Centre Number

Student Number

CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NEW SOUTH WALES

2009
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

Morning Session
Monday 17 August 2009

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided on the back page
- All necessary working should be shown in every question
- Write your Centre Number and Student Number at the top of this page AND on the separate Answer Booklets provided for each question

Total marks – 120

- Attempt Questions 1-10
- All questions are of equal value

6200-1

Total marks – 120
Attempt Questions 1-10
All questions are of equal value.

Answer each question in a SEPARATE writing booklet.

Marks

Question 1 (12 marks) Use a SEPARATE writing booklet.

(a) Evaluate $\dfrac{2+\sqrt{2}}{7(e^2-4)}$ correct to three significant figures. **2**

(b) Solve $(y-2)^2=9$. **2**

(c) Find a primitive of $\dfrac{x}{3}+\dfrac{1}{x^2}$. **2**

(d) Solve $|5a+3|\le 13$. **2**

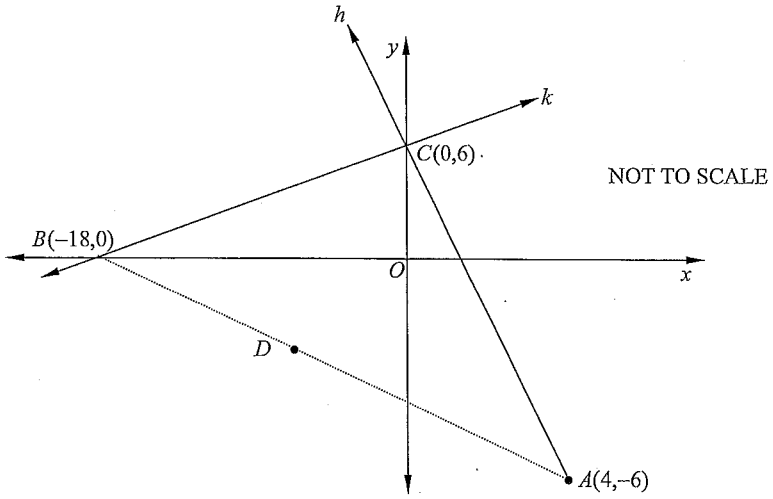
(e) Find the limiting sum of the series $20+4+\dfrac{4}{5}+\ldots$ **2**

(f) If $g(x)=7x^3-3x+1$ evaluate $g'(2)$. **2**

Marks

Question 2 (12 marks) Use a SEPARATE writing booklet.

(a) In the diagram, the lines h and k are drawn. The coordinates of A, B and C are $(4,-6)$, $(-18,0)$ & $(0,6)$ respectively. D is the midpoint of AB.



(i) Show that D has coordinates $(-7,-3)$. **1**

(ii) Calculate the length of DC. Leave your answer in exact form. **1**

(iii) Show that the equation of the line h is given by $3x+y-6=0$. **2**

(iv) Show that the line h is perpendicular to the line k. **2**

(v) AB is the diameter of a circle which passes through the points A, B and C. Show that the equation of the circle is given by $(x+7)^2+(y+3)^2=130$. **2**

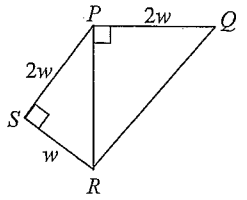
(vi) Find the area of the circle which passes through A, B and C. Leave your answer in terms of π. **1**

(b) Find the values of q if $3qx^2-5x+3q=0$ is negative definite. Leave your answer in exact form. **3**

Marks

Question 3 (12 marks) Use a SEPARATE writing booklet.

(a) The diagram shows two right angled triangles PQR and PRS.



NOT TO SCALE

Let $SR=w$. The lengths SP and PQ are each twice the length of SR.

(i) Prove that $QR=3w$. **2**

(ii) Find the area of quadrilateral $PQRS$ in terms of w. **2**

(b) Differentiate with respect to x and leave your answers in simplest form:

(i) $\dfrac{\ln x}{x}$. **2**

(ii) $(x-5)^2e^x$. **2**

(c) (i) Find $\displaystyle\int \frac{3x}{x^2-9}\,dx$. **2**

(ii) Evaluate $\displaystyle\int_0^3 \sqrt{x}\,dx$. **2**

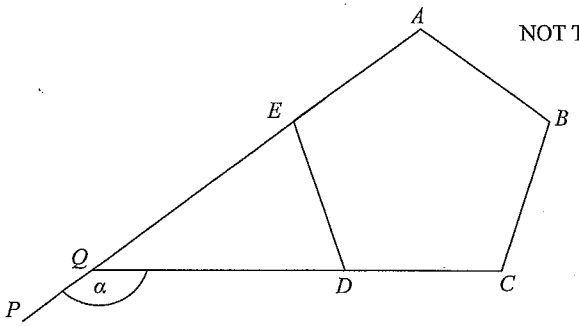
Question 4 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Evaluate $\sum_{n=2}^{16}(13-5n)$. **2**

(b) Prove that **3**

$$\frac{\sin\theta}{1-\cos\theta}+\frac{\sin\theta}{1+\cos\theta}=2\operatorname{cosec}\theta$$

(c) $ABCDE$ is a regular pentagon. The points P, Q, E and A are collinear. The points Q, D and C are also collinear.



Find the size of angle α giving reasons. **2**

(d) A six-sided die is biased so that the number 3 occurs twice as often as any other number.

(i) The die is rolled once. Show that the probability that an odd number occurs is $\frac{4}{7}$. **1**

(ii) If the biased die is rolled twice, find the probability of the sum of the uppermost numbers being six. **2**

This biased die is now rolled together with TWO fair six-sided dice.

(iii) What is the probability that at least two odd numbers are uppermost? **2**

Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Solve for x: $\log_{10}x^6-8=3\log_{10}x$. **2**

Give your answer correct to 1 decimal place.

(b) Consider the curve given by $y=2x^3-9x^2+12x$.

