(c) (i) (1 mark)
Outcomes Assessed: H3
Targeted Performance Bands: 2-3

Criteria	Mark
• Correctly completes the required value ($x^2 \log_e x = 2.773$)	1

Sample Answer

x	2	2.5	3	3.5	4
$x^2 \log_e x$	2.773	5.727	9.888	15.346	22.181

(c) (ii) (2 marks)
Outcomes Assessed: H3, H5
Targeted Performance Bands: 2-4

Criteria	Marks
• Substitutes the correct values in the correct Simpson's formula	1
• Correctly calculates the answer in decimal form (e.g. 21.504)	1

Sample Answer
Using Simpson's Formula: $\int_a^b y\,dx \approx \frac{h}{3}\left[y_0 + 4(y_1 + y_3 + ...) + 2(y_2 + y_4 + ...) + y_n\right]$

$$\int_2^4 x^2 \log_e x\,dx = \frac{0.5}{3}\left[2.773 + 4(5.727 + 15.346) + 2 \times 9.888 + 22.181\right]$$

$$\therefore \int_2^4 x^2 \log_e x\,dx \cong 21.504$$

(d) (i) (1 mark)
Outcomes Assessed: H4
Targeted Performance Bands: 2-3

Criteria	Mark
• Correctly gives answer for $\angle AOB$ in radians	1

Sample Answer
$l = r\theta$
$22.5 = 12.5 \times \theta \Rightarrow \theta = 1.8$ radians

(d) (ii) (2 marks)
Outcomes Assessed: H4
Targeted Performance Bands: 2-4

Criteria	Marks
• Correctly substitutes θ and r into segment formula	1
• Gives correct answer	1

Sample Answer
$$A = \frac{1}{2}\theta(r_1^2 - r_2^2) = \frac{1}{2}(1.8)(9^2 - 3.5^2) = 61.875 \text{ cm}^2$$

Question 7 (12 marks)

(a) (i) (1 mark)
Outcomes Assessed: H3, H4
Targeted Performance Band: 3-4

Criteria	Mark
• Finds the correct solution	1

Sample Answer

$$P = 10^6 \times e^{kt}$$

$$\frac{dP}{dt} = k(10^6 \times e^{kt})$$

$$= kP$$

(a) (ii) (2 marks)
Outcomes Assessed: H3, H4 , H5.
Targeted Performance Band: 3-4

Criteria	Marks
• Arrives at the expression $1.7 = e^{10k}$	1
• Finds the correct value of k	1

Sample Answer

$$1.7 \times 10^6 = 10^6 \times e^{10k}$$

$$\ln(1.7) = \ln(e^{10k})$$

$$10k = \ln(1.7)$$

$$k = 0.05306 = 0.053$$

(a) (iii) (2 marks)
Outcomes Assessed: H3, H4
Targeted Performance Band: 2-4

Criteria	Marks
• Arrives at the expression $P = 10^6 \times e^{27(0.053)}$	1
• Finds the correct rounded value of P	1

Sample Answer

$$t = 27$$

$$P = 10^6 \times e^{27(0.053)}$$

$$P = 4\,182\,879.98 = 4\,180\,000 = 4.18\,\text{million}$$

(a) (iv) (2 marks)
Outcomes Assessed: H3, H4
Targeted Performance Band: 2-4

Criteria	Marks
• Arrives at the expression $8.5 = e^{0.053t}$	1
• Finds the correct answer	1

Sample Answer

$$8.5 \times 10^6 = 10^6 \times e^{kt}$$

$$\ln(8.5) = \ln(e^{kt})$$

$$\ln(8.5) = kt$$

$$t = 40.3$$

$$\therefore \text{Year 2020}$$

(b) (i) (1 mark)
Outcomes Assessed: H4, H5
Targeted Performance Band: 2-3

Criteria	Mark
• Finds the correct answer	1

Sample Answer

$$A_1 = P(1.005) - 4000$$

(b) (ii) (2 marks)
Outcomes Assessed: H4, H5
Targeted Performance Band: 3-5

Criteria	Marks
• Finds an expression for A_2 with working	1
• Correctly arrives at A_3 from generalising A_2	1

Sample Answer

$$A_1 = P \times (1.005) - 4000$$

$$A_2 = (A_1 \times 1.005) - 4000 = [(P \times 1.005) - 4000] \times 1.005 - 4000$$

$$= P(1.005)^2 - 4000(1 + 1.005)$$

$$A_3 = (A_2 \times 1.005) - 4000 = [P(1.005)^2 - 4000(1 + 1.005)] \times 1.005 - 4000$$

$$= P(1.005)^3 - 4000(1 + 1.005 + 1.005^2)$$

(b) (iii) (2 marks)
Outcomes Assessed: H4, H5
Targeted Performance Band: 3-5

Criteria	Marks
• Writes a numerical solution to A_{36} and equates $A_{36} = 0$	1
• Finds the correct answer	1

Sample Answer

Continuing the pattern,

$A_4 = A_3 \times 1.0075 - 4800$

$A_5 = [A_3(1.0075) - 4800]1.0075 - 4800 = A_3(1.0075)^2 - 4800(1 + 1.0075)$

$A_6 = [A_3(1.0075)^3 - 4800(1 + 1.0075 + 1.0075^2)$

...

