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Centre Number

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Student Number



CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NEW SOUTH WALES

2008
TRIAL HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics

Morning Session
Monday 11 August 2008

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided on the back page
- All necessary working should be shown in every question
- Write your Centre Number and Student Number at the top of this page

Total marks – 120

- Attempt Questions 1-10
- All questions are of equal value

Disclaimer

Every effort has been made to prepare these 'Trial' Higher School Certificate Examinations in accordance with the Board of Studies documents, *Principles for Setting HSC Examinations in a Standards-Referenced Framework* (BOS Bulletin, Vol 8, No 9, Nov/Dec 1999), and *Principles for Developing Marking Guidelines Examinations in a Standards Referenced Framework* (BOS Bulletin, Vol 9, No 3, May 2000). No guarantee or warranty is made or implied that the 'Trial' Examination papers mirror in every respect the actual HSC Examination question paper in any or all courses to be examined. These papers do not constitute 'advice' nor can they be construed as authoritative interpretations of Board of Studies intentions. The CSSA accepts no liability for any reliance use or purpose related to these 'Trial' question papers. Advice on HSC examination issues is only to be obtained from the NSW Board of Studies.

3101-1

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \quad \text{if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln \left(x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln \left(x + \sqrt{x^2 + a^2} \right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Total marks – 120
 Attempt Questions 1-10
 All questions are of equal value.

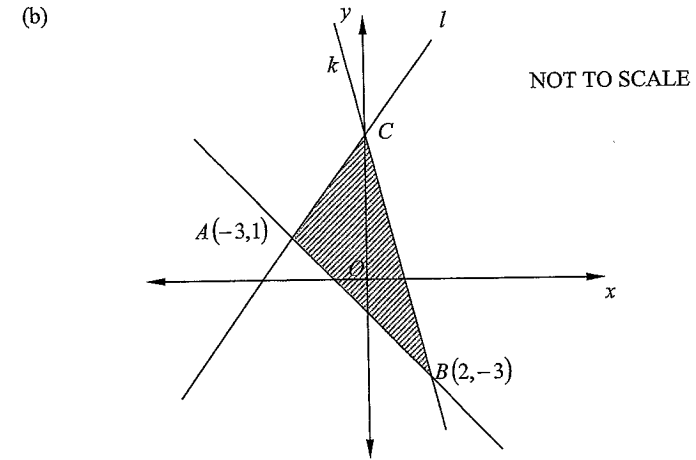
Answer each question in a SEPARATE writing booklet.

Question 1 (12 marks) Use a SEPARATE writing booklet. Marks

- | | |
|--|---|
| (a) Evaluate $e^{\pi+1}$ correct to two significant figures. | 2 |
| (b) Factorise fully $12x^2 - 20x - 8$. | 2 |
| (c) Differentiate $3x^4 + \sin x$ with respect to x . | 2 |
| (d) Solve $8 - \frac{2x+1}{3} = \frac{8-x}{5}$. | 2 |
| (e) If $a + \sqrt{b} = 4(7 + \sqrt{5})$ and a and b are integers, find a and b . | 2 |
| (f) Sketch the graph of $y = 6 - 3x $. | 2 |

Question 2 (12 marks) Use a SEPARATE writing booklet. Marks

- | | |
|---|---|
| (a) Evaluate $\lim_{h \rightarrow 3} \frac{3-h}{9-h^2}$. | 2 |
|---|---|



The point $A(-3, 1)$ lies on the line l given by the equation $4x - 3y + 15 = 0$ and the point $B(2, -3)$ lies on the line k given by the equation $4x + y - 5 = 0$.

- | | |
|--|---|
| (i) Show that C , which is the point of intersection of the lines l and k , has coordinates $(0, 5)$. | 1 |
| (ii) Find the gradient of the line joining A and B . | 1 |
| (iii) Find the equation of the line AB . | 2 |
| (iv) Find the perpendicular distance from point A to the line k . | 2 |
| (v) Find the area of triangle ABC . | 2 |
| (vi) Write down the inequations which define the shaded region. | 2 |

Question 3 (12 marks) Use a SEPARATE writing booklet.

Marks

(a) Differentiate with respect to x :

(i) $\log_e 2x$.

1

(ii) $\frac{7-6x}{e^x}$.

2

(b) (i) Find $\int \frac{1-2x^5}{x} dx$.

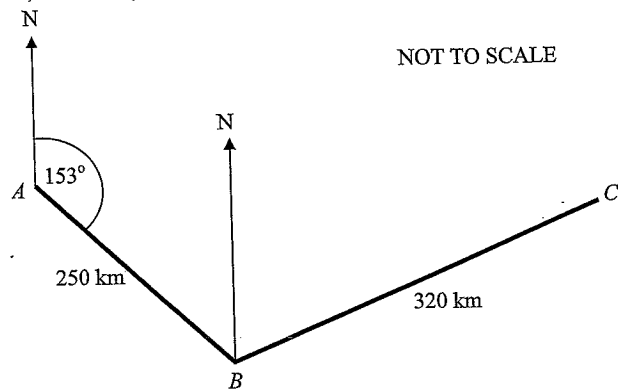
2

(ii) Evaluate $\int_{-\frac{\sqrt{3}}{2}}^1 2\sec^2 x dx$.

2

Give your answer correct to 1 decimal place.

(c) A ship sailed 250 km from port A on a bearing of 153° and arrived at Port B to pick up some passengers. It then progressed to its destination Port C , a distance of 320 km on a bearing of 071° .



Copy or trace this diagram into your writing booklet.

(i) Show that $\angle ABC = 98^\circ$.

1

(ii) Show that the distance AC is approximately 433 kilometres.

2

(iii) The return trip is a straight line back to Port A and not passing through Port B . Find the bearing the ship must take to go straight from Port C to Port A .

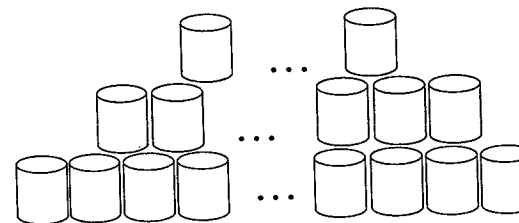
2

Marks

Question 4 (12 marks) Use a SEPARATE writing booklet

(a) Paint cans are stacked such that there are 38 cans on the bottom row, 35 cans on the next row, 32 on the next row and so on until a total of 253 cans are stacked.

NOT TO SCALE



(i) Write down a formula for the number of cans in the n th row.

1

(ii) How many rows are there in this stack?

2

(iii) How many cans are there in the final row of this stack?

1

(b) The numbers 0, 1, 2, 3 and 4 are written on 5 discs. These discs are placed in a bag. One disc is drawn out at random and the number recorded, then replaced. A second disc is then drawn out and its number also recorded.

(i) What is the probability that the product of the numbers on the 2 discs is a MULTIPLE of 4?

2

(ii) What is the probability that the product of the numbers on the 2 discs is AT LEAST 4?

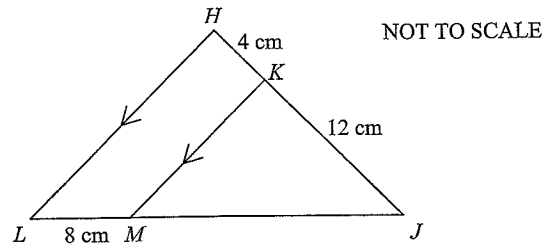
1

Question 4 (continued)

(c) A function is defined by the rule: $g(x) = \begin{cases} 3 & \text{for } x \leq 0 \\ 3 \cos x & \text{for } x > 0 \end{cases}$

- (i) State the domain of $g(x)$. 1
 (ii) State the range of $g(x)$. 1

(d) In the diagram HL is parallel to KM . $HK = 4$ cm, $KJ = 12$ cm and $LM = 8$ cm.