(i) Find the stationary points and determine their nature. **3**

(ii) Show that a point of inflexion occurs at $x=\frac{3}{2}$. **1**

(iii) Sketch the graph of $y=f(x)$ indicating clearly the stationary points and point of inflexion. **2**

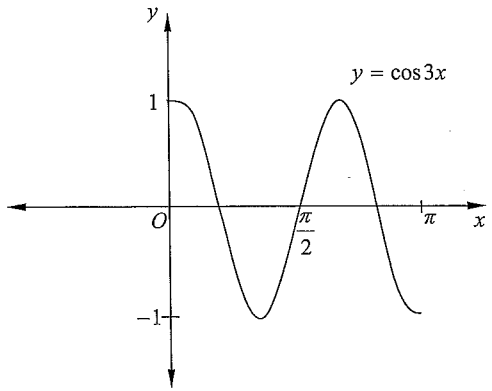
(iv) For what values of x is the curve concave up? **1**

(c) Solve $2(x^2+1)^2-19(x^2+1)-10=0$. **3**

Marks

Question 6 (12 marks) Use a SEPARATE writing booklet

(a) The graph of $y = \cos 3x$ is shown below.



NOT TO SCALE

(i) Solve $\cos 3x = 0$ for $0 \leq x \leq \pi$. **2**

(ii) State the amplitude and period of $y = \cos 3x$. **2**

(iii) Copy this diagram into your answer booklet showing the x-intercepts. **2**

Hence sketch the graph of $y = \sec 3x$ in the domain $0 \leq x \leq \pi$ showing any asymptotes.

$y = \frac{1}{\cos 3x}$

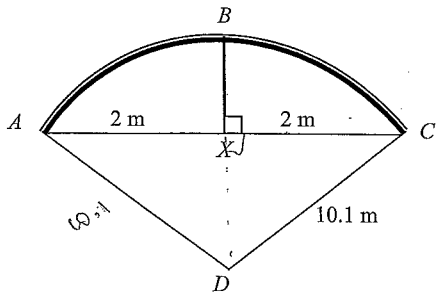
(iv) Using (iii), find the number of solutions to $\sec 3x = x$ in the domain $0 \leq x \leq \pi$. **2**

Question 6 continues on page 8

Marks

Question 6 (continued)

(b) A bridge's steel arch ABC is part of a circle of radius 10.1 metres. BX bisects the chord AC which is 4 metres long.



NOT TO SCALE

(i) Find the size of angle ADC correct to the nearest degree. **2**

(ii) Find the length of steel needed to make the arch ABC. **2**

End of Question 6

Question 7 (12 marks) Use a SEPARATE writing booklet

Marks

(a) Consider the parabola $x^2 = 8(y+2)$.

(i) Find the coordinates of the vertex. **1**

(ii) Find the coordinates of the focus. **1**

(iii) Find the equation of the tangent to the parabola at the point $\left(2, -\frac{3}{2}\right)$. **2**

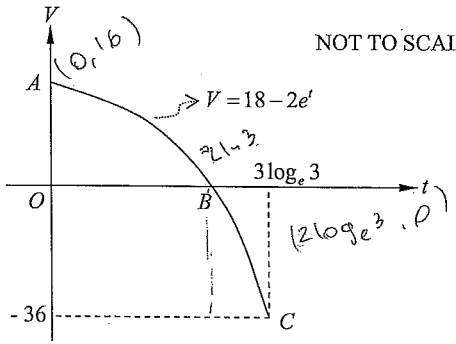
(iv) Find the coordinates of the point where the tangent meets the directrix. **1**

(b) The velocity, V, in m/s of a particle moving in a straight line is given by $V = 18 - 2e^t$, where t is the time in seconds.

(i) Find the initial velocity of the particle. **1**

(ii) Show that the time at which the particle comes to rest is $2\log_e 3$ seconds. **2**

(iii) The graph of the velocity V of the particle as a function of t is given below.



The coordinates of point C are $(3\log_e 3, -36)$.
Write down the coordinates of points A and B. **1**

(iv) Hence, or otherwise, find the distance travelled by the particle between $t = 0$ and $t = 3\log_e 3$. **3**

Marks

Question 8 (12 marks) Use a SEPARATE writing booklet

(a) Kenny begins his retirement with \$500 000 at the beginning of 2009. The annual interest rate is 8% p.a. Interest is calculated annually on the balance at the beginning of the year and added to the remaining balance. Kenny plans to withdraw \$56 000 annually, with the first withdrawal at the end of 2009.

Let A_n be the remaining balance after the nth withdrawal.

(i) Show that $A_2 = (5\times10^5)(1.08)^2 - 5.6\times10^4(1.08+1)$. **2**

(ii) Show that $A_n = 10^5\left[7 - 2(1.08)^n\right]$. **2**

(iii) In which year will Kenny's fund reach zero? **3**

(b) Populations cannot increase indefinitely. Environmental and economic factors such as limited food, weather and space control the size of the population.

Two thousand kangaroos, each aged 2 years old, are released into the wild on an island. After 3 years there are approximately 1800 kangaroos that inhabit the island. The size of the population, N, after t years is predicted by the equation

$$N = N_0\, e^{-kt}.$$

(i) Show that the size of the kangaroo population decreases at a rate proportional to the size of the population. **1**

(ii) Find the value of N_0 and k. **2**

(iii) After how many years will the kangaroo population halve? **2**

Question 9 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) The equation below refers to the filtering cycle of a pump in Helen's garden.

The flow rate of the volume of water that the filter pumps water into and out of a pond in litres per minute, is given by

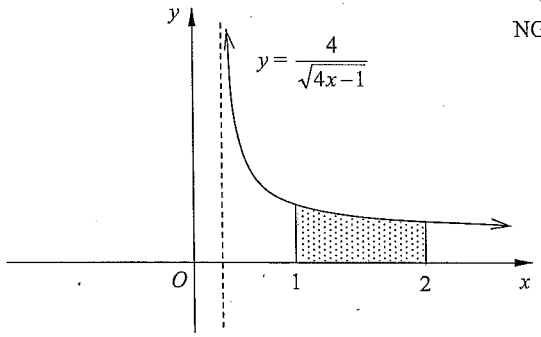
$$\frac{dV}{dt} = 20\sin\frac{\pi}{35}t.$$

(i) If the pump started at 8.55 pm, what is the first time after 8.55 pm at which the flow rate is zero? **2**

(ii) If the pond is initially empty find an expression for the volume, V, of water in the pond after t minutes. **3**

(iii) Find the maximum volume of water in the pond during the filtering cycle. Leave your answer in terms of π. **2**

(b)



NOT TO SCALE

The area enclosed by the curve $y = \frac{4}{\sqrt{4x-1}}$ the lines $x = 1$ and $x = 2$ is shaded as shown in the diagram above.

