Centre Number

Student Number

CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NEW SOUTH WALES

2009
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics
Extension 1

Afternoon Session
Thursday, 20 August 2009

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks – 84
- Attempt Questions 1-7
- All questions are of equal value

6300 - 1

Total marks – 84
Attempt Questions 1–7
All questions are of equal value

Answer each question in a SEPARATE writing booklet.

Marks

Question 1 (12 marks) Use a SEPARATE writing booklet.

(a) Find the remainder when $P(x)=x^3-3x^2+3x-5$ is divided by $x-2$. **2**

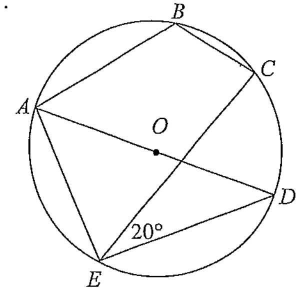
(b) Find $\int \sin^2 6x\,dx$. **2**

(c) Sketch the graph of $y=3\sin^{-1}(2x)$, clearly indicating the domain and range. **3**

(d) (i) Find the Cartesian equation of the curve with parametric equations $x=\cos t$ and $y=3+\sin t$. **2**

(ii) Describe this locus geometrically. **1**

(e) In the diagram AOD and EC are straight lines, O is the centre of the circle, and $\angle CED=20^\circ$. **2**



NOT TO SCALE

Find $\angle ABC$, giving reasons for your answer.

Marks

Question 2 (12 marks) Use a SEPARATE writing booklet.

(a) Find $\lim\limits_{x \to 0} \dfrac{\sin 3x}{x}$. **1**

(b) Use the substitution $u = 3x - 1$ to evaluate $\displaystyle\int_1^2 \frac{x}{3x-1}\,dx$. **3**

(c) Find all real numbers such that $\ln(2x+3) + \ln(x-2) = 2\ln(x+4)$. **4**

(d) (i) From a group of 7 girls and 6 boys, 3 girls and 2 boys are chosen. How many different groups of 5 are possible? **2**

(ii) If the group of 5 stands in a line what is the probability that the boys stand together? **2**

Marks

Question 3 (12 marks) Use a SEPARATE writing booklet.

(a) Solve the inequality $\dfrac{x^2 - 4}{x+3} < x - 4$ for x. **3**

(b) Prove by Mathematical Induction that $3^{3n} + 2^{n+2}$ is divisible by 5, for all positive integers n. **4**

(c) A particle, P, moves on the x-axis for time $t \geq 0$, in seconds, with velocity $v = \dfrac{2}{1+3x}$ cms^{-1}, where x, in centimetres, is the displacement from the origin $x = 0$.

(i) Find an expression for the acceleration, a cms^{-2}, and show that a varies directly with v^3. **3**

(ii) If the particle was initially at the origin, describe the motion both initially and as $t \to \infty$. **2**

Marks

Question 4 (12 marks) Use a SEPARATE writing booklet.

(a) The function $f(x) = e^x - x - 2$ has a zero near $x = 1.2$. **2**

Use one application of Newton's method to find a second approximation to the zero. Write your answer correct to three significant figures.

(b) A function is defined by $f(x) = e^{3x} - 1$ for all real x.

(i) Draw the graph of $y = f(x)$ and state the range of the function. **2**

(ii) Find the inverse function, $f^{-1}(x)$, clearly indicating any restrictions. **3**

(c) A particle moves in a straight line so that its displacement x cm from the origin at time $t \geq 0$, in seconds, is given by $x = \sqrt{3}\cos 3t - \sin 3t$.

(i) Show that the particle moves in simple harmonic motion. **2**

(ii) Find the velocity when the particle is 1 cm from the origin on its first oscillation. **3**

Marks

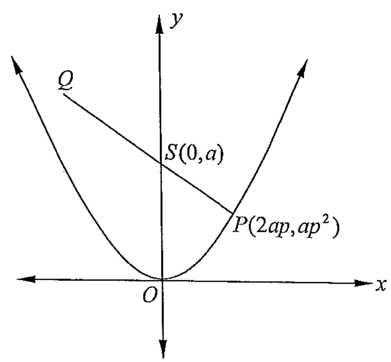
Question 5 (12 marks) Use a SEPARATE writing booklet.

(a) (i) If the roots of $x^3 - 6x^2 + 3x + k = 0$ are consecutive terms of an arithmetic series show that one of the roots is 2. **2**

(ii) Hence find the value of k and the other two roots. **3**

(b) Show that $\dfrac{\tan 2\theta - \tan\theta}{\tan 2\theta + \cot\theta} = \tan^2\theta$. **3**

(c) $P(2ap,\ ap^2)$ is a point on the parabola $x^2 = 4ay$ with focus $S(0,\ a)$. The point Q lies on PS produced and Q divides PS so that $PQ:QS = -4:3$.