$A_{36} = A_3(1.0075)^{33} - 4800(1 + 1.0075 + 1.0075^2 + ... + 1.0075^{32})$

but $A_{36} = 0$ as the loan is repaid in full

$$\therefore 0 = A_3(1.0075)^{33} - 4800\left[\frac{1(1.0075^{33} - 1)}{1.0075 - 1}\right]$$

$$\therefore A_3 = \frac{640000(1.0075^{33} - 1)}{1.0075^{33}} \text{ but } A_3 = P(1.005)^3 - 12060.10$$

$$\therefore P(1.005)^3 = \frac{640000(1.0075^{33} - 1)}{1.0075^{33}} + 12060.10$$

$$P = \$149\,662.11$$

Question 8 (12 marks)
(a) (1 mark)
Outcomes Assessed: H5
Targeted Performance Band: 2-3

Criteria	Mark
• Finds the correct answer	1

Sample Answer

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

(b) (i) (1 mark)
Outcomes Assessed: H4, H5
Targeted Performance Band: 2-3

Criteria	Mark
• Finds the correct answer	1

Sample Answer

$x = 16 - 4\log_e(4 - 1) + 5$

$\quad = 21 - 4\ln 3$

(b) (ii) (2 marks)
Outcomes Assessed: H4, H5
Targeted Performance Band: 3-5

Criteria	Marks
• Finds an equation for v	1
• Finds the correct answer	1

Sample Answer

$$v = 2t - \frac{4}{t-1} = 0$$

$2t^2 - 2t - 4 = 0$

$(t-2)(t+1) = 0$

$\therefore t = 2$

(b) (iii) (2 marks)
Outcomes Assessed: H4, H5
Targeted Performance Band: 3-4

Criteria	Marks
• Finds an equation for a	1
• Correctly shows $a > 0$ for $t > 1$	1

Sample Answer

$$a = 2 + \frac{4}{(t-1)^2}$$

For $t > 1$, $\frac{4}{(t-1)^2} > 0 \quad \therefore a > 0$

(b) (iv) (2 marks)
Outcomes Assessed: H4, H5
Targeted Performance Band: 3-5

Criteria	Marks
• Finds an expression that defines the distance travelled	1
• Finds the correct answer	1

Sample Answer

$$\int_2^4 2t - \frac{4}{t-1} \, dt = \left[t^2 - 4\log_e(t-1)\right]_2^4$$

$$= (16 - 4\ln 3) - (4 - 4\ln 1)$$

$$= 12 - 4\ln 3$$

(c) (i) (2 marks)
Outcomes Assessed: H3, H8
Targeted Performance Band: 3-4

Criteria	Marks
• Correctly makes x the subject	1
• Correctly substitutes $x^2 = \left(1-\frac{1}{y}\right)^2$ in the volume formula and deduces the answer	1

Sample Answer

$$V_y = \pi \int_1^2 x^2 \, dy \; ; \; x = 1-\frac{1}{y} \therefore x^2 = \left(1-\frac{1}{y}\right)^2$$

$$V_y = \pi \int_1^2 \left(1-\frac{1}{y}\right)^2 dy = \pi \int_1^2 \left(1-\frac{2}{y}+\frac{1}{y^2}\right) dy$$

(c) (ii) (2 marks)
Outcomes Assessed: H3, H5
Targeted Performance Band: 2-4

Criteria	Marks
• Finds the correct primitive	1
• Finds the correct volume using Leibnitz-Newton Formula	1

Sample Answer

$$V_y = \pi \left[y - 2\ln y - \frac{1}{y} \right]_1^2$$

$$= \pi \left[\left(2 - 2\ln 2 - \frac{1}{2}\right) - (1 - 2\ln 1 - 1) \right]$$

$$\cong 0.357 \text{ units}^3$$

Question 9 (12 marks)
(a) (i) (1 mark)
Outcomes Assessed: P3, P4
Targeted Performance Band: 2-3

Criteria	Mark
• Finds the correct answer	1

Sample Answer

$$g(-x) = \frac{2}{(-x)^2 - 1} = \frac{2}{x^2 - 1} = g(x) \quad \therefore \text{even function}$$