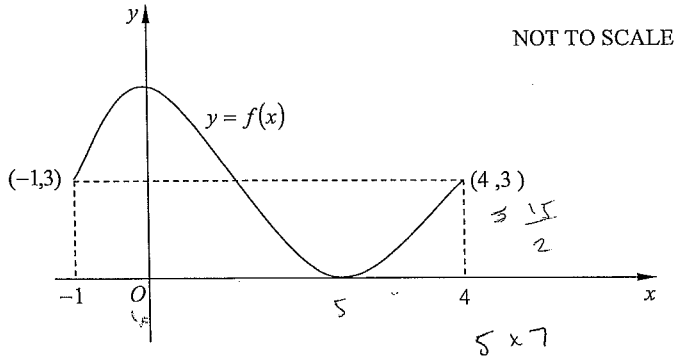
(i) Show that the volume, V, of the solid formed when this shaded region is rotated about the x-axis is given by: **2**

$$V = 4\pi\int_1^2 \frac{4}{4x-1}\,dx.$$

(ii) Hence calculate the volume, V. Leave your answer in exact form. **3**

Question 10 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) The graph below represents the function $y = f(x)$.



NOT TO SCALE

If $\int_{-1}^{4} f(x)\,dx = \frac{15}{2}$, find the value of $\int_{-1}^{4} [f(x)+4]\,dx$. **2**

(b) Given $\frac{d}{dx}(b^x) = b^x \log_e b$, **3**

Evaluate $\int_0^{\pi} \pi^x\,dx$ correct to one decimal place.

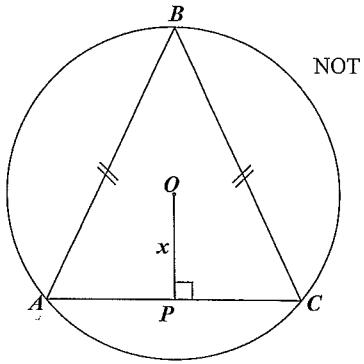
Question 10 continues on page 13

Question 10 (continued)

Marks

(c) An isosceles triangle ABC, where $AB = BC$, is inscribed in a circle of radius 10cm.

$OP = x$ and OP bisects AC, such that $AC \perp OP$.



NOT TO SCALE

(i) Show that the area, A, of ΔABC is given by $A = (10 + x)\sqrt{100 - x^2}$. **2**

(ii) Show that $\dfrac{dA}{dx} = \dfrac{100 - 10x - 2x^2}{\sqrt{100 - x^2}}$. **2**

(iii) Hence prove that the triangle with maximum area is equilateral. **3**

End of Paper

CATHOLIC SECONDARY SCHOOLS ASSOCIATION

2009 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

MATHEMATICS – SUGGESTED SOLUTIONS

These marking guidelines show the criteria to be applied to responses along with the marks to be awarded in line with the quality of responses. These guidelines are suggested and not prescriptive. This is not intended to be an exhaustive list but rather an indication of the considerations that students could include in their responses.

Question 1 (12 marks)

(a) (2 marks)

Outcomes Assessed: P3, H3

Targeted Performance Bands: 2-3

Criteria	Marks
• Gives the correct answer.	1
• Correctly rounds to THREE significant figures.	1

Sample answer:

$$\frac{2+\sqrt{2}}{7(e^2-4)} = 0.1439175925$$

$= 0.144$ (3 significant figures)

(b) (2 marks)

Outcomes Assessed: P3, P4

Targeted Performance Bands: 2-3

Criteria	Marks
• Progress towards the correct answer.	1
• Gives the correct answer.	1

Sample answer:

$(y-2)^2 = 9 \quad \therefore y-2 = \pm 3$

$y = 5 \quad , \quad y = -1$

(c) (2 marks)

Outcomes Assessed: H8

Targeted Performance Bands: 3-4

Criteria	Marks
• Correct primitive of ONE of the terms.	1
• Correct primitive of the other term.	1

Sample answer:

$$\text{Primitive } \frac{x^2}{6} + \frac{x^{-1}}{-1} = \frac{x^2}{6} - \frac{1}{x} + C$$

(d) (2 marks)

Outcomes Assessed: P3, P4

Targeted Performance Bands: 2-4

Criteria	Marks
• Writes TWO correct inequations.	1
• Gives the correct answer.	1

Sample answer:

$5a+3 \le 13$ *or* $-(5a+3) \le 13$

$5a \le 10$ $\quad -5a-3 \le 13$

$a \le 2$ $\quad -5a \le 16$

$a \ge -3.2$

(e) (2 marks)

Outcomes Assessed: H5

Targeted Performance Bands: 3-4

Criteria	Marks
• Correctly finds the value of r.	1
• Gives the correct answer.	1

Sample answer:

$$20+4+\frac{4}{5}+\ldots = \frac{20}{1-\frac{1}{5}} = 25$$

DISCLAIMER
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(f) (2 marks)
Outcomes Assessed: P7, H5
Targeted Performance Bands: 2-3

Criteria	Marks
• Correct derivative.	1
• Gives the correct answer.	1

Sample answer:

$$g(x) = 7x^3 - 3x + 1$$
$$g'(x) = 21x^2 - 3$$
$$g'(2) = 21(2^2) - 3 = 81$$

Question 2 (12 marks)
(a) (i) (1 mark)
Outcomes Assessed: P3, P4, H5
Targeted Performance Bands: 2-3

Criteria	Mark
• Gives the correct answer.	1

Sample answer:

$$x = \frac{-18+4}{2} = -7 \qquad y = \frac{0+-6}{2} = -3$$

(a) (ii) (1 mark)
Outcomes Assessed: P3, P4 , H5
Targeted Performance Bands: 2-3

Criteria	Mark
• Gives the correct answer.	1

Sample answer:

$$DC = \sqrt{(-7-0)^2 + (-3-6)^2} = \sqrt{130}$$

(a) (iii) (2 marks)
Outcomes Assessed: P3, P4, H5
Targeted Performance Bands: 2-3

Criteria	Marks
• Finds the correct gradient AC.	1
• Gives the correct equation of line.	1

Sample answer:

$$m = \frac{6--6}{0-4} = -3$$
$$\therefore y - 6 = -3(x-0)$$
$$y - 6 = -3x$$
$$3x + y - 6 = 0$$

(a) (iv) (2 marks)
Outcomes Assessed: P3, P4, H5
Targeted Performance Bands: 2-3

Criteria	Marks
• Finds the correct gradient BC.	1
• Correctly shows $m_1 \times m_2 = -1$.	1

Sample answer:

$m_{BC} = \frac{6-0}{0--18} = \frac{1}{3}$, $\therefore$ h is perpendicular to k since $\frac{1}{3} \times -3 = -1$.