NOT TO SCALE

(i) Show that Q has coordinates $\left(-6ap,\ a(4-3p^2)\right)$. **2**

(ii) Show that as P varies, the locus of Q is a parabola. **2**

Marks

Question 6 (12 marks) Use a SEPARATE writing booklet.

(a) Simplify $\dfrac{2^{4n}\times 3^{2n}}{8^{n}\times 6^{n}}+3^{n}$. **3**

(b) A balloon in the shape of a cylinder, with height h and radius r, expands so that h is always proportional to r, that is $h=kr$ for some constant k.
When $r=4\,\text{cm}$, the volume is expanding at the rate of $0.2\,\text{cm}^3\,\text{s}^{-1}$.

(i) Show that when $r=4\,\text{cm}$ the rate of change of the radius is given by $\dfrac{dr}{dt}=\dfrac{1}{240\pi k}$. **2**

(ii) If the surface area of the balloon is expanding at the rate of $0.1\,\text{cm}^2\,\text{s}^{-1}$ when $r=4\,\text{cm}$, find the constant of proportionality, k. **3**

(c) (i) Differentiate both sides of the expansion $(1+x)^{2n}=\sum_{k=0}^{2n}{}^{2n}C_k\,x^k$. **2**

(ii) Hence show that $\sum_{k=1}^{2n}k\,{}^{2n}C_k=n\times 4^n$. **2**

Marks

Question 7 (12 marks) Use a SEPARATE writing booklet.

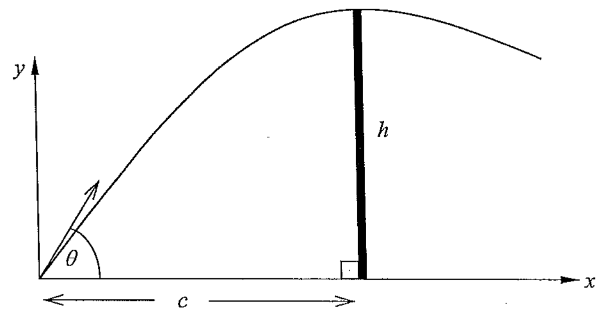
(a) A student is taking a test with 50 multiple-choice questions and guesses the answer to each one. The probability of guessing a question correctly is 0.3.

(i) What is the probability that the student answers 25 questions correctly? **2**

(ii) What is the most likely number of questions answered correctly? **3**

(b) A vertical wall, height h metres, stands on horizontal ground. When a projectile is fired, in a vertical plane which is at right angles to the wall, from a point on the ground c metres from the wall, it just clears the wall at the highest point of its path.
The equations of motion for the projectile with angle of projection, θ, are:

$x=Vt\cos\theta \qquad y=Vt\sin\theta-\frac{1}{2}gt^2$ (Do not prove these.)



NOT TO SCALE

(i) Show that the particle reaches the highest point on its path when $t=\dfrac{V\sin\theta}{g}$. **2**

(ii) Show that the speed of projection is given by $V^2=\dfrac{g}{2h}\left(4h^2+c^2\right)$. **3**

(iii) Find the angle of projection, θ, in terms of h and c. **2**

End of paper

CATHOLIC SECONDARY SCHOOLS ASSOCIATION

2009 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

MATHEMATICS EXTENSION 1

Question 1 (12 marks)

(a) (2 marks)

Outcomes assessed: PE3

Targeted Performance Bands: E2-E3

Criteria	Marks
• applies the Remainder Theorem or equivalent progress towards solution	1
• finds correct remainder	1

Sample Answer:

$P(x) = x^3 - 3x^2 + 3x - 5$

By the Remainder Theorem $P(2) = \text{remainder}$

$\therefore \text{remainder} = 8 - 12 + 6 - 5$

$= -3$

OR

Correct division of polynomial.

(b) (2 marks)

Outcomes assessed: HE6, HE7

Targeted Performance Bands: E2-E3

Criteria	Marks
• correct trigonometric substitution in integral	1
• finds a correct primitive (+C not necessary)	1

Sample Answer:

$$\int \sin^2 6x\,dx = \frac{1}{2}\int (1-\cos 12x)\,dx$$
$$= \frac{1}{2}\left(x - \frac{1}{12}\sin 12x\right) + C$$
$$= \frac{x}{2} - \frac{\sin 12x}{24} + C$$

(c) (3 marks)

Outcomes assessed: HE4

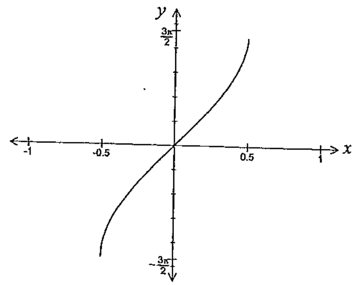
Targeted Performance Bands: E2-E3

Criteria	Marks
• draws correctly shaped graph	1
• identifies correct domain	1
• identifies correct range	1