20

(a) (ii) (1 mark)
Outcomes Assessed: P3, P4
Targeted Performance Band: 2-3

Criteria	Mark
• Finds the correct answer	1

Sample Answer
Domain: All real x: $x \neq \pm 1$

(b) (i) (1 mark)
Outcomes Assessed: H5
Targeted Performance Band: 3-5

Criteria	Mark
• Finds the correct answer	1

Sample Answer
$y = 1 - 3\cos 2x \quad \therefore$ Amplitude = 3

(b) (ii) (2 marks)
Outcomes Assessed: P4, H5
Targeted Performance Band: 2-4

Criteria	Marks
• Draws a negative cosine curve	1
• Draws the correct graph that includes amplitude and period	1

Sample Answer
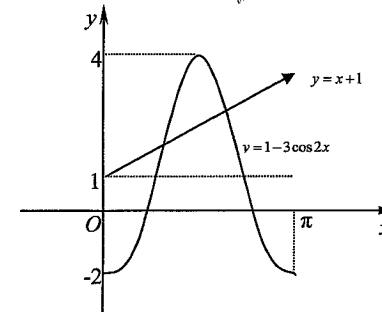
(b) (iii) (2 marks)
Outcomes Assessed: P4, H5
Targeted Performance Band: 2-4

Criteria	Marks
• Draws the straight line $y = x+1$	1
• Finds the correct number of solutions	1

Sample Answer
See graph above for $y = x+1$
From the graph, $x + 3\cos 2x = 0$ has 2 solutions

21

(c) (i) (2 marks)
Outcomes Assessed: P3
Targeted Performance Band: 2-4

Criteria	Marks
• Correctly finds the distances *SX*, *XY* and *YF*	1
• Links all three expressions to give the correct answer	1

Sample Answer

$$SX = \sqrt{a^2 + 4} \quad \therefore T = \frac{D}{S} = \frac{\sqrt{a^2+4}}{6} \text{ hours}$$

$$XY = 4 - a \quad \therefore T = \frac{4-a}{12} \text{ hours}$$

$$YF = \frac{2\pi(2)}{4} = \pi \quad \therefore T = \frac{\pi}{8} \text{ hours}$$

$$\Rightarrow \therefore T = \frac{\sqrt{a^2+4}}{6} + \frac{4-a}{12} + \frac{\pi}{8}$$

$$T = \frac{4\sqrt{a^2+4} - 2a + (8 + 3\pi)}{24}$$

(c) (ii) (3 marks)
Outcomes Assessed: P3
Targeted Performance Band: 2-4

Criteria	Marks
• Correctly finds the derivative of *T* and equates to zero to find *a*	2
• Correctly shows the value of *a* is a minimum	1

Sample Answer

$$\frac{dT}{da} = \frac{2(a^2+4)^{-\frac{1}{2}} \times 2a - 2}{24} = 0 \text{ for stationary points}$$

$$4a = 2\sqrt{a^2 + 4}$$

$$4a^2 = a^2 + 4$$

$$a = \frac{2}{\sqrt{3}}$$

a	$\dfrac{2}{\sqrt{3}}^{-}$	$\dfrac{2}{\sqrt{3}}$	$\dfrac{2}{\sqrt{3}}^{+}$
$\dfrac{dT}{da}$	–	0	+

$\therefore$ a minimum time occur

when $a = \dfrac{2}{\sqrt{3}}$

_____ MINIMUM _____/

Question 10 (12 marks)
(a) (i) (2 marks)
Outcomes Assessed: H4, H5
Targeted Performance Band: 3-4

Criteria	Marks
• Finds the correct primitive expression with constant *C*	1
• Finds the correct solution	1

Sample Answer

$$V = \frac{-bt^2}{2} + C \text{ when } t = 0 , V = 250\,000$$

$$250\,000 = 0 + C$$

$$\therefore V = 250\,000 - \frac{1}{2}bt^2$$

(a) (ii) (2 marks)
Outcomes Assessed: H4, H5, H6
Targeted Performance Band: 3-5

Criteria	Marks
• Finds the correct time for $V = 85\,000$	1
• Finds the correct answer	1

Sample Answer

$$85\,000 = 250\,000 - \frac{1}{2}(0.431)\,t^2$$

$$\frac{1}{2}t^2(0.431) = 165\,000$$

$$t^2 = 765\,661.25$$

$$t = 875.02$$

$$\therefore \frac{dV}{dt} = -0.431 \times 875.02$$

$$= -377.1 \text{ litres per minute}$$

(b) (i) (1 mark)
Outcomes Assessed: P2, P3
Targeted Performance Band: 2-3

Criteria	Mark
• Finds the correct answer	1

Sample Answer
By Pythagoras,

$$BD^2 = 15^2 + 15^2 = 450$$

$$\therefore BD = 15\sqrt{2}$$