(a) (v) (2 marks)
Outcomes Assessed: P3, P4, H5
Targeted Performance Bands: 3-4

Criteria	Marks
• Finds correct distance DA or DB.	1
• Gives the correct answer.	1

Sample answer:

$$DA = \sqrt{(-7-4)^2 + (-3--6)^2} = \sqrt{130}$$

Using $(x-h)^2 + (y-k)^2 = r^2$, the centre of the circle is $(-7,-3)$ with radius $\sqrt{130}$.
Then by substitution $(x+7)^2 + (y+3)^2 = 130$

(a) (vi) (1 mark)
Outcomes Assessed: P3, P4, H5
Targeted Performance Bands: 2 - 3

Criteria	Mark
• Gives the correct answer.	1

Sample answer:

$A = \pi r^2 = \pi \times (\sqrt{130})^2 = 130\pi$ units^2

(b) (3 marks)
Outcomes Assessed: P3, H2
Targeted Performance Bands: 3 - 4

Criteria	Marks
• Establishes that $\Delta < 0$.	1
• Solves the quadratic inequation correctly.	1
• Gives the correct answer.	1

Sample answer:

$3qx^2 - 5x + 3q = 0$. For negative definite, $3q < 0$ and $(-5)^2 - 4(3q)(3q) < 0$

$(-5)^2 - 4(3q)(3q) < 0$

$25 - 36q^2 < 0 \quad \therefore (5-6q)(5+6q) < 0$

$q < \frac{-5}{6}$ & $q > \frac{5}{6}$

But $3q < 0$

$\therefore q < \frac{-5}{6}$

Question 3 (12 marks)
(a) (i) (2 marks)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 2-4

Criteria	Marks
• Finds a correct expression for PR.	1
• Gives the correct proof.	1

Sample answer:

$PR^2 = (2w)^2 + w^2 = 5w^2$

$QR^2 = (2w)^2 + 5w^2 = 9w^2$

$QR = \pm 3w \quad \therefore QR = 3w$

(a) (ii) (2 marks)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 2-3

Criteria	Marks
• Finds the correct area of triangle PSR.	1
• Finds the correct area of triangle PQR.	1

Sample answer:

$A = \frac{1}{2}(w)(2w) + \frac{1}{2}(2w)(\sqrt{5}w)$

$A = w^2 + w^2\sqrt{5} = w^2(1+\sqrt{5}) \ \ u^2$

(b) (i) (2 marks)
Outcomes Assessed: P7, H5
Targeted Performance Bands: 3-4

Criteria	Marks
• Differentiates using the quotient rule with ONE mistake.	1
• Gives the correct answer.	1

Sample answer:

Let $y = \frac{\ln x}{x}$ $\quad u = \ln x \therefore u' = \frac{1}{x}$ & $v = x \therefore v' = 1$

$\frac{dy}{dx} = \frac{vu' - uv'}{v^2} = \frac{x.\frac{1}{x} - \ln x.1}{x^2} = \frac{1 - \ln x}{x^2}$

(b) (ii) (2 marks)
Outcomes Assessed: P7, H5
Targeted Performance Bands: 3-4

Criteria	Marks
• Correctly uses the product rule but has ONE mistake.	1
• Gives the correct answer.	1

Sample answer:

Let $y = (x-5)^2 e^x \quad u = (x-5)^2 \therefore u' = 2(x-5)$ & $v = e^x \therefore v' = e^x$

$\frac{dy}{dx} = uv' + vu' = (x-5)^2.e^x + e^x.2(x-5) = e^x(x-5)(x-3)$

(c) (i) (2 marks)
Outcomes Assessed: H8
Targeted Performance Bands: 3-4

Criteria	Marks
• Finds the correct logarithmic primitive but has ONE mistake	1
• Gives the correct answer.	1

Sample answer:

$$\int \frac{3x}{x^2-9}\,dx = \frac{3}{2}\int\left(\frac{2x}{x^2-9}\right)dx$$
$$= \frac{3}{2}\log_e\left(x^2-9\right) + C$$

(c) (ii) (2 marks)
Outcomes Assessed: H8
Targeted Performance Bands: 3-4

Criteria	Marks
• Finds the primitive of $\sqrt{x}$ but has an error in calculating the integral.	1
• Correctly applies Newton–Leibnitz formula to obtain the answer.	1

Sample answer:

$$\int_0^3 \sqrt{x}\,dx$$
$$= \left[\frac{2x^{\frac{3}{2}}}{3}\right]_0^3 = \left(2\left(\frac{3^{1.5}}{3}\right) - 2\left(\frac{0^{1.5}}{3}\right)\right) = \frac{2\sqrt{27}}{3} = 3.464$$

Question 4 (12 marks)
(a) (2 marks)
Outcomes Assessed: H4
Targeted Performance Band: 3-4

Criteria	Marks
• Gives correct a, d and n	1
• Gives the correct answer.	1

Sample answer:

$$\sum_{n=2}^{16} 13 - 5n = (13-10) + (13-15) + (13-20) + \ldots + (13-80)$$
$$= 3 - 2 - 7 - \ldots - 67$$

This is an arithmetic series with $a = 3$, $d = -5$ and $n = 15$.

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right) \qquad S_n = \frac{n}{2}(a + l)$$
$$= \frac{15}{2}\left(2\times 3 + 14\times(-5)\right) \quad \text{or} \quad S_n = \frac{15}{2}(3 - 67)$$
$$= -480 \qquad = -480$$

(b) (3 marks)
Outcomes Assessed: P3
Targeted Performance Band: 3-4

Criteria	Marks
• Finds correct denominator and numerator.	1
• Changes $(1 - \cos^2\theta)$ to $\sin^2\theta$.	1
• Gives the correct proof.	1

Sample Answer:

$$\text{LHS} = \frac{\sin\theta}{1-\cos\theta} + \frac{\sin\theta}{1+\cos\theta}$$
$$= \frac{(1+\cos\theta)\sin\theta + \sin\theta(1-\cos\theta)}{1-\cos^2\theta}$$
$$= \frac{\sin\theta + \sin\theta\cos\theta + \sin\theta - \sin\theta\cos\theta}{1-\cos^2\theta}$$
$$= \frac{2\sin\theta}{\sin^2\theta}$$
$$= \frac{2}{\sin\theta}$$
$$= 2\operatorname{cosec}\theta$$