Sample Answer:

$y = 3\sin^{-1}(2x)$

domain: $\frac{-1}{2} \le x \le \frac{1}{2}$ range: $\frac{-3\pi}{2} \le y \le \frac{3\pi}{2}$



(d) (i) (2 marks)

Outcomes assessed: PE3

Targeted Performance Bands: E2-E3

Criteria	Marks
• uses correct trigonometric identity	1
• substitutes correctly and determines correct equation	1

Sample Answer:

$x = \cos t$

$y = 3 + \sin t \quad \Rightarrow \quad \sin t = y - 3$

substitute into $\cos^2 t + \sin^2 t = 1$

$x^2 + (y-3)^2 = 1$

(d) (ii) (1 mark)

Outcomes assessed: PE3

Targeted Performance Bands: E2-E3

Criteria	Marks
• correctly describes locus	1

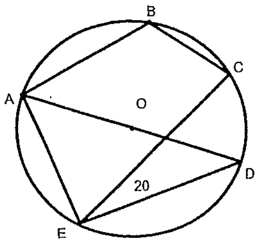
Sample Answer:

Geometrically the locus is a circle with centre (0, 3) and radius 1.

(e) (2 marks)
Outcomes assessed: PE2, PE3
Targeted Performance Bands: E2-E3

Criteria	Marks
• finds $\angle AED$, giving correct reason	1
• finds $\angle ABC$, giving correct reason	1

Sample Answer:



$\angle AED = 90^0$ (angle in a semicircle, AD is a diameter)

$\therefore \angle AEC = 70^0$

$\angle ABC = 110^0$ (opposite angles of cyclic quadrilateral $ABCE$ are supplementary)

Question 2 (12 marks)
(a) (1 mark)
Outcomes assessed: PE2
Targeted Performance Bands: E2-E3

Criteria	Marks
• gives correct result	1

Sample Answer:

$$\lim_{x\to 0}\frac{\sin 3x}{x} = 3\lim_{x\to 0}\frac{\sin 3x}{3x}$$
$$= 3\times 1 \qquad \text{using } \lim_{x\to 0}\tfrac{\sin x}{x} = 1$$
$$= 3$$

(b) (3 marks)
Outcomes assessed: HE6
Targeted Performance Bands: E2-E3

Criteria	Mark
• rewrites the integral using the substitution	1
• finds the new limits	1
• evaluates the integral correctly (correct numerical equivalence)	1

Sample Answer:

$$\int_1^2 \frac{x}{3x-1}\,dx = \frac{1}{9}\int_1^2 \frac{3x}{3x-1}\times 3dx$$
$$= \frac{1}{9}\int_2^5 \frac{u+1}{u}\,du$$
$$= \frac{1}{9}\int_2^5 \left(1+\frac{1}{u}\right)du$$
$$= \frac{1}{9}\left[u+\ln u\right]_2^5$$
$$= \frac{1}{9}\left[5+\ln 5-(2+\ln 2)\right]$$
$$= \frac{1}{9}\left(3+\ln\tfrac{5}{2}\right)$$
$$= \tfrac{1}{3}+\tfrac{1}{9}\ln\tfrac{5}{2}$$

$u = 3x-1 \qquad 3x = u+1$

$\dfrac{du}{dx} = 3$

Limits $\quad x=2 \Rightarrow u=5$

$\qquad x=1 \Rightarrow u=2$

(c) (4 marks)
Outcomes assessed: HE7
Targeted Performance Bands: E2-E3

Criteria	Mark
• uses logarithmic laws	1
• establishes the quadratic equation	1
• solves the quadratic equation	1
• gives correct solution	1

Sample Answer

$$\ln(2x+3)+\ln(x-2) = 2\ln(x+4)$$
for valid solutions $x > 2$
$$\ln(2x+3)(x-2) = \ln(x+4)^2$$
$$2x^2 - x - 6 = x^2 + 8x + 16$$
$$x^2 - 9x - 22 = 0$$
$$(x+2)(x-11) = 0$$
$$\therefore x = -2 \text{ or } x = 11$$

but $x = -2$ is not valid $\therefore x = 11$ is the only solution

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(d) (i) (2 marks)
Outcomes assessed: PE3
Targeted Performance Bands: E2-E3

Criteria	Mark
• uses combinations correctly or significant progress towards answer	1
• gives correct answer	1

Sample Answer:

Girls can be selected in $^7C_3 = 35$ ways

Boys can be selected in $^6C_2 = 15$ ways

There are $^7C_3 \times {^6C_2} = 525$ groups of 5.

