



CATHOLIC SECONDARY SCHOOLS  
ASSOCIATION OF NSW

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Centre Number

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Student Number

**2016**  
TRIAL HIGHER SCHOOL CERTIFICATE  
EXAMINATION

# Mathematics

Morning Session  
Thursday, 4 August 2016

### General Instructions

- Reading time – 5 mins
- Working time – 3 hours
- Write using black pen
- Use Multiple Choice Answer Sheet provided
- Board-approved calculators may be used
- A reference sheet is provided on a SEPARATE sheet
- In Questions 11-16, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number at the top of this page

Total marks - 100

**Section I** Pages 2 - 5

10 marks

- Attempt Questions 1 - 10
- Allow about 15 minutes for this section

**Section II** Pages 6 - 14

90 marks

- Attempt Questions 11 - 16
- Allow about 2 hours and 45 minutes for this section

## Section I

10 marks

Attempt Questions 1–10

Allow about 15 minutes for this section

Use the Multiple-Choice Answer Sheet for Questions 1–10.

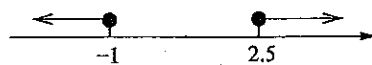
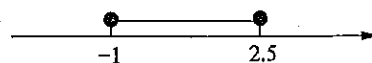
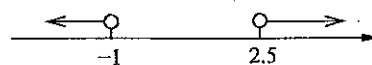
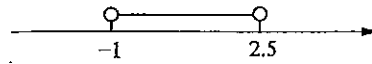
1 Solve  $3 - 2(x + 1) = 7$ .

- (A) -3
- (B) -2.5
- (C) -2
- (D) -1

2 What is the gradient of the line  $ax + by + c = 0$ ?

- (A)  $\frac{a}{b}$
- (B)  $\frac{b}{a}$
- (C)  $-\frac{a}{b}$
- (D)  $-\frac{b}{a}$

3 Which of the following is the solution of the inequality  $|3 - 4x| < 7$ ?

- (A) 
- (B) 
- (C) 
- (D) 

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Every effort has been made to prepare these 'Trial' Higher School Certificate Examinations in accordance with the NSW Board of Studies documents, Principles for Setting HSC Examinations in a Standards-Referenced Framework ([www.boardofstudies.nsw.edu.au/principles-for-setting-exams.html](http://www.boardofstudies.nsw.edu.au/principles-for-setting-exams.html)), and Principles for Developing Marking Guidelines Examinations in a Standards-Referenced Framework ([www.boardofstudies.nsw.edu.au/units/principles\\_hsc.html](http://www.boardofstudies.nsw.edu.au/units/principles_hsc.html)). No guarantee or warranty is made or implied that the 'Trial' Examination papers mirror in every respect the actual HSC Examination question paper in any or all courses to be examined. These papers do not constitute 'advice' nor can they be construed as authoritative interpretations of Board of Studies intentions. The CSSA accepts no liability for any reliance use or purpose related to these 'Trial' question papers. Advice on HSC examination issues is only to be obtained from the NSW BOS.

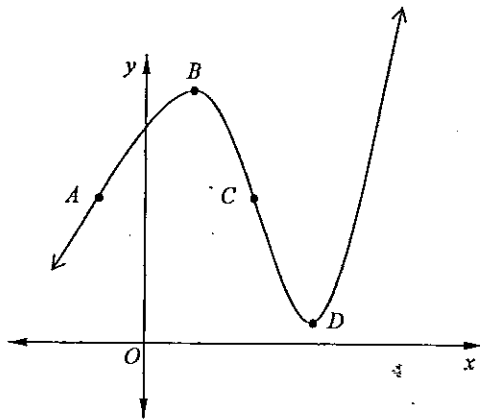
4 Find the number of sides of a regular polygon whose exterior angles are  $24^\circ$ .

- (A) 13
- (B) 14
- (C) 15
- (D) 16

5 How many solutions of the equation  $(\tan x - 2)(2 \sin x - 1) = 0$  lie in the domain  $0 \leq x \leq \pi$ ?

- (A) 1
- (B) 2
- (C) 3
- (D) 4

6



The diagram shows the points  $A$ ,  $B$ ,  $C$  and  $D$  on a curve.