Copy or trace the diagram into your writing booklet.

- (i) Prove that $\triangle HJL$ is similar to $\triangle HKM$. 2
 (ii) Hence, or otherwise, calculate the length of MJ . 1

End of Question 4

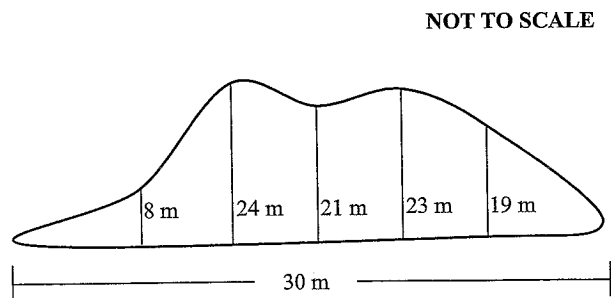
Marks

Question 5 (12 marks) Use a SEPARATE writing booklet

- (a) (i) Prove that $\sec^2 \theta - 2 \tan \theta = (\tan \theta - 1)^2$. 2
 (ii) Hence, or otherwise, solve $\sec^2 \theta - 2 \tan \theta = 0$ for $0 \leq \theta \leq 2\pi$. 2
- (b) Let $f(x) = 15 + 12x + 3x^2 - 2x^3$.
- (i) Find the coordinates of the stationary points of $y = f(x)$ and determine their nature. 3
 (ii) Find the coordinates of the point of inflexion. 1
 (iii) Sketch the graph of $y = f(x)$ indicating clearly the stationary points and point of inflexion. 2
- (c) The roots of the equation $px^2 - x + q = 0$ are -2 and 5 . 2
 Find p and q .

Question 6 (12 marks) Use a SEPARATE writing booklet

- (a) The diagram shows the width of a frog pond at 5 metre intervals.



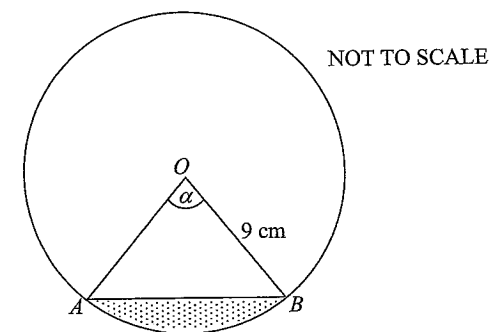
- (i) Use the trapezoidal rule with six sub-intervals to estimate the surface area of the pond. 2
- (ii) The average depth of the pond is 1.2 m. Calculate the volume of the pond. 1
- (iii) The optimum number of lily pads for a healthy frog population is 5 lily pads per 25 m^2 . How many lily pads should be planted in this pond? 1

Marks

Question 6 (continued)

- (b) (i) Show that $x = 3.2$ is an approximate solution of the equation $\log_{10}(x+3) = \frac{1}{4}x$. 1
- (ii) On the same diagram accurately draw the graphs of $y = \log_{10}(x+3)$ and $y = \frac{1}{4}x$ for $-3 \leq x \leq 4$. 2
- (iii) Hence, or otherwise, determine the number of solutions to the equation $\log_{10}(x+3) - \frac{1}{4}x = 0$, $x > -3$. 1

(c)



- (i) The area of the sector OAB is $\frac{135\pi}{8} \text{ cm}^2$. Given the radius of the circle is 9 cm, find the acute angle α , leaving your answer in exact form. 2
- (ii) Hence, or otherwise, find the shaded area. 2

End of Question 6

Question 7 (12 marks) Use a SEPARATE writing booklet.

- (a) **Train A** is travelling towards a railway station along the route shown.



The displacement, S (in kilometres), of train A from the railway station is given by the expression $S_A(t) = 2 - 3t + 3t^2 - t^3$ where t is the time in minutes.

The following table shows the displacement of train A from the railway station at the given times.

t (minutes)	0	0.5	1.0	1.5	2.0	2.5
$S_A(t)$ (km)	2	1.125	1.0	0.875	0	-2.375

- (i) Interpret the value of $S_A(t)$ when $t = 2.5$ minutes. 1
- (ii) Find in terms of t the velocity, $V_A(t)$, of train A . 1
- (iii) At what time is train A at rest? 1

Train B is travelling in the opposite direction to train A . Train B is approaching the railway station at a velocity, $V_B(t)$, given by the function $V_B(t) = 6t - 3$.

- (iv) If train B is initially 8 kilometres to the left of the railway station, find its displacement $S_B(t)$ in terms of t . 2
- (v) Show that the trains will pass each other at $t = \sqrt[3]{10}$ minutes. 1

Marks

Question 7 (continued)

- (b) Antonia and Melina recently received an inheritance windfall of \$20 000 each. After speaking to a financial adviser, both decide to invest their money with 'Gauss Investments Australia'.

Antonia decides to invest the \$20 000 in a savings policy at 7.5% p.a. and at the end of the first month she makes a contribution of \$180 to this policy. Antonia continues to contribute \$180 per month at the same interest rate. The savings policy is compounded monthly with interest paid just before each contribution.

Let $\$A_n$ be the amount Antonia's savings policy grows to by the end of the n th contribution.

- (i) Show that after the second contribution, 2
 $A_2 = 20\,000(1.00625)^2 + 180(1 + 1.00625)$.
- (ii) Antonia's intention is to collect her savings when the policy reaches \$50 000. For how many months must Antonia contribute to reach her goal? 2

Melina decides to invest her \$20 000 in a term deposit for fifteen years with interest compounded monthly.

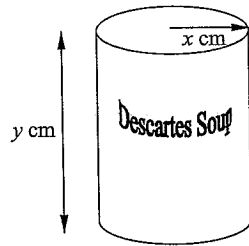
- (iii) At what annual interest rate must Melina invest her money if she is to also receive \$50 000? 2

End of Question 7

Marks

Question 8 (12 marks) Use a SEPARATE writing booklet.

(a)



A closed cylindrical can of radius x cm and height y cm is to be made from a sheet of metal with area 435π cm². There is 20% wastage of the sheet metal in manufacturing the can.

- (i) Find the area of sheet metal required to make the can. 1
- (ii) Show that $y = \frac{174}{x} - x$. 2
- (iii) Hence show that the volume, V , of the can is given by $V = 174\pi x - \pi x^3$. 1
- (iv) Find, correct to one decimal place, the value of x which gives a maximum volume. 2
- (v) Hence calculate the maximum volume of the can correct to the nearest cm³. 1

(b) The decay of the mass M , in kg, of a radioactive substance can be modelled by $M = M_0 e^{-kt}$, where t is the time in years and k is a constant.