(d) (ii) (2 marks)
Outcomes assessed: PE3
Targeted Performance Bands: E2-E3

Criteria	Marks
• calculates the number of ways that the boys can stand together	1
• finds the correct probability	1

Sample Answer:

If the boys stand together then there are $2! = 2$ ways to arrange themselves.
In the line there are 3 girls and the group of boys to be arranged $\Rightarrow 4! = 24$ arrangements.
$\therefore 2! \times 4! = 48$ ways of the boys standing together in the line.

If no restrictions the 5 can be arranged in $5! = 120$ ways in a line.

$$\text{P(boys stand together)} = \frac{48}{120} = \frac{2}{5}.$$

Question 3 (12 marks)
(a) (3 marks)
Outcomes assessed: PE3
Targeted Performance Bands: E3-E4

Criteria	Marks
• establishes correct quadratic or other correct significant step towards solution	1
• further significant step towards solution	1
• finds solution	1

Sample Answer:

$$\frac{x^2-4}{x+3} < x-4 \qquad \times (x+3)^2 \qquad x \neq -3$$

$$(x+3)(x^2-4) < (x-4)(x+3)^2$$

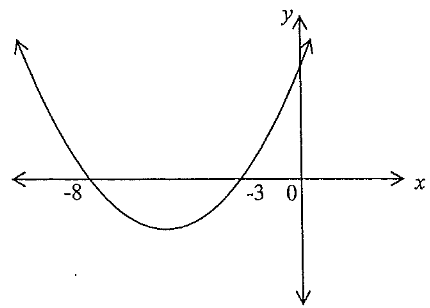
$$(x+3)(x^2-4) - (x-4)(x+3)^2 < 0$$

$$(x+3)(x^2-4-(x-4)(x+3)) < 0$$

$$(x+3)(x^2-4-(x^2-x-12)) < 0$$

$$(x+3)(x+8) < 0$$

$$-8 < x < -3$$



(b) (4 marks)
Outcomes assessed: HE2
Targeted Performance Bands: E2-E3

Criteria	Marks
• establishes the truth of $S(1)$	1
• establishes the result for $S(k)$	1
• substitutes result in $S(k+1)$	1
• deduces the required result	1

Sample Answer:

Let $S(n)$ be the statement $3^{3n}+2^{n+2}$ is divisible by 5

Consider $S(1)$: $3^3+2^3=35$ which is divisible by 5.
Hence $S(1)$ is true

If $S(k)$ is true: $3^{3k}+2^{k+2}=5M$ where M is an integer *

RTP $S(k+1)$ is true i.e. prove $3^{3(k+1)}+2^{(k+1)+2}=5Q$ where Q is an integer

$$\begin{aligned}LHS &= 3^{3k+3}+2^{k+3}\\ &= 3^3\times 3^{3k}+2\times 2^{k+2}\\ &= 27(5M-2^{k+2})+2\times 2^{k+2} \quad \text{if } S(k) \text{ is true using } *\\ &= 27\times 5M-27\times 2^{k+2}+2\times 2^{k+2}\\ &= 5\times 27M-25\times 2^{k+2}\\ &= 5(27M-5\times 2^{k+2})\\ &= 5Q \text{ where } Q \text{ is an integer since } M \text{ and } k \text{ are integers}\end{aligned}$$

Hence if $S(k)$ then $S(k+1)$ is true. Thus since $S(1)$ is true it follows by induction that $S(n)$ is true for positive integral n.

OR

$$\begin{aligned}LHS &= 3^{3k+3}+2^{k+3}\\ &= 3^3\times 3^{3k}+2\times 2^{k+2}\\ &= 25\times 3^{3k}+2\times 3^{3k}+2\times 2^{k+2}\\ &= 25\times 3^{3k}+2(3^{3k}+2^{k+2})\\ &= 25\times 3^{3k}+2\times 5M \quad \text{if } S(k) \text{ is true using } *\\ &= 5(5\times 3^{3k}+2M)\\ &= 5Q \text{ where } Q \text{ is an integer since } M \text{ and } k \text{ are integers}\end{aligned}$$

Conclusion as above

(c) (i) (3 marks)
Outcomes assessed: HE5
Targeted Performance Bands: E3-E4

Criteria	Marks
• progress towards correct differentiation	1
• finds a correct expression for acceleration	1
• shows correct relationship	1

Sample Answer:

$$v=\frac{2}{1+3x}$$

$$\begin{aligned}\frac{1}{2}v^2 &= \frac{1}{2}\frac{4}{(1+3x)^2}\\ &= 2(1+3x)^{-2}\end{aligned}$$

Now

$$\begin{aligned}a &= \frac{d}{dx}\left(\frac{1}{2}v^2\right)\\ &= 2\times -2(1+3x)^{-3}\times 3\\ &= \frac{-12}{(1+3x)^3}\\ &= -12\times\frac{8}{(1+3x)^3}\times\frac{1}{8}\\ &= -\frac{12}{8}v^3\\ &= -\frac{3}{2}v^3\end{aligned}$$

$\therefore a$ varies directly as v^3

(c) (ii) (2 marks)
Outcomes assessed: HE7
Targeted Performance Bands: E2-E3

Criteria	Marks
• describes initial motion	1
• describes motion as $t\to\infty$	1

Sample Answer:

Initially $v=2$ cms^{-1} $\therefore$ the particle moves in a positive direction from the origin.