(c) (2 marks)
Outcomes Assessed: P4
Targeted Performance Band: 3-4

Criteria	Marks
• Gives the angle in the regular pentagon.	1
• Gives the correct angle giving reasons.	1

Sample Answer:

Angle in pentagon $= (n-2)\times 180^\circ$
$= 3\times 180^\circ$
$= 540^\circ$

Size of each angle $= 540 \div 5 = 108^\circ$

$\angle DEA = 108^\circ$ and $\angle EDC = 108^\circ$

$\therefore \angle QED = 72^\circ$ (straight angle) also $\angle EDQ = 72^\circ$ (similarly)

$\angle PQD = 72^\circ + 72^\circ$ (exterior angle of a triangle is equal to the sum of the two interior opposite angles)

$\therefore \alpha = 144^\circ$

(d) (i) (1 mark)
Outcomes Assessed: H5
Targeted Performance Band: 3-4

Criteria	Mark
• Gives the correct answer.	1

Sample Answer:

Sample space is {1, 2, 3, 3, 4, 5, 6}

$P(E) = \frac{4}{7}$

(d) (ii) (2 marks)
Outcomes Assessed: H5
Targeted Performance Band: 3-4

Criteria	Marks
• Uses the correct outcomes.	1
• Gives the correct answer with required working	1

Sample Answer:

$P(E) = P(1,5) + P(2,4) + P(3,3) + P(4,2) + P(1,5)$

$= 4\times\frac{1}{7}\times\frac{1}{7} + \frac{2}{7}\times\frac{2}{7}$

$= \frac{8}{49}$

(d) (iii) (2 marks)
Outcomes Assessed: H5
Targeted Performance Band: 4-5

Criteria	Marks
• Uses the correct outcomes.	1
• Gives the correct answer with required working	1

Sample Answer:

P(at least 2 odd numbers) $= P$(3 odd numbers) + P(2 odd and 1 even number)

P(odd number on a fair die) $= \frac{1}{2}$ P(odd number on biased die) $= \frac{4}{7}$

P(even number on a fair die) $= \frac{1}{2}$ P(even number on biased die) $= \frac{3}{7}$

$P(E) = \frac{1}{2}\times\frac{1}{2}\times\frac{4}{7}$ + P((odd, odd, even) or (odd, even, odd) or (even, odd, odd))

$= \frac{1}{7} + \frac{4}{7}\times\frac{1}{2}\times\frac{1}{2} + \frac{4}{7}\times\frac{1}{2}\times\frac{1}{2} + \frac{3}{7}\times\frac{1}{2}\times\frac{1}{2}$

$= \frac{15}{28}$

Question 5 (12 marks)
(a) (2 marks)
Outcomes Assessed: H3
Targeted Performance Band: 4-5

Criteria	Marks
• Correct solution for $\log_{10} x$.	1
• Gives the correct answer.	1

Sample Answer:

$\log_{10} x^6 - 8 = 3\log_{10} x$

$6\log_{10} x - 8 = 3\log_{10} x$

$3\log_{10} x = 8$

$\log_{10} x = \frac{8}{3}$

$10^{\frac{8}{3}} = x$

$x = 464.2$

(b) (i) (3 marks)

Outcomes Assessed: P7, H6

Targeted Performance Band: 3-5

Criteria	Marks
• Finds the stationary points	1
• Finds the nature of ONE stationary point	1
• Finds the nature of the other stationary point	1

Sample Answer :

$y = 2x^3 - 9x^2 + 12x \quad \therefore \frac{dy}{dx} = 6x^2 - 18x + 12$

For stationary points $\frac{dy}{dx} = 0 \quad \therefore 6(x-2)(x-1) = 0 \quad \therefore x = 1 \text{ or } x = 2$

$\therefore$ the stationary points are $(1, 5)$ & $(2, 4)$

Also for the nature of the stationary points, $\frac{d^2y}{dx^2} = 12x - 18$

At $x = 1$, $\frac{d^2y}{dx^2} = -6 < 0 \quad \therefore (1, 5)$ is a MAXIMUM stationary point

At $x = 2$, $\frac{d^2y}{dx^2} = 6 > 0 \quad \therefore (2, 4)$ is a MINIMUM stationary point

(b) (ii) (1 mark)

Outcomes Assessed: P6, H6, H7, H9

Targeted Performance Band: 3-5

Criteria	Mark
• $\frac{d^2y}{dx^2} = 0$ and shows a change in concavity at $x = \frac{3}{2}$	1

Sample Answer :

For points of inflexion, $\frac{d^2y}{dx^2} = 12x - 18 = 0 \quad \therefore x = \frac{3}{2}$

x	1	$\frac{3}{2}$	2
$\frac{d^2y}{dx^2}$	-6	0	6
	concave down		concave up

(b) (iii) (2 marks)

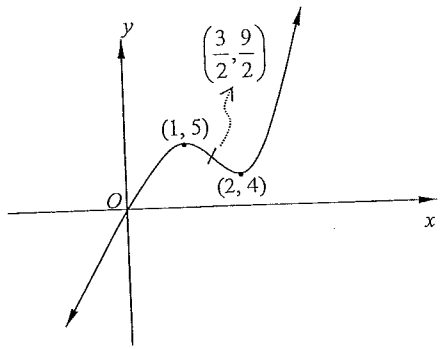
Outcomes Assessed: P6, H6, H7, H9

Targeted Performance Band: 3-5

Criteria	Marks
• Draws the correct cubic curve	1
• Plots all important points	1

Sample Answer :

$y = 2x^3 - 9x^2 + 12x$



(b) (iv) (1 mark)

Outcomes Assessed: P6, H6, H7

Targeted Performance Band: 2-4

Criteria	Mark
• Correctly solves the equation $\frac{d^2y}{dx^2} > 0$ or gives correct answer from graph.	1

Sample Answer :

$\frac{d^2y}{dx^2} = 12x - 18$

For the curve to be concave up, $\frac{d^2y}{dx^2} > 0 \quad \therefore 12x - 18 > 0$

$\therefore x > \frac{3}{2}$

(c) (3 marks)
Outcomes Assessed: P4
Targeted Performance Band: 3-4

Criteria	Marks
• Correctly solves the equation $2A^2 - 19A - 10 = 0$.	1
• Correctly substitutes $(x^2 + 1)$ and then solves for x^2.	1
• Gives the correct solution.	1

Sample Answer :

Let $A = x^2 + 1$

$2A^2 - 19A - 10 = 0$

$(2A + 1)(A - 10) = 0$

$A = -\frac{1}{2}$ or 10

$\therefore x^2 + 1 = -\frac{1}{2}$ or $x^2 + 1 = 10$

$x^2 = -\frac{3}{2}$ No solution $x^2 = 9$

$x = \pm 3$

Question 6 (12 marks) (a) (i) (2 marks)
Outcomes Assessed: P4, H5
Targeted Performance Band: 2-4

Criteria	Marks
• Gives ONE correct answer in radians OR THREE correct answers in degrees.	1
• Gives THREE correct answers in radians.	1

Sample Answer:

$\cos 3x = 0$

Basic angle is $\frac{\pi}{2}$ (First Quadrant).