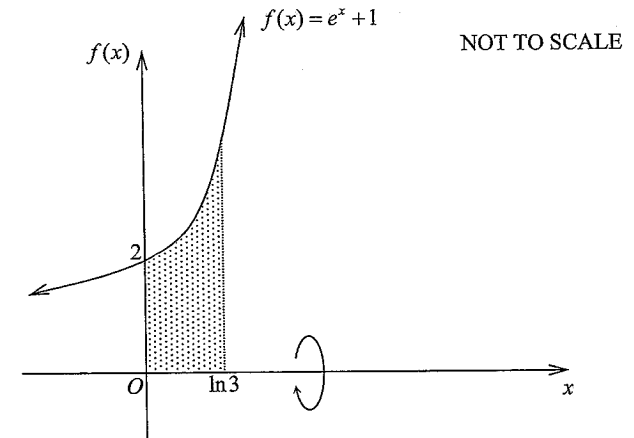
88kg of the substance takes 9 years to decay to 73kg.

- (i) Find the values of M_0 and k . 2
- (ii) Find the mass of the substance after 18 years. 1
- (iii) Find the rate of decay after 18 years. 2

Marks

Question 9 (12 marks) Use a SEPARATE writing booklet

(a)



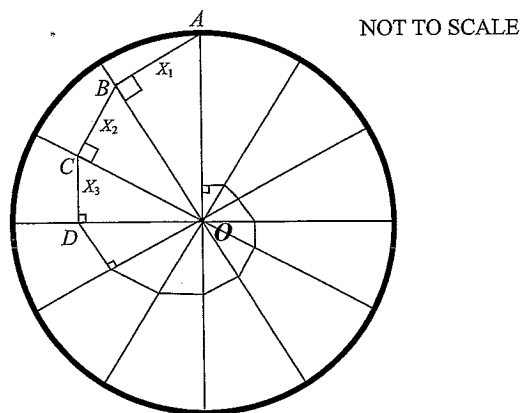
The shaded region bounded by the graph $f(x) = e^x + 1$, the lines $x = 0$, $x = \ln 3$ and the x -axis is rotated about the x -axis to form a solid of revolution.

- (i) Show that the volume V of the solid is given by: 1

$$V = \pi \int_0^{\ln 3} e^{2x} + 2e^x + 1 \, dx.$$
- (ii) Find the volume V of the solid formed. Leave your answer in exact form. 3

Question 9 (continued)

(b)



The diagram above shows a circular cartwheel with radius $OA = 4$ cm. The cartwheel is divided into 12 equal sectors.

Perpendiculars AB, BC, CD, \dots have been drawn and have respective lengths X_1, X_2, X_3, \dots etc as shown.

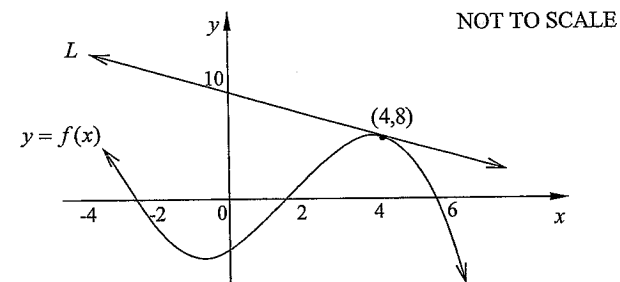
- | | | |
|-------|--|---|
| (i) | By considering triangle OAB , show the distance $X_1 = 2$ cm. | 1 |
| (ii) | Show that the distances X_1, X_2, X_3, \dots form a geometric sequence. | 3 |
| (iii) | Show that $X_{12} = 2 \left(\frac{\sqrt{3}}{2} \right)^{11}$ cm. | 1 |
| (iv) | Calculate the length of one spiral, i.e. $(X_1 + X_2 + X_3 + \dots + X_{12})$. | 2 |
| (v) | If the spiral is continued indefinitely, show that its total length will not exceed 15 cm. | 1 |

End of Question 9

Marks

Question 10 (12 marks) Use a SEPARATE writing booklet.

(a)



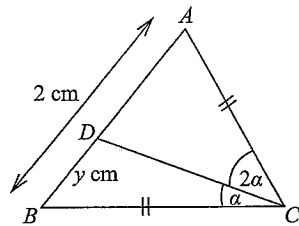
The line L is a tangent to the curve $y = f(x)$ at the point $(4, 8)$ and also crosses the y -axis at 10.

2

What is the value of $f'(4)$?

Question 10 (continued)

(b)



The diagram above shows an isosceles triangle ABC with $AC = BC$ and $AB = 2$ cm.

D is a point on AB so that $\angle ACD = 2\alpha$, $\angle BCD = \alpha$ and $BD = y$ cm.

(i) Show that $\angle BAC = \left(\frac{\pi}{2} - \frac{3}{2}\alpha\right)$. 2

(ii) Use the Sine rule in triangle ACD to show $CD = \frac{(2-y)\cos\left(\frac{3}{2}\alpha\right)}{\sin 2\alpha}$. 2

(iii) Similarly, find another expression for the length of CD in triangle BCD . 2

(iv) Given the formula $\sin 2\alpha = 2\sin\alpha\cos\alpha$, (Do NOT prove this).

Show that $y = \frac{2}{1+2\cos\alpha}$. 2

(v) Hence prove that as α varies, $\frac{2}{3} < BD < 1$ for $0 < \alpha < \frac{\pi}{3}$. 2

End of Paper



CATHOLIC SECONDARY SCHOOLS ASSOCIATION
2008 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

MATHEMATICS – SUGGESTED SOLUTIONS

These marking guidelines show the criteria to be applied to responses along with the marks to be awarded in line with the quality of responses. These guidelines are suggested and not prescriptive. This is not intended to be an exhaustive list but rather an indication of the considerations that students could include in their responses.

Question 1 (12 marks)

(a) (2 marks)

Outcomes Assessed: P3, H3

Targeted Performance Bands: 2-3

Criteria	Mark
• Gives the correct answer.	1
• Correctly rounds to TWO significant figures.	1

Sample answer

$$e^{\pi+1} = 62.90292428$$

$$= 63 \text{ (2 significant figures)}$$

(b) (2 marks)

Outcomes Assessed: P3, P4

Targeted Performance Bands: 2-3

Criteria	Mark
• Factorises by common factor correctly or equivalent.	1
• Completes the factorisation into the HCF and TWO brackets.	1

Sample answer

$$12x^2 - 20x - 8 = 4(3x^2 - 5x - 2)$$

$$= 4(3x + 1)(x - 2)$$

(c) (2 marks)

Outcomes Assessed: P7, H5

Targeted Performance Bands: 2-3

Criteria	Mark
• Correctly differentiates one of the terms.	1
• Differentiates the expression correctly.	1

Sample answer

$$\frac{d}{dx}(3x^4 + \sin x) = 12x^3 + \cos x$$

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(d) (2 marks)

Outcomes Assessed: P3, P4

Targeted Performance Bands: 2-3

Criteria	Mark
• Multiplies each term by 15 correctly or equivalent progress.	1
• Gives the correct answer.	1

Sample answer

$$8 \frac{2x+1}{3} = \frac{8-x}{5}$$

$$120 - 5(2x+1) = 3(8-x)$$

$$120 - 10x - 5 = 24 - 3x$$

$$120 - 5 - 24 = 7x$$

$$91 = 7x$$

$$x = 13$$

(e) (2 marks)

Outcomes Assessed: P3, P4

Targeted Performance Bands: 2-3

Criteria	Mark
• Correctly expands and finds the value of a.	1
• Finds the correct value for b.	1