As t increases, x increases and v decreases.
As $t\to\infty$, the particle continues in a positive direction with $v\to 0$.

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Question 4 (12 marks)
(a) (2 marks)
Outcomes assessed: PE3, HE7
Targeted Performance Bands: E2-E3

Criteria	Marks
• progress towards solution	1
• finds correct approximation (correct numerical equivalence)	1

Sample Answer:

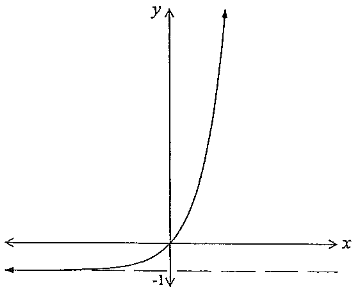
$$f(x) = e^x - x - 2$$
$$\therefore f'(x) = e^x - 1$$
$$\text{Let } x_1 = 1.2$$
$$f(x_1) = e^{1.2} - 1.2 - 2 = 0.1201169...$$
$$f'(x_1) = e^{1.2} - 1 = 2.3201169...$$
$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$
$$= 1.2 - \frac{0.1201169...}{2.3201169...}$$
$$= 1.14822...$$
$$= 1.15$$

(b) (i) (2 marks)
Outcomes assessed: PE6
Targeted Performance Bands: E2-E3

Criteria	Marks
• draws correct graph	1
• states correct range	1

Sample Answer:

$$y = e^{3x} - 1$$
Range: $y > -1$



(b) (ii) (3 marks)
Outcomes assessed: HE4
Targeted Performance Bands: E2-E3

Criteria	Marks
• interchanges variables or progress towards solution	1
• changes subject of equation or further progress towards solution	1
• states inverse function with correct restriction	1

Sample Answer:

$$y = e^{3x} - 1$$
Swap x and y
$$x = e^{3y} - 1$$
$$e^{3y} = x + 1$$
$$3y = \ln(x+1)$$
$$y = \frac{1}{3}\ln(x+1)$$
$$f^{-1}(x) = \frac{1}{3}\ln(x+1), \quad x > -1$$

(c) (i) (2 marks)
Outcomes assessed: HE3
Targeted Performance Bands: E2-E3

Criteria	Marks
• differentiates correctly	1
• shows motion is simple harmonic	1

Sample Answer:

$$x = \sqrt{3}\cos 3t - \sin 3t$$
$$v = \frac{dx}{dt}$$
$$= -3\sqrt{3}\sin 3t - 3\cos 3t$$
$$a = \frac{dv}{dt}$$
$$= -9\sqrt{3}\cos 3t + 9\sin 3t$$
$$= -9(\sqrt{3}\cos 3t - \sin 3t)$$
$$= -9x$$
which is of the form $a = -n^2 x$ where $n = 3$
$\therefore$ motion is simple harmonic

(c) (ii) (3 marks)
Outcomes assessed: HE3
Targeted Performance Bands: E3-E4

Criteria	Marks
• establishes result using auxiliary angle or other progress toward solution	1
• solves correctly for time	1
• finds correct velocity (correct numerical equivalence)	1

Sample Answer:

when $x=1$, $\sqrt{3}\cos 3t-\sin 3t=1$

Let $\sqrt{3}\cos 3t-\sin 3t=R\cos(3t+\alpha)$

$R\cos(3t+\alpha)=R\cos 3t\cos\alpha-R\sin 3t\sin\alpha$
$\therefore R\cos\alpha=\sqrt{3}$
$R\sin\alpha=1$
i.e. $\tan\alpha=\frac{1}{\sqrt{3}} \quad\Rightarrow\quad \alpha=\frac{\pi}{6}$
$R^2=1+3 \quad\Rightarrow\quad R=2$
$\sqrt{3}\cos 3t-\sin 3t=2\cos\left(3t+\frac{\pi}{6}\right)$

i.e. solve $2\cos\left(3t+\frac{\pi}{6}\right)=1$

$\cos\left(3t+\frac{\pi}{6}\right)=\frac{1}{2}$

$3t+\frac{\pi}{6}=\frac{\pi}{3}$ (first oscillation)

$t=\frac{\pi}{18}$ seconds

When $t=\frac{\pi}{18}$ $\quad v=-3\sqrt{3}\sin\frac{\pi}{6}-3\cos\frac{\pi}{6}$

$=-3\sqrt{3}\times\frac{1}{2}-3\times\frac{\sqrt{3}}{2}$

$=-3\sqrt{3}\ \text{cms}^{-1}$

Question 5 (12 marks)
(a) (i) (2 marks)
Outcomes assessed: PE3
Targeted Performance Bands: E3-E4