$\therefore 3x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$ $\quad 0 \le 3x \le 3\pi$

$x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$ $\quad 0 \le x \le \pi$

(a) (ii) (2 marks)
Outcomes Assessed: H5
Targeted Performance Band: 2-4

Criteria	Marks
• Gives correct amplitude.	1
• Gives correct period.	1

Sample Answer:

Amplitude = 1

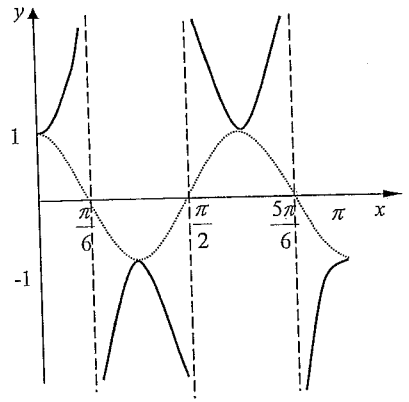
Period $= \frac{2\pi}{n}$

$= \frac{2\pi}{3}$

(a) (iii) (2 marks)
Outcomes Assessed: H5
Targeted Performance Band: 2-4

Criteria	Marks
• Shows x-intercepts as asymptotes on $y = \cos 3x$.	1
• Gives correct reciprocal curve.	1

Sample Answer:

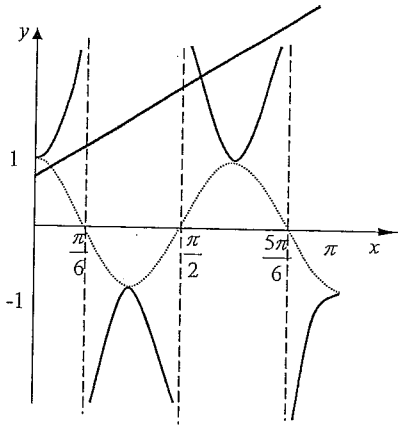


(a) (iv) (2 marks)
Outcomes Assessed: H5
Targeted Performance Band: 2-4

Criteria	Marks
• Correctly draws $y=x$ on graph.	1
• Shows TWO solutions.	1

Sample Answer:

2 solutions



(b) (i) (2 marks)
Outcomes Assessed: P4
Targeted Performance Band: 2-4

Criteria	Marks
• Correctly substitutes into cosine rule.	1
• Gives correct solution.	1

Sample Answer:

$$\cos \angle ADC = \frac{10.1^2 + 10.1^2 - 4^2}{2\times 10.1\times 10.1}$$

$$= 0.92$$

$$\angle ADC = 23^\circ$$

(b) (ii) (2 marks)
Outcomes Assessed: H5
Targeted Performance Band: 2-3

Criteria	Marks
• Gives angle in radians.	1
• Gives correct solution.	1

Sample Answer:

$$23^\circ = \frac{\pi}{180} \times 23 \text{ radians}$$

$$= 0.4 \text{ radians}$$

$$l = r\theta$$

$$=10.1\times 0.4$$

$$= 4.04 \text{ metres}$$

Question 7 (12 marks)
(a)(i) (1 mark)
Outcomes Assessed: H4, H5
Targeted Performance Band: 4

Criteria	Mark
• Gives the correct answer.	1

Sample Answer:
Vertex = (0, –2)

(a)(ii) (1 mark)
Outcomes Assessed: H4, H5
Targeted Performance Band: 4

Criteria	Mark
• Gives the correct answer.	1

Sample Answer:
Focus = (0, 0)

(a) (iii) (2 marks)
Outcomes Assessed: H4, H5
Targeted Performance Band: 4-5

Criteria	Marks
• Correctly differentiates to find the gradient.	1
• Gives the correct equation.	1

Sample Answer:

$x^2 = 8(y+2)$

$y = \frac{x^2}{8} - 2 \qquad \therefore \frac{dy}{dx} = \frac{2x}{8} = \frac{x}{4} \qquad \therefore \text{ at } \left(2, -\frac{3}{2}\right) \Rightarrow m = \frac{1}{2}$

$\therefore y + \frac{3}{2} = \frac{1}{2}(x-2) \quad \Rightarrow x - 2y - 5 = 0$

$\therefore$ The tangent to the parabola at $\left(2, -\frac{3}{2}\right)$ is $x - 2y - 5 = 0$.

(a) (iv) (1 mark)
Outcomes Assessed: H4, H5
Targeted Performance Band: 4-5

Criteria	Mark
• Shows that the tangent meets the directrix at $(-3,-4)$.	1

Sample Answer:

Directrix has equation $y = -4$

Substituting into the tangent, $x - 2(-4) - 5 = 0$, $x = -3$

Therefore the tangent cuts the directrix at (–3, –4).

(b) (i) (1 mark)
Outcomes Assessed: H5
Targeted Performance Band: 4

Criteria	Mark
• Determines the initial velocity correctly.	1

Sample Answer:

When $t = 0$, $V = 18 - 2e^0$

$\therefore V = 18 - 2 = 16 m/s.$

(b) (ii) (2 marks)
Outcomes Assessed: H4
Targeted Performance Band: 4-5

Criteria	Marks
• Substitutes $V = 0$ to determine t with some progress towards the answer.	1
• Correctly solves for t.	1

Sample Answer:

At rest when $V = 0$

$\therefore 0 = 18 - 2e^t$

$e^t = 9$

$\therefore t = \log_e 9$

$= 2\log_e 3$

(b) (iii) (1 mark)
Outcomes Assessed: H4, H5
Targeted Performance Band: 4

Criteria	Mark
• Correctly determines the coordinates of A and B.	1

Sample Answer:

$A = (0, 16)$ and $B = (2\log_e 3, 0)$

(b) (iv) (3 marks)
Outcomes Assessed: H4, H5
Targeted Performance Band: 4-5

Criteria	Marks
• Splits the integral to allow for the absolute value or equivalent working.	1
• Integrates the expression for velocity correctly.	1
• Correctly substitutes to find the distance travelled.	1

Sample Answer:

$$\text{Distance} = \int_0^{2\log_e 3} (18 - 2e^t)\,dt + \left| \int_{2\log_e 3}^{3\log_e 3} (18 - 2e^t)\,dt \right|$$

$$= \left[18t - 2e^t\right]_0^{2\log_e 3} + \left| \left[18t - 2e^t\right]_{2\log_e 3}^{3\log_e 3} \right|$$

$$= \left[36\log_e 3 - 2e^{2\log_e 3}\right] - [0 - 2] + \left| \left[54\log_e 3 - 2e^{3\log_e 3}\right] - \left[36\log_e 3 - 2e^{2\log_e 3}\right] \right|$$

$$= 36\log_e 3 - 18 + 2 + \left| 54\log_e 3 - 54 - 36\log_e 3 + 18 \right|$$

$$= 36\log_e 3 - 16 + \left| 18\log_e 3 - 36 \right|$$

$$= 36\log_e 3 - 16 + 36 - 18\log_e 3$$

$$= 18\log_e 3 + 20$$

Question 8 (12 marks)

(a) (i) (2 marks)

Outcomes Assessed: H1, H5

Targeted Performance Band: 4-5

Criteria	Marks
• Progress towards A_2	1
• Shows that $A_2 = 5\times10^5(1.08)^2 - 5.6\times10^4(1.08+1)$.	1

Sample Answer:

$A_1 = 500000(1.08) - 56000$

$= 5\times10^5(1.08) - 5.6\times10^4$

$A_2 = A_1(1.08) - 5.6\times10^4$

$= \left[5\times10^5(1.08) - 5.6\times10^4\right](1.08) - 5.6\times10^4$

$= 5\times10^5(1.08)^2 - 5.6\times10^4(1.08+1)$ as required.

(a)(ii) (2 marks)

Outcomes Assessed: H4, H5

Targeted Performance Band: 4-6

Criteria	Marks
• Gives the correct expression for A_n.	1
• Correctly simplifies A_n to give the expression as required.	1

Sample Answer:

$A_n = 5\times10^5(1.08)^n - 5.6\times10^4\left(1+1.08+...+1.08^{n-1}\right)$

$= 5\times10^5(1.08)^n - 5.6\times10^4\left[\dfrac{1.08^n-1}{0.08}\right]$

$= 5\times10^5(1.08)^n - 7\times10^5(1.08^n-1)$

$= 5\times10^5(1.08)^n - 7\times10^5(1.08)^n + 7\times10^5$

$= 7\times10^5 - 2\times10^5(1.08)^n$

$= 10^5\left[7-2(1.08)^n\right]$ as required.

(a) (iii) (3 marks)

Outcomes Assessed: H3, H4, H5

Targeted Performance Band: 4-5

Criteria	Marks
• Equates $A_n = 0$	1
• Progress towards finding n.	1
• Finds the correct value for n and hence the year 2025.	1

Sample Answer:

When $A_n = 0$, the fund will have reached zero.

$0 = 10^5\left[7-2(1.08)^n\right]$

$0 = \left[7-2(1.08)^n\right]$

$2(1.08)^n = 7$

$n = \dfrac{\log_e 3.5}{\log_e(1.08)}$

$= 16.27788...$

During the 17th year the fund will reach zero.
Therefore during 2025 the fund will reach zero.

(b) (i) (1 mark)

Outcomes Assessed: H3, H4

Targeted Performance Band: 4-5

Criteria	Mark
• Correctly determines $\dfrac{dN}{dt}$ as a rate proportional to the number of kangaroos alive.	1

Sample Answer:

$N = N_0e^{-kt}$

$\therefore \dfrac{dN}{dt} = N_0 \times -ke^{-kt}$

$= -k \times N_0e^{-kt}$

$= -kN$

$\therefore$ The number of kangaroos decreases at a rate proportional to the number of kangaroos alive.

(b) (ii) (2 marks)

Outcomes Assessed: H3, H4

Targeted Performance Band: 4-5

Criteria	Marks
• Correctly shows $N_0 = 2000$.	1
• Correctly shows $k = 0.0351$.	1

Sample Answer:

$2000 = N_0 e^{-k(0)} \quad \therefore N_0 = 2000$

$1800 = 2000e^{-k(3)} \quad \therefore e^{-3k} = 0.9$

$-3k = \log_e 0.9$

$k = 0.0351$

(b) (iii) (2 marks)

Outcomes Assessed: H3, H4

Targeted Performance Band: 4-5

Criteria	Marks
• Correctly substitutes 1000 with progress towards the answer.	1
• Correctly determines the time when the population halves.	1

Sample Answer:

$1000 = 2000e^{-0.0351t} \quad \therefore e^{-0.0351t} = 0.5$

$-0.0351t = \log_e 0.5$

$t = 19.7$ *years*

Question 9 (12 marks)

(a) (i) (2 marks)

Outcomes Assessed: H4, H9

Targeted Performance Band: 4-5

Criteria	Marks
• Solves for t correctly.	1
• Correctly determines the first time after 8.55 pm	1

Sample Answer:

When $\frac{dV}{dt} = 0$

$20\sin\frac{\pi}{35}t = 0$

$\sin\frac{\pi}{35}t = 0$

$\frac{\pi}{35}t = \pi$

$\therefore t = 35$ minutes

Therefore the first time the flow rate is zero after 8.55 pm is $8.55 + 35$ minutes $= 9.30$ pm

(a) (ii) (3 marks)

Outcomes Assessed: H8, H9

Targeted Performance Band: 4-5

Criteria	Marks
• Correctly integrates the given rate.	1
• Correctly determines the value of the constant.	1
• Derives the correct expression for the volume of water in the pond after t minutes.	1

Sample Answer:

$\frac{dV}{dt} = 20\sin\frac{\pi}{35}t$

$V = 20\int\left(\sin\frac{\pi}{35}t\right)dt$

$= 20\left(\frac{-\cos\frac{\pi}{35}t}{\frac{\pi}{35}}\right) + C \quad = \frac{-700}{\pi}\cos\frac{\pi}{35}t + C$

At $t = 0, V = 0 \quad \therefore 0 = \frac{-700}{\pi}\cos\frac{\pi}{35}(0) + C$

$\therefore C = \frac{700}{\pi}$

$\therefore$ Volume $V = \frac{700}{\pi} - \frac{700}{\pi}\cos\frac{\pi}{35}t$

(a) (iii) (2 marks)