Sample answer

$$a + \sqrt{b} = 4(7 + \sqrt{5})$$

$$RHS = 28 + 4\sqrt{5}$$

$$= 28 + \sqrt{16} \times \sqrt{5}$$

$$= 28 + \sqrt{80}$$

$$\therefore a = 28, b = 80$$

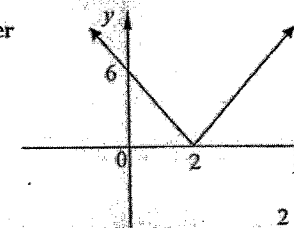
(f) (2 marks)

Outcomes Assessed: P4

Targeted Performance Bands: 2-3

Criteria	Mark
• Correct shape of the function.	1
• Correct x and y intercepts.	1

Sample answer



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Question 2 (12 marks)

(a) (2 marks)

Outcomes Assessed: H5

Targeted Performance Bands: 3-4

Criteria	Mark
• Arrives at the expression $\lim_{h \rightarrow 3} \frac{1}{3+h}$	1
• Gives the correct answer.	1

Sample answer

$$\lim_{h \rightarrow 3} \frac{3-h}{9-h^2} = \lim_{h \rightarrow 3} \frac{3-h}{(3-h)(3+h)} = \lim_{h \rightarrow 3} \frac{1}{3+h} = \frac{1}{6}$$

$$\therefore \lim_{h \rightarrow 3} \frac{3-h}{9-h^2} = \frac{1}{6}$$

(b) (i) (1 mark)

Outcomes Assessed: P3, P4, H5

Targeted Performance Bands: 2-3

Criteria	Mark
• Substitutes the point (0, 5) into both equations.	1

Sample answer

$$4x - 3y + 15 = 0 \qquad 4x + y - 5 = 0$$

$$4(0) - 3(5) + 15 = -15 + 15 = 0 \qquad 4(0) + 5 - 5 = 0 \text{ (as required)}$$

(b) (ii) (1 mark)

Outcomes Assessed: P3, P4, H5

Targeted Performance Bands: 2-3

Criteria	Mark
• Finds correct gradient.	1

Sample answer

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{1 - -3}{-3 - -2}$$

$$= -\frac{4}{5}$$

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(b) (iii) (2 marks)

Outcomes Assessed: P3, P4, H5

Targeted Performance Bands: 2-3

Criteria	Mark
• Uses point-gradient formula.	1
• Gives the correct equation of line.	1

Sample answer

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -\frac{4}{5}(x - -3)$$

$$5y - 5 = -4x - 12$$

$$4x + 5y + 7 = 0$$

(b) (iv) (2 marks)

Outcomes Assessed: P3, P4, H5

Targeted Performance Bands: 3-4

Criteria	Mark
• Substitutes correctly.	1
• Gives the correct answer.	1

Sample answer

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} \qquad 4x + y - 5 = 0$$

$$= \frac{|4(-3) + 1(1) - 5|}{\sqrt{4^2 + 1^2}} \qquad (-3, 1)$$

$$= \frac{|-12 + 1 - 5|}{\sqrt{17}} = \frac{16}{\sqrt{17}}$$

(b) (v) (2 marks)

Outcomes Assessed: P3, P4, H5

Targeted Performance Bands: 3-4

Criteria	Mark
• Finds correct distance BC.	1
• Gives the correct answer.	1

Sample answer

$$BC = \sqrt{(0-2)^2 + (5-(-3))^2}$$

$$= \sqrt{68}$$

$$\text{Area} = \frac{1}{2} \times \sqrt{68} \times \frac{16}{\sqrt{17}}$$

$$= 16 \text{ u}^2$$

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(b) (vi) (2 marks)

Outcomes Assessed: P3, P4, H5

Targeted Performance Bands: 3-4

Criteria	Mark
• Gives one correct inequality.	1
• Gives the correct answer.	1

Sample answer

$$4x - 3y + 15 \geq 0$$

$$4x + y - 5 \leq 0$$

$$4x + 5y + 7 \geq 0$$

Question 3 (12 marks)

(a) (i) (1 mark)

Outcomes Assessed: P7, H5

Targeted Performance Bands: 3-4

Criteria	Mark
• Gives the correct answer.	1

Sample answer

$$\frac{d}{dx}(\log_e 2x) = \frac{2}{2x} = \frac{1}{x}$$

(a) (ii) (2 marks)

Outcomes Assessed: P7, H5

Targeted Performance Bands: 3-4

Criteria	Mark
• Correctly uses the quotient rule but has ONE mistake.	1
• Gives the correct answer.	1

Sample answer

$$\text{Let } y = \frac{7 - 6x}{e^x}$$

$$\frac{dy}{dx} = \frac{-6e^x - (7 - 6x)e^x}{e^{2x}} = \frac{-13 + 6x}{e^x}$$

(b) (i) (2 marks)

Outcomes Assessed: H8

Targeted Performance Bands: 3-4

Criteria	Mark
• Finds the primitive of one of the terms $\frac{1}{x}$ or $2x^4$.	1
• Gives the correct answer.	1

Sample answer

$$\int \frac{1 - 2x^5}{x} dx = \int \left(\frac{1}{x} - 2x^4 \right) dx$$
$$= \log_e x - \frac{2x^5}{5} + C$$

(b) (ii) (2 marks)

Outcomes Assessed: H8

Targeted Performance Bands: 3-4

Criteria	Mark
• Finds the primitive of $2\sec^2 x$ but has an error in calculating the integral.	1
• Correctly applies Newton-Leibnitz formula to obtain the answer to 1 decimal place.	1

Sample answer

$$\int_{\frac{\sqrt{3}}{2}}^1 2 \sec^2 x dx = [2 \tan x]_{\frac{\sqrt{3}}{2}}^1$$
$$= 5.46643 = 5.5$$

(c) (i) (1 mark)

Outcomes Assessed: P4

Targeted Performance Bands: 2-3

Criteria	Mark
• Gives the correct answer.	1

Sample answer

$$\angle ABC = 27^\circ + 71^\circ \text{ (bearing of } 071^\circ \text{, and alternate angles)}$$
$$= 98^\circ$$

(c) (ii) (2 marks)

Outcomes Assessed: P4

Targeted Performance Bands: 2-3

Criteria	Mark
• Correctly substitutes into the Cosine rule.	1
• Gives the correct answer.	1

Sample answer

$$AC^2 = 250^2 + 320^2 - 2 \times 250 \times 320 \times \cos 98^\circ$$

$$AC \cong 432.6 \text{ km}$$

$$AC \cong 433 \text{ km}$$

(c) (iii) (2 marks)

Outcomes Assessed: P4, H5

Targeted Performance Bands: 3-4

Criteria	Mark
• Correctly finds angle ACB.	1
• Gives the correct bearing.	1

Sample answer

$$\frac{\sin C}{250} = \frac{\sin 98^\circ}{433}$$

$$\sin C \cong 0.5717483076$$

$$\therefore \angle ACB = 34^\circ 52'$$

$$\begin{aligned} \text{Bearing} &= 180^\circ + 71^\circ + 34^\circ 52' \\ &= 285^\circ 52' \end{aligned}$$

Question 4 (12 marks)

(a)(i) (1 mark)

Outcomes Assessed: H5

Targeted Performance Band: 3-4

Criteria	Mark
• Gives the correct answer.	1

Sample Answer

Using the formula $T_n = a + (n-1)d$, where $a = 38$ and $d = -3$:

$$T_n = 38 + (n-1) \times -3$$

$$= 38 - 3n + 3$$

$$= 41 - 3n$$

(a) (ii) (2 marks)

Outcomes Assessed: H5

Targeted Performance Band: 3-5

Criteria	Mark
• Correctly substitutes into the appropriate formula.	1
• Determines the correct answer.	1

Sample Answer

Substituting into the formula $S_n = \frac{n}{2}[2a + (n-1)d]$, where $S_n = 253$, $a = 38$ and $d = -3$:

$$253 = \frac{n}{2}[2 \times 38 + (n-1) \times -3]$$

$$506 = n(79 - 3n)$$

$$3n^2 - 79n + 506 = 0$$

Using the quadratic formula:

$$n = \frac{79 \pm \sqrt{6241 - 6072}}{6}$$

$$= 15.3 \text{ or } 11$$

\therefore there are 11 rows of paint tins.