Criteria	Marks
• defines roots in arithmetic series	1
• uses sum of roots to show result	1

Sample Answer:

Let the roots be $\alpha-d$, α and $\alpha+d$
$x^3-6x^2+3x+k=0$
sum of roots $=\frac{-b}{a}=6$
Also sum of roots $=\alpha-d+\alpha+\alpha+d=3\alpha$
$\therefore 3\alpha=6$
$\alpha=2$
i.e. one of the roots is 2

(a) (ii) (3 marks)
Outcomes assessed: PE3
Targeted Performance Bands: E2-E3

Criteria	Mark
• finds correct value for k	1
• progress toward solution	1
• finds correct roots	1

Sample Answer:
Since one root is 2 substitute into equation to find k.
$2^3-6\times 2^2+3\times 2+k=0$
$\therefore k=10$
i.e. equation is $x^3-6x^2+3x+10=0$
product of roots $=\frac{-d}{a}=-10$
product of roots $=\alpha(\alpha-d)(\alpha+d)$ from (i)
$=\alpha(\alpha^2-d^2)$
$\therefore -10=2\times(2^2-d^2)$
$-5=4-d^2$
$d^2=9$
$d=\pm 3$

$\therefore$ roots are $-1, 2, 5$

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(b) (3 marks)
Outcomes assessed: PE2
Targeted Performance Bands: E3-E4

Criteria	Marks
• establishes correct t-formula or other progress towards result	1
• significant progress toward the result	1
• completes the proof	1

Sample Answer:

Let $t=\tan\theta$, $\therefore \tan 2\theta=\dfrac{2t}{1-t^2}$

$$\begin{aligned}\text{LHS}&=\frac{\tan 2\theta-\tan\theta}{\tan 2\theta+\cot\theta}\\&=\left(\frac{2t}{1-t^2}-t\right)\div\left(\frac{2t}{1-t^2}+\frac{1}{t}\right)\\&=\frac{2t-t+t^3}{1-t^2}\div\left(\frac{2t^2+1-t^2}{t(1-t^2)}\right)\\&=\frac{t(1+t^2)}{1-t^2}\times\frac{t(1-t^2)}{t^2+1}\\&=t^2\\&=\tan^2\theta\\&=\text{RHS}\end{aligned}$$

OR

$$\begin{aligned}\text{LHS}&=\frac{\tan 2\theta-\tan\theta}{\tan 2\theta+\cot\theta}\\&=\left(\frac{2\tan\theta}{1-\tan^2\theta}-\tan\theta\right)\div\left(\frac{2\tan\theta}{1-\tan^2\theta}+\frac{1}{\tan\theta}\right)\\&=\left(\frac{2\tan\theta-\tan\theta+\tan^3\theta}{1-\tan^2\theta}\right)\times\left(\frac{\tan\theta(1-\tan^2\theta)}{2\tan^2\theta+1-\tan^2\theta}\right)\\&=\tan\theta(1+\tan^2\theta)\times\frac{\tan\theta}{\tan^2\theta+1}\\&=\tan^2\theta\\&=\text{RHS}\end{aligned}$$

(c) (i) (2 marks)
Outcomes assessed: PE4
Targeted Performance Bands: E3-E4

Criteria	Mark
• uses correct formula for division of interval or progress using other correct method	1
• finds correct coordinates from working	1

Sample Answer:

$P(2ap,\ ap^2)$, $S(0,\ a)$ and $PQ:QS=-4:3$

Let Q have coordinates $(x_q,\ y_q)$

$$x_q=\frac{3\times 2ap-4\times 0}{-4+3}=\frac{6ap}{-1}=-6ap \qquad y_q=\frac{3\times ap^2-4\times a}{-4+3}=\frac{3ap^2-4a}{-1}=a\left(4-3p^2\right)$$

$\therefore$ Q has coordinates $\left(-6ap,\ a(4-3p^2)\right)$

(c) (ii) (2 marks)
Outcomes assessed: PE4
Targeted Performance Bands: E3-E4

Criteria	Marks
• makes progress to finding the locus	1
• shows locus is a parabola	1

Sample Answer:

From (i) $x=-6ap$

$\therefore p=\dfrac{-x}{6a}$ and $p^2=\dfrac{x^2}{36a^2}$

$$\begin{aligned}\therefore y&=a\left(4-3p^2\right)\\&=a\left(4-\frac{3x^2}{36a^2}\right)\\&=4a-\frac{x^2}{12a}\\\frac{x^2}{12a}&=4a-y\\x^2&=48a^2-12ay\\&=-12a(y-4a)\end{aligned}$$

which is the form of a parabola [with vertex $(0,\ 4a)$]