Outcomes Assessed: H4, H5

Targeted Performance Band: 4-5

Criteria	Marks
• Correctly substitutes 35 minutes into the expression for volume.	1
• Correctly determines the maximum volume.	1

Sample Answer:

From part (i), the filtering cycle is 35 minutes.

$$\therefore V = \frac{700}{\pi} - \frac{700}{\pi}\cos\frac{\pi}{35}\times 35 = \frac{700}{\pi} + \frac{700}{\pi} = \frac{1400}{\pi} \text{ litres}$$

(b) (i) (2 marks)

Outcomes Assessed: H8, H9

Targeted Performance Band: 4-6

Criteria	Marks
• Correctly applying the volume formula.	1
• Gives the expression.	1

Sample Answer:

Use the formula $V = \pi \int y^2 dx$

$$V = \pi \int_1^2 \left[\frac{4}{\sqrt{4x-1}}\right]^2 dx = \pi \int_1^2 \frac{16}{4x-1} dx = 4\pi \int_1^2 \frac{4}{4x-1} dx$$

(b) (ii) (3 marks)

Outcomes Assessed: H8, H9

Targeted Performance Band: 4-6

Criteria	Marks
• Correctly integrates the volume.	1
• Correctly applies Newton–Leibnitz formula/substitution.	1
• Finds the volume of the solid, in exact form.	1

Sample Answer:

$$V = 4\pi \int_1^2 \frac{4}{4x-1} dx$$

$$= 4\pi \left[\log_e(4x-1)\right]_1^2 = 4\pi\left[(\log_e 7) - (\log_e 3)\right]$$

$$= 4\pi \log_e\left(\frac{7}{3}\right) u^3$$

Question 10 (12 marks)

(a) (2 marks)

Outcomes Assessed: H8, H9

Targeted Performance Band: 5-6

Criteria	Marks
• Performs the integration correctly	1
• Correctly substitutes into the integral to find the area.	1

Sample Answer :

$$\int_{-1}^{4}[f(x)+4]dx = \int_{-1}^{4} f(x)dx + \int_{-1}^{4} 4\,dx$$

$$= 7\frac{1}{2} + [4x]_{-1}^{4}$$

$$= 7\frac{1}{2} + [4\times 4 - 4\times -1]$$

$$= 7\frac{1}{2} + [16+4]$$

$$= 27\frac{1}{2}$$

(b) (3 marks)

Outcomes Assessed: H3, H9

Targeted Performance Band: 5-6

Criteria	Marks
• Correctly integrates the expression.	1
• Correctly substitutes into the integral.	1
• Correctly determines the value of the integral.	1

Sample Answer:

$$\int_0^{\pi} \pi^x dx$$

$$= \left[\frac{\pi^x}{\ln \pi}\right]_0^{\pi}$$

$$= \left[\frac{\pi^{\pi}}{\ln \pi}\right] - \left[\frac{\pi^0}{\ln \pi}\right]$$

$$= \frac{\pi^{\pi}}{\ln \pi} - \frac{1}{\ln \pi}$$

$$= 30.9786 = 31.0$$

(c) (i) (2 marks)
Outcomes Assessed: H1, H4
Targeted Performance Band: 5-6

Criteria	Marks
• Correct application of Pythagoras' Theorem.	1
• Gives the correct expression for area.	1

Sample Answer:

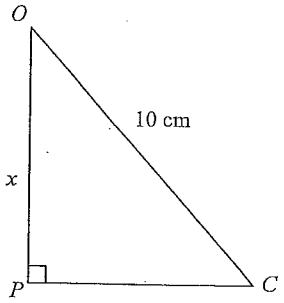
In ΔOPC, $OC = 10$ (radius of circle)

Therefore, by Pythagoras' Theorem $PC = \sqrt{100 - x^2}$

In ΔABC, height $= 10 + x$ and base $= 2 \times \sqrt{100 - x^2}$

$\therefore$ Area $A = \frac{1}{2} \times 2\sqrt{100 - x^2} \times (10 + x)$

$= (10 + x)\sqrt{100 - x^2}$ (as required)



(c) (ii) (2 marks)
Outcomes Assessed: H1, H4
Targeted Performance Band: 5-6

Criteria	Marks
• Correctly applying the product rule.	1
• Correctly simplifying to the required expression.	1

Sample Answer:

$$\frac{dA}{dx} = (100 - x^2)^{\frac{1}{2}} - \frac{x(10 + x)}{(100 - x^2)^{\frac{1}{2}}}$$

$$= \frac{(100 - x^2) - x(10 + x)}{\sqrt{100 - x^2}}$$

$$= \frac{100 - x^2 - 10x - x^2}{\sqrt{100 - x^2}}$$

$$= \frac{100 - 10x - 2x^2}{\sqrt{100 - x^2}} \text{ as required}$$

(c) (iii) (3 marks)
Outcomes Assessed: H1, H4, H9
Targeted Performance Band: 5-6

Criteria	Marks
• Determines the maximum correctly.	1
• Shows a side is $10\sqrt{3}$.	1
• Shows that ΔABC is equilateral when $x = 5$.	1

Sample Answer:

For maximum area, $\frac{dA}{dx} = 0$

$$\frac{100 - 10x - 2x^2}{\sqrt{100 - x^2}} = 0$$

$$2x^2 + 10x - 100 = 0$$

$$x^2 + 5x - 50 = 0$$

$$(x - 5)(x + 10) = 0$$

$\therefore x = 5$ (Cannot have a negative value for a side length)

Show that it is a maximum area at $x = 5$

x	4	5	6
$\frac{dA}{dx}$	>0	= 0	<0

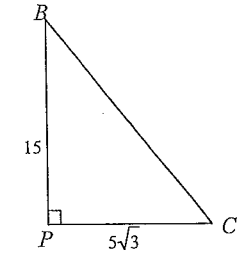
Therefore a maximum area occurs at $x = 5$.
Dimensions of ΔABC are as follows for maximum area,

$$AC = 2 \times \sqrt{100 - 5^2}$$

$$= 2\sqrt{75}$$

$$= 10\sqrt{3}$$

By Pythagoras' Theorem
$BC = 10\sqrt{3}$
Similarly $AC = 10\sqrt{3}$



Therefore the triangle with maximum area is equilateral with side length $10\sqrt{3}$ cm.