(a) (iii) (1 mark)

Outcomes Assessed: H5

Targeted Performance Bands: 3-4

Criteria	Mark
• Gives the correct answer.	1

Sample Answer

$$T_n = 41 - 3(11)$$

$$= 8$$

Therefore there are 8 paint tins in the final row.

(b)(i) (2 marks)

Outcomes Assessed: H4, H5

Targeted Performance Bands: 3-4

Criteria	Mark
• Determines that there are 3 outcomes that give the product of 4.	1
• Correctly determines the probability of the multiples of 4.	1

Sample Answer

List of the sample space

	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	6	8
3	0	3	6	9	12
4	0	4	8	12	16

From the table, there are 8 outcomes that give a multiple of 4.

$$\therefore P(4) = \frac{8}{25}$$

(b)(ii) (1 mark)

Outcomes Assessed: H4, H5

Targeted Performance Bands: 3-4

Criteria	Mark
• Gives the correct answer.	1

Sample Answer

	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	6	8
3	0	3	6	9	12
4	0	4	8	12	16

From the table, there are 11 outcomes that are at least 4:

$$\therefore P(\text{at least } 4) = \frac{11}{25}$$

(c)(i) (1 mark)

Outcomes Assessed: P3, P5

Targeted Performance Bands: 3-5

Criteria	Mark
• Gives the correct answer.	1

Sample Answer

Domain: all real x .

(c)(ii) (1 mark)

Outcomes Assessed: P3, P5

Targeted Performance Bands: 4-5

Criteria	Mark
• Gives the correct answer.	1

Sample Answer

Range: $-3 \leq y \leq 3$.

(d)(i) (2 marks)

Outcomes Assessed: H2, H5

Targeted Performance Bands: 3-5

Criteria	Mark
• Correctly proves one pair of equal angles.	1
• Correctly shows that the triangles are equiangular and therefore similar.	1

Sample Answer

$\angle HJL = \angle KJM$ (common angle).

$\angle HLJ = \angle KMJ$ (corresponding angles in parallel lines, $HL \parallel KM$).

$\angle LHJ = \angle MKJ$ (corresponding angles in parallel lines, $HL \parallel KM$).

Therefore $\triangle HJL \sim \triangle KJM$ (equiangular).

(d)(ii) (2 marks)

Outcomes Assessed: H2, H5

Targeted Performance Bands: 3-5

Criteria	Mark
• Correctly sets up a ratio with the correct reasoning.	1
• Correctly determines the length of MJ .	1

Sample Answer

Let $MJ = x$

$$\therefore \frac{x}{8} = \frac{12}{4} \text{ (ratio of intercepts in parallel lines, } HL \parallel KM \text{)}$$

$$4x = 96$$

$$\therefore x = 24 \text{ cm}$$

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Question 5 (12 marks)

(a) (i) (2 marks)

Outcomes Assessed: P4

Targeted Performance bands: 3-4

Criteria	Mark
• Substitutes the correct trigonometric identity.	1
• Gives the correct answer.	1

Sample Answer

$$\sec^2 \theta - 2 \tan \theta = (\tan \theta - 1)^2$$

$$LHS = \sec^2 \theta - 2 \tan \theta$$

$$= 1 + \tan^2 \theta - 2 \tan \theta$$

$$= \tan^2 \theta - 2 \tan \theta + 1$$

$$= (\tan \theta - 1)^2$$

$$= RHS$$

(a) (ii) (2 marks)

Outcomes Assessed: P4, HS

Targeted Performance Band: 3-4

Criteria	Mark
• Gives ONE correct answer in radians.	1
• Gives TWO correct answers in radians.	1

Sample Answer

$$\sec^2 \theta - 2 \tan \theta = 0$$

$$(\tan \theta - 1)^2 = 0 \quad (\text{from part (i)})$$

$$\tan \theta = 1$$

Basic angle (1st quadrant is $\frac{\pi}{4}$)

$$\therefore \theta = \frac{\pi}{4}, \frac{5\pi}{4}$$

(b) (i) (3 marks)

Outcomes Assessed: P7, H6

Targeted Performance Band: 3-5

Criteria	Mark
• Finds the stationary points.	1
• Finds the nature of ONE stationary point.	1
• Finds the nature of the other stationary point.	1

Sample Answer

$$f(x) = 15 + 12x + 3x^2 - 2x^3 \quad \therefore f'(x) = 12 + 6x - 6x^2$$

$$\text{For stationary points } f'(x) = 0 \quad \therefore 2 + x - x^2 = 0$$

$$(2-x)(x+1) = 0$$

$$x = 2 \quad \text{or } x = -1$$

\therefore the stationary points are (2,35) and (-1,8).

Also for the nature of the stationary points, $f''(x) = 6 - 12x$

At $x = 2$, $f''(2) = -18 < 0 \quad \therefore (2, 35)$ is a MAXIMUM stationary point.

At $x = -1$, $f''(-1) = 18 > 0 \quad \therefore (-1, 8)$ is a MINIMUM stationary point.

(b) (ii) (1 mark)

Outcomes Assessed: P7, H6

Targeted Performance Bands: 3-4

Criteria	Mark
• Correctly determines the point of inflexion.	1

Sample Answer

Point of inflexion occurs at $f''(x) = 0$ and a change in sign occurs,

$$6 - 12x = 0$$

$$x = \frac{1}{2}$$

x	$\frac{1^-}{2}$	$\frac{1}{2}$	$\frac{1^+}{2}$
$f''(x)$	+6	0	-6

Therefore point of inflexion occurs at $(\frac{1}{2}, \frac{43}{2})$.

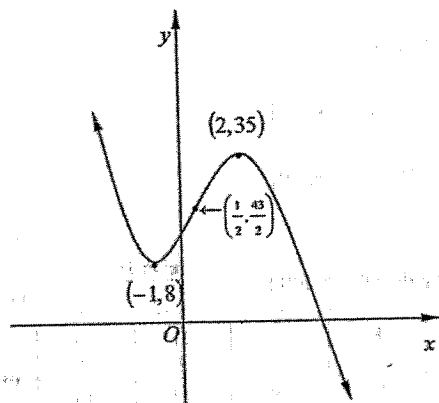
(b) (iii) (2 marks)

Outcomes Assessed: P6, H6, H7

Targeted Performance Bands: 3-5

Criteria	Mark
• Draws the correct cubic curve.	1
• Plots the stationary points and point of inflexion.	1

Sample Answer



(c) (2 marks)

Outcomes Assessed: P4, H4

Targeted Performance Band: 4-5

Criteria	Mark
• Correctly sets up two equations to evaluate p and q using the sums and products or equivalent working.	1
• Correctly evaluates p and q .	1