Question 6 (12 marks)
(a) (3 marks)
Outcomes assessed: PE2
Targeted Performance Bands: E2-E3

Criteria	Marks
• simplifies some indices	1
• further progress with simplifying indices	1
• gives correct expression	1

Sample Answer:

$$\begin{aligned}\frac{2^{4n}\times 3^{2n}}{8^n\times 6^n}+3^n &= \frac{2^{4n}\times 3^{2n}}{2^{3n}\times 2^n\times 3^n}+3^n\\ &= \frac{2^{4n}\times 3^n}{2^{4n}}+3^n\\ &= 3^n+3^n\\ &= 2\times 3^n\end{aligned}$$

(b) (i) (2 marks)
Outcomes assessed: PE5, HE7
Targeted Performance Bands: E3-E4

Criteria	Marks
• establishes correct derivative	1
• shows the result	1

Sample Answer:

$$\begin{aligned}V &= \pi r^2 h\\ &= \pi r^3 k \qquad \text{since } h = kr\end{aligned}$$

$$\frac{dV}{dt}=\frac{dV}{dr}\times\frac{dr}{dt}$$

$$\frac{dV}{dt}=3\pi r^2 k\times\frac{dr}{dt}$$

$$\frac{dV}{dt}=0.2 \text{ when } r=4$$

$$\therefore 0.2 = 3\pi\times 4^2 k\times\frac{dr}{dt}$$

$$\begin{aligned}\frac{dr}{dt} &= \frac{0.2}{48\pi k}\\ &= \frac{1}{240\pi k}\end{aligned}$$

(b) (ii) (3 marks)
Outcomes assessed: PE5, HE7
Targeted Performance Bands: E3-E4

Criteria	Marks
• finds expression for $\frac{dr}{dt}$ using surface area or progress toward result	1
• equates expressions using (i) or significant progress toward result	1
• finds correct value of k	1

Sample Answer:

$$\begin{aligned}S &= 2\pi rh+2\pi r^2\\ &= 2\pi r^2 k+2\pi r^2 \qquad \text{since } h=kr\\ &= 2\pi r^2(k+1)\end{aligned}$$

$$\frac{dS}{dt}=\frac{dS}{dr}\times\frac{dr}{dt}$$

$$\frac{dS}{dt}=4\pi r(k+1)\times\frac{dr}{dt}$$

$$\frac{dS}{dt}=0.1 \text{ when } r=4$$

$$\therefore 0.1=4\pi\times 4(k+1)\times\frac{dr}{dt}$$

$$\begin{aligned}\frac{dr}{dt} &= \frac{0.1}{16\pi(k+1)}\\ &= \frac{1}{160\pi(k+1)}\end{aligned}$$

$$\therefore \frac{1}{160\pi(k+1)}=\frac{1}{240\pi k} \qquad \text{from (i)}$$

$$\begin{aligned}240k &= 160k+160\\ 80k &= 160\\ k &= 2\end{aligned}$$

(c) (i) (2 marks)
Outcomes assessed: HE7
Targeted Performance Bands: E3-E4

Criteria	Marks
• differentiate LHS correctly	1
• differentiate RHS correctly	1

Sample Answer:

$$(1+x)^{2n} = \sum_{k=0}^{2n} {}^{2n}C_k\, x^k = {}^{2n}C_0 + {}^{2n}C_1\, x + {}^{2n}C_2\, x^2 + \ldots + {}^{2n}C_k\, x^k + \ldots + {}^{2n}C_{2n}\, x^{2n}$$

Differentiate both sides with respect to x.

$\text{LHS} = 2n(1+x)^{2n-1}$

$RHS = {}^{2n}C_1 + {}^{2n}C_2\, 2x + \ldots + {}^{2n}C_k\, kx^{k-1} + \ldots + {}^{2n}C_{2n}\, 2nx^{2n-1}$

$= \sum_{k=1}^{2n} {}^{2n}C_k\, kx^{k-1}$

$\left[\therefore 2n(1+x)^{2n-1} = \sum_{k=1}^{2n} k\, {}^{2n}C_k\, x^{k-1}\right]$

(c) (ii) (2 marks)
Outcomes assessed: HE7
Targeted Performance Bands: E3-E4

Criteria	Marks
• correct substitution into equation	1
• gives correct conclusion	1

Sample Answer:

Let $x = 1$ in the expansion of $2n(1+x)^{2n-1} = \sum_{k=1}^{2n} k\, {}^{2n}C_k\, x^{k-1}$.