Sample Answer

$$\alpha + \beta = \frac{1}{p} \quad \text{and} \quad \alpha\beta = \frac{q}{p}$$

$$\therefore -2 + 5 = \frac{1}{p} \Rightarrow \frac{1}{p} = 3 \therefore p = \frac{1}{3}$$

$$\text{Also } -10 = \frac{q}{1/3} \therefore q = -\frac{10}{3}$$

$$\text{Hence } p = \frac{1}{3}, q = -\frac{10}{3}$$

$$\text{Alternatively, } 4p + 2 + q = 0 \quad \dots \textcircled{1}$$

$$25p - 5 + q = 0 \quad \dots \textcircled{2}$$

$$\therefore 21p - 7 = 0$$

$$\therefore p = \frac{1}{3} \text{ \& } q = -\frac{10}{3}$$

Question 6

(a) (i) (2 marks)

Outcomes Assessed: H5, H8

Targeted Performance Band: 3-4

Criteria	Mark
• Correct substitution or equivalent working.	1
• Gives the correct answer.	1

Sample Answer

Using the trapezoidal rule:

$$A = \frac{h}{2} [y_0 + 2(y_1 + y_2 + \dots + y_{n-1}) + y_n]$$

$$A = \frac{5}{2} [0 + 2(8 + 24 + 21 + 23 + 19) + 0]$$

$$\approx 475 \text{ m}^2$$

(Alternatively, using the table below)

y	0	8	24	21	23	19	0	
weight	1	2	2	2	2	2	1	
$y \times \text{wt}$	0	16	48	42	46	38	0	$\Sigma = 190$

$$A \approx \frac{5}{2} \times 190$$

$$\approx 475 \text{ m}^2$$

(a) (ii) (1 mark)

Outcomes Assessed: H5, H8

Targeted Performance Band: 3-4

Criteria	Mark
• Gives the correct answer.	1

Sample Answer

$$V = 475 \times 1.2$$

$$= 570 \text{ m}^3$$

(a) (iii) (1 mark)

Outcomes Assessed: H8

Targeted Performance Band: 3-4

Criteria	Mark
• Correctly applies the ratio to find the answer.	1

Sample Answer

$$5 \text{ lily pads} / 25 \text{ m}^2$$

$$x \text{ lily pads} / 475 \text{ m}^2$$

$$\therefore 25x = 5 \times 475$$

$$x = \frac{5 \times 475}{25}$$

$$= 95 \text{ lily pads.}$$

(b) (i) (1 mark)

Outcomes Assessed: H3

Targeted Performance Band: 3-4

Criteria	Mark
• Gives the correct answer.	1

Sample Answer

$$\text{When } x = 3.2,$$

$$LHS = \log_{10}(6.2) \approx 0.79239...$$

$$RHS = 0.25 \times 3.2 = 0.8 \approx LHS$$

$$\therefore x = 3.2 \text{ is an approximate solution of } \log_{10}(x+3) = \frac{1}{4}x.$$

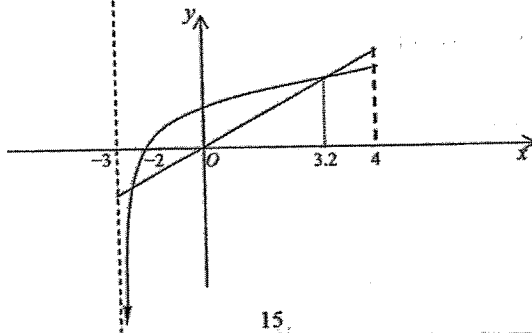
(b) (ii) (2 marks)

Outcomes Assessed: P5, H3

Targeted Performance Band: 4-5

Criteria	Mark
• Correctly sketches $y = \log_{10}(x+3)$, including asymptote at $x = -3$.	1
• Correctly sketches $y = \frac{1}{4}x$, intersecting $y = \log_{10}(x+3)$ at two points.	1

Sample Answer



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(b) (iii) (1 mark)

Outcomes Assessed: H3

Targeted Performance Band: 3-4

Criteria	Mark
• Gives the correct answer.	1

Sample Answer

$$\text{From the graph, there are two solutions to the equation } \log_{10}(x+3) - \frac{1}{4}x = 0, x > -3$$

(c) (i) (2 marks)

Outcomes Assessed: H5

Targeted Performance Band: 3-5

Criteria	Mark
• Correctly substitutes into formula.	1
• Determines α correctly, leaving in exact form.	1

Sample Answer

$$\frac{135\pi}{8} = \frac{1}{2} \times 81 \times \alpha$$

$$\alpha = \frac{5\pi}{12}$$

(c) (ii) (2 marks)

Outcomes Assessed: H5

Targeted Performance Bands: 3-5

Criteria	Mark
• Correctly substitutes in formula.	1
• Determines the area of the minor segment.	1

Sample Answer

$$A = \frac{1}{2} \times 81 \left(\frac{5\pi}{12} - \sin \frac{5\pi}{12} \right)$$

$$\approx \frac{1}{2} \times 81 (1.308996 - 0.965926)$$

$$\approx 13.894335...$$

$$= 13.9 \text{ to one decimal place.}$$

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Question 7 (12 marks)

(a)(i) (1 mark)

Outcomes Assessed: H4, H5

Targeted Performance Band: 3-5

Criteria	Mark
• Gives the correct answer.	1

Sample Answer

When $t = 2.5$ minutes, the displacement is negative because the train is now to the left of the train station.

(a)(ii) (1 mark)

Outcomes Assessed: H4, H5

Targeted Performance Band: 3-5

Criteria	Mark
• Gives the correct answer.	1

Sample Answer

$$V_A(t) = S'_A(t)$$

$$= -3 + 6t - 3t^2$$

∴ the velocity of train A is $(-3 + 6t - 3t^2)$ km/min.

(a)(iii) (1 mark)

Outcomes Assessed: H4, H5

Targeted Performance Band: 3-5

Criteria	Mark
• Gives the correct answer.	1

Sample Answer

Train A will be at rest when $V_A(t) = 0$.

$$-3 + 6t - 3t^2 = 0$$

$$t^2 - 2t + 1 = 0$$

$$(t-1)^2 = 0$$

$$\therefore t = 1$$

∴ Train A is at rest at $t = 1$.

(a)(iv) (2 marks)

Outcomes Assessed: H4, H5

Targeted Performance Band: 4-5

Criteria	Mark
• Gives correct primitive.	1
• Gives correct constant of integration.	1

Sample Answer

$$V_B(t) = 6t - 3$$

$$S_B(t) = \int(6t - 3)dt$$

$$= 3t^2 - 3t + c$$

$$\text{at } t = 0, s = -8$$

$$\therefore -8 = c$$

$$\therefore S_B(t) = 3t^2 - 3t - 8$$

(a)(v) (1 mark)

Outcomes Assessed: H4, H5

Targeted Performance Band: 5-6

Criteria	Mark
• Gives the correct answer.	1

Sample Answer

$$S_A(t) = S_B(t)$$

$$2 - 3t + 3t^2 - t^3 = 3t^2 - 3t - 8$$

$$-t^3 = -10$$

$$\therefore t = \sqrt[3]{10}$$

(b)(i) (2 marks)

Outcomes Assessed: H5, H9

Targeted Performance Band: 4-5

Criteria	Mark
• Derives the correct expression for A_1 .	1
• Shows that A_2 is the given expression.	1