$\text{LHS} = 2n \times 2^{2n-1}$

$= n \times 2^{2n}$

$= n \times 4^n$

$\text{RHS} = \sum_{k=1}^{2n} k\, {}^{2n}C_k$

$\therefore \sum_{k=1}^{2n} k\, {}^{2n}C_k = n \times 4^n$

Question 7 (12 marks)
(a) (i) (2 marks)
Outcomes assessed: HE3
Targeted Performance Bands: E2-E3

Criteria	Marks
• establishes correct binomial probability	1
• gives correct answer (correct numerical equivalence)	1

Sample Answer:

Let probability of correct guess, $p = 0.3$ and incorrect guess, $q = 0.7$

Binomial probability; $(0.7+0.3)^{50}$

$P(\text{25 correct}) = {}^{50}C_{25}(0.7)^{25}(0.3)^{25}$

$[= 0.0014]$

(a) (ii) (3 marks)
Outcomes assessed: H5
Targeted Performance Bands: E3-E4

Criteria	Marks
• applies greatest coefficient method or some progress towards solution	1
• further progress towards solution (e.g. solution of inequality)	1
• gives correct answer	1

Sample Answer:

Most likely number correct ⇒ find the greatest term in $(0.7+0.3)^{50}$

Find k such that $\frac{T_{k+1}}{T_k} \geq 1$

$\frac{T_{k+1}}{T_k} = \frac{50-k+1}{k} \times \frac{0.3}{0.7}$

i.e. $\frac{153-3k}{7k} \geq 1$

$153 - 3k \geq 7k$

$10k \leq 153$

$k \leq 15.3$

$\therefore k = 15$

Most likely number correct is 15. $\left[T_{16} = {}^{50}C_{15}(0.3)^{15}(0.7)^{35} = 0.122\right]$

(b)(i) (2 marks)
Outcomes assessed: HE3
Targeted Performance Bands: E2-E3

Criteria	Marks
• differentiates and equates to zero	1
• shows correct result	1

Sample Answer:

Particle reaches maximum height when $y' = 0$

$$y = Vt\sin\theta - \frac{1}{2}gt^2 \quad \Rightarrow \quad y' = V\sin\theta - gt$$

when $y' = 0$, $gt = V\sin\theta$ i.e. $t = \dfrac{V\sin\theta}{g}$

(b) (ii) (3 marks)
Outcomes assessed: HE3
Targeted Performance Bands: E3-E4

Criteria	Marks
• some progress toward solution	1
• further progress toward solution	1
• substitutes and simplifies to obtain desired result	1

Sample Answer:

At maximum height $t = \dfrac{V\sin\theta}{g}$, $x = c$ and $y = h$

$$h = \frac{V^2\sin^2\theta}{g} - \frac{1}{2}g\frac{V^2\sin^2\theta}{g^2} \quad \text{and} \quad c = \frac{V^2\cos\theta\sin\theta}{g}$$

$$h = \frac{V^2\sin^2\theta}{2g}$$

$$\therefore \sin^2\theta = \frac{2gh}{V^2} \qquad (1)$$

$$c^2 = \frac{V^4\cos^2\theta\sin^2\theta}{g^2}$$

$$= \frac{V^4\sin^2\theta(1-\sin^2\theta)}{g^2}$$

$$= \frac{V^4\frac{2gh}{V^2}\left(1-\frac{2gh}{V^2}\right)}{g^2} \quad \text{substituting for } \sin^2\theta \text{ from (1)}$$

$$= \frac{2h(V^2-2gh)}{g}$$

$$\therefore V^2 = 2gh + \frac{c^2g}{2h}$$

$$= \frac{4gh^2 + c^2g}{2h}$$

$$= \frac{g}{2h}\left(4h^2+c^2\right)$$

(b) (iii) (2 marks)
Outcomes assessed: HE3
Targeted Performance Bands: E3-E4

Criteria	Marks
• significant progress towards solutions	1
• finds a correct expression for θ	1

Sample Answer:

$$c = \frac{V^2\cos\theta\sin\theta}{g} \qquad h = \frac{V^2\sin^2\theta}{2g}$$

$$\frac{h}{c} = \frac{V^2\sin^2\theta}{2g} \times \frac{g}{V^2\cos\theta\sin\theta}$$

$$\frac{h}{c} = \frac{\sin\theta}{2\cos\theta}$$

$$\frac{2h}{c} = \tan\theta$$

$$\therefore \theta = \tan^{-1}\left(\frac{2h}{c}\right)$$

OR

$$V^2 = \frac{g}{2h}\left(4h^2+c^2\right) \qquad h = \frac{V^2\sin^2\theta}{2g} \quad \text{i.e. } \sin^2\theta = \frac{2gh}{V^2}$$

$$\sin^2\theta = \frac{2gh}{\frac{g}{2h}\left(4h^2+c^2\right)}$$

$$\sin^2\theta = \frac{4h^2}{\left(4h^2+c^2\right)}$$

$$\sin\theta = \frac{2h}{\sqrt{4h^2+c^2}} \qquad (\theta \text{ acute})$$

$$\theta = \sin^{-1}\left(\frac{2h}{\sqrt{4h^2+c^2}}\right)$$

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