Sample Answer

$$A_1 = 20000(1.00625) + 180$$

$$A_2 = A_1(1.00625) + 180$$

$$= (20000(1.00625) + 180)(1.00625) + 180$$

$$= 20000(1.00625)^2 + 180(1 + 1.00625)$$

(b)(ii) (2 marks)

Outcomes Assessed: H5, H9

Targeted Performance Band: 4-6

Criteria	Mark
• Correctly derives an expression of A_n , and substituting $A_n = 50000$.	1
• Determines the correct number of months.	1

Sample Answer

Let $A_n = 50000$.

$$50000 = 20000(1.00625)^n + 180(1 + 1.00625 + \dots + 1.00625^{n-1})$$

$$50000 = 20000(1.00625)^n + 180 \left[\frac{(1.00625)^n - 1}{0.00625} \right]$$

$$50000 = 20000(1.00625)^n + 28800(1.00625)^n - 28800$$

$$78800 = 48800(1.00625)^n$$

$$(1.00625)^n = \frac{788}{488}$$

$$\log_e(1.00625)^n = \log_e\left(\frac{788}{488}\right)$$

$$n = 76.9 \text{ months}$$

$$= 77 \text{ months}$$

∴ after 77 months/payments Antonia will have reached her goal.

(b)(iii) (2 marks)

Outcomes Assessed: H5, H9

Targeted Performance Band: 4-5

Criteria	Mark
• Substitutes correctly into the compound interest formula.	1
• Gives the correct answer.	1

Sample Answer

Using the formula: $A = P(1+r)^n$ where $n = 15 \times 12$

$$50000 = 20000(1+r)^{180}$$

$$\frac{5}{2} = (1+r)^{180}$$

$$(1+r) = \left(\frac{5}{2}\right)^{\frac{1}{180}}$$

$$1+r = 1.005103$$

$$r = 0.005103$$

$$= 0.51\% \text{ per month}$$

$$= 6.12\% \text{ p.a.}$$

Question 8 (12 marks)

(a) (i) (1 mark)

Outcomes Assessed: P3, P4

Targeted Performance Band: 2-3

Criteria	Mark
• Finds the correct answer.	1

Sample Answer

$$435\pi - \frac{20}{100} \times 435\pi = 348\pi \text{ cm}^2$$

(a) (ii) (2 marks)

Outcomes Assessed: H4, H5

Targeted Performance Band: 3-4

Criteria	Mark
• Correctly equates $348\pi = 2\pi x^2 + 2\pi xy$.	1
• Finds the correct answer.	1

Sample Answer

The surface area of the can $SA = 2\pi x^2 + 2\pi xy$

$$348\pi = 2\pi x^2 + 2\pi xy \Rightarrow 174 = x^2 + xy$$

$$\therefore y = \frac{174 - x^2}{x} = \frac{174}{x} - x$$

(a) (iii) (1 mark)

Outcomes Assessed: H4, H5

Targeted Performance Band: 3-4

Criteria	Mark
• Finds the correct answer.	1

Sample Answer

$$V = \pi \times x^2 \times \left(\frac{174}{x} - x\right) = 174\pi x - \pi x^3$$

(a) (iv) (2 marks)

Outcomes Assessed: H5

Targeted Performance Band: 4-5

Criteria	Mark
• Correctly shows $x = 7.6$ cm.	1
• Shows $x = 7.6$ cm gives a maximum volume.	1

Sample Answer

$$V = 174\pi x - \pi x^3 \Rightarrow \frac{dV}{dx} = 174\pi - 3\pi x^2$$

$$\therefore \text{when } \frac{dV}{dx} = 0, x^2 = 58 \therefore x \approx 7.6 \text{ cm}$$

$$\text{Also, } \frac{d^2V}{dx^2} = -6\pi x = -6 \times \pi \times 7.6 < 0 \therefore \text{maximum when } x = 7.6 \text{ cm.}$$

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(a) (v) (1 mark)

Outcomes Assessed: H5

Targeted Performance Band: 2-3

Criteria	Mark
• Finds the correct answer.	1

Sample Answer

$$V = 174\pi \times 7.6 - \pi \times 7.6^3 = 2775 \text{ cm}^3$$

(b) (i) (2 marks)

Outcomes Assessed: H3, H4, H5

Targeted Performance Band: 3-5

Criteria	Mark
• Correctly shows $M_0 = 88$.	1
• Correctly shows $k = 0.020764$.	1

Sample Answer

$$\text{When } t = 0, M = 88 \therefore M_0 = 88$$

$$\text{When } t = 9, M = 73$$

$$73 = 88e^{-k(9)} \Rightarrow \ln\left(\frac{73}{88}\right) = \ln(e^{-9k})$$

$$\therefore -9k = \ln\left(\frac{73}{88}\right) \therefore k = 0.020764$$

(b) (ii) (1 mark)

Outcomes Assessed: H3, H4

Targeted Performance Band: 3-4

Criteria	Mark
• Finds the correct answer.	1

Sample Answer

$$M = 88e^{-0.020764(18)}$$

$$M = 60.6 \text{ kg}$$

(b) (iii) (2 marks)

Outcomes Assessed: H3, H4, H5

Targeted Performance Band: 3-5

Criteria	Mark
• Finds an correct expression for $\frac{dM}{dt}$.	1
• Finds the correct answer.	1

Sample Answer

$$\frac{dM}{dt} = -k \times 88e^{-kt}$$

$$= -0.020764 \times 60.6$$

$$= -1.3 \text{ kg/year}$$

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Question 9 (12 marks)

(a) (i) (1 mark)

Outcomes Assessed: H5, H8

Targeted Performance Band: 3-5

Criteria	Mark
• Substitutes correctly and simplifies to derive the given expression.	1

Sample Answer

$$V = \pi \int_0^{\ln 3} (e^x + 1)^2 dx \therefore V = \pi \int_0^{\ln 3} (e^{2x} + 2e^x + 1) dx$$

(a) (ii) (3 marks)

Outcomes Assessed: H5, H8

Targeted Performance Band: 3-5

Criteria	Mark
• Finds the primitive of $\int_0^{\ln 3} (e^{2x} + 2e^x + 1) dx$ and correctly applies the Newton-Leibnitz formula.	2
• Gives correct answer $\pi[8 + \ln 3]$ or equivalent.	1

Sample Answer

$$V = \pi \int_0^{\ln 3} e^{2x} + 2e^x + 1 dx$$

$$\therefore V = \pi \left[\frac{e^{2x}}{2} + 2e^x + x \right]_0^{\ln 3} = \pi \left[\left(\frac{e^{2 \ln 3}}{2} + 2e^{\ln 3} + \ln 3 \right) - \left(\frac{1}{2} + 2 + 0 \right) \right]$$

$$= \pi \left[\left(\frac{9}{2} + 6 + \ln 3 \right) - 2\frac{1}{2} \right] = \pi(8 + \ln 3)$$

(b) (i) (1 mark)

Outcomes Assessed: H5

Targeted Performance Band: 3-4

Criteria	Mark
• Gives correct answer.	1

Sample Answer

$$\text{Angle of each sector} = 360^\circ \div 12 = 30^\circ \therefore \text{In } \triangle AOB, \sin 30^\circ = \frac{X_1}{4} \therefore \frac{1}{2} = \frac{X_1}{4}$$

$$\therefore X_1 = 4 \times \frac{1}{2} = 2 \text{ cm.}$$

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(b) (ii) (3marks)

Outcomes Assessed: H5

Targeted Performance Band: 3-5

Criteria	Mark
• Correctly obtains the length of X_2 in exact form.	1
• Correctly obtains the length of X_3 in exact form.	1
• Correctly calculates the value of r to prove X_1, X_2, X_3, \dots form a geometric series.	1

Sample Answer

$$\text{In } \triangle OBA, OB^2 = 4^2 - 2^2 = 12 \quad \therefore OB = 2\sqrt{3} \text{ cm.}$$

$$\therefore \text{In } \triangle OCB, \sin 30^\circ = \frac{X_2}{OB} = \frac{X_2}{2\sqrt{3}} \quad \therefore \frac{1}{2} = \frac{X_2}{2\sqrt{3}} \quad \therefore X_2 = \sqrt{3}$$

$$\text{Similarly } X_3 = \frac{3}{2} \quad \therefore X_1, X_2, X_3, \dots = 2, \sqrt{3}, \frac{3}{2}, \dots$$

$$\therefore \frac{\sqrt{3}}{2} = \frac{1}{2}\sqrt{3} \text{ and } \frac{3}{2} \div \sqrt{3} = \frac{1}{2}\sqrt{3}$$

$\therefore X_1, X_2, X_3, \dots$ forms a geometric sequence with a common ratio equal to $\frac{1}{2}\sqrt{3}$.

(b) (iii) (1mark)

Outcomes Assessed: H5

Targeted Performance Band: 3-5

Criteria	Mark
• Correctly uses the general term formula to obtain the distance X_{12} in exact form.	1

Sample Answer

$$X_{12} = ar^{11} = 2 \times \left[\frac{\sqrt{3}}{2} \right]^{11}$$

(b) (iv) (2marks)

Outcomes Assessed: H5

Targeted Performance Band: 3-5

Criteria	Mark
• Correctly uses the sum of n terms formula to obtain the length one spiral.	1
• Correctly calculates the value of S_{12} .	1

Sample Answer

$$S_{12} = \frac{2 \left[1 - \left(\frac{\sqrt{3}}{2} \right)^{12} \right]}{1 - \frac{\sqrt{3}}{2}} = \frac{2 \left[1 - \frac{729}{4096} \right]}{\frac{2 - \sqrt{3}}{2}} = \frac{4 \times \frac{3367}{4096}}{2 - \sqrt{3}} = \frac{3367}{1024} \div (2 - \sqrt{3}) = 12.27 \text{ cm}$$

(b) (v) (1mark)

Outcomes Assessed: H5, H9

Targeted Performance Band: 3-5

Criteria	Mark
• Correctly uses the limiting sum formula to obtain the total length.	1

Sample Answer

$$S_{\infty} = \frac{2}{1 - \frac{\sqrt{3}}{2}} = \frac{4}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}} = 4(2 + \sqrt{3}) = 14.9$$

\therefore the total length will not exceed 15 cm.

Question 10 (12 marks)

(a) (2 marks)

Outcomes Assessed: H7

Targeted Performance Band: 3-4

Criteria	Mark
• Finds the gradient of the tangent.	1
• Gives correct answer.	1

Sample Answer

(a) The gradient = $\frac{\text{rise}}{\text{run}} = \frac{-2}{4} = -\frac{1}{2} \therefore f'(4) = -\frac{1}{2}$.

Alternatively:

Equation of the line L is given by: $y = mx + b$ sub (4, 8) $\therefore 8 = 4m + 10$

$\therefore m = -\frac{1}{2} \therefore f'(4) = -\frac{1}{2}$.

(b)(i) (1 mark)

Outcomes Assessed: H5

Targeted Performance Band: 4-5

Criteria	Mark
• Establishes base angles of isosceles triangle are equal.	1
• Gives correct answer.	1

Sample Answer

$AC = BC \therefore \angle BAC = \frac{\pi - 3\alpha}{2} = \left(\frac{\pi}{2} - \frac{3}{2}\alpha\right)$ (angle sum of isosceles triangle with base \angle 's equal)

(b)(ii) (2marks)

Outcomes Assessed: H5

Targeted Performance Band: 4-6

Criteria	Mark
• Correctly uses the Sine Rule in ΔACD realising that $AD = (2 - y)$.	1
• Correctly finds the required value by realising that $\sin\left(\frac{\pi}{2} - \frac{3}{2}\alpha\right) = \cos\left(\frac{3}{2}\alpha\right)$.	1

Sample Answer

$BD = y$ and since $AB = 2$ then $AD = (2 - y)$.

Using ΔACD , $\frac{(2-y)}{\sin 2\alpha} = \frac{CD}{\sin\left(\frac{\pi}{2} - \frac{3}{2}\alpha\right)} \therefore \frac{(2-y)}{\sin 2\alpha} = \frac{CD}{\cos\left(\frac{3}{2}\alpha\right)}$

$\therefore CD = \frac{(2-y)\cos\left(\frac{3}{2}\alpha\right)}{\sin 2\alpha} \dots \textcircled{1}$

(b)(iii) (2marks)

Outcomes Assessed: H5

Targeted Performance Band: 4-6

Criteria	Mark
• Correctly uses the Sine Rule in ΔBCD .	1
• Gives correct answer.	1

Sample Answer

In ΔBCD , $\frac{y}{\sin \alpha} = \frac{CD}{\sin\left(\frac{\pi}{2} - \frac{3}{2}\alpha\right)} \therefore CD = \frac{y\cos\left(\frac{3}{2}\alpha\right)}{\sin \alpha} \dots \textcircled{2}$

(b)(iv) (2marks)

Outcomes Assessed: H5

Targeted Performance Band: 5-6

Criteria	Mark
• Writes the expression $1 = \frac{(2-y)}{y} \times \frac{\sin \alpha}{2 \sin \alpha \cos \alpha}$ or equivalent.	1
• Correctly calculates the value of BD .	1

Sample Answer

$$\begin{aligned} \therefore \textcircled{1} \div \textcircled{2} \quad 1 &= \frac{(2-y)}{y} \times \frac{\sin \alpha}{2 \sin \alpha \cos \alpha} \\ 1 &= \frac{(2-y)}{2y \cos \alpha} \quad \therefore y(1+2 \cos \alpha) = 2 \quad \therefore (BD=y) = \frac{2}{1+2 \cos \alpha} \end{aligned}$$

(b)(v) (2marks)

Outcomes Assessed: H2, H5

Targeted Performance Band: 5-6

Criteria	Mark
• Writes the expression $1 < 2 \cos \alpha < 2$.	1
• Gives correct answer.	1

Sample Answer

$$\begin{aligned} \text{Since } 0 < \alpha < \frac{\pi}{3} \quad \therefore \cos 0 > \cos \alpha > \cos \frac{\pi}{3} \quad \text{i.e. } 1 > \cos \alpha > \frac{1}{2} \\ \therefore \frac{1}{2} < \cos \alpha < 1 \quad \rightarrow \quad 1 < 2 \cos \alpha < 2 \quad \rightarrow \quad 1+1 < 1+2 \cos \alpha < 2+1 \\ \therefore 2 < 1+2 \cos \alpha < 3 \quad \rightarrow \quad \frac{1}{2} > \frac{1}{1+2 \cos \alpha} > \frac{1}{3} \\ \therefore 1 > \frac{2}{1+2 \cos \alpha} > \frac{2}{3} \\ \therefore \frac{2}{3} < \frac{2}{1+2 \cos \alpha} < 1 \\ \text{i.e. } \frac{2}{3} < BD < 1 \end{aligned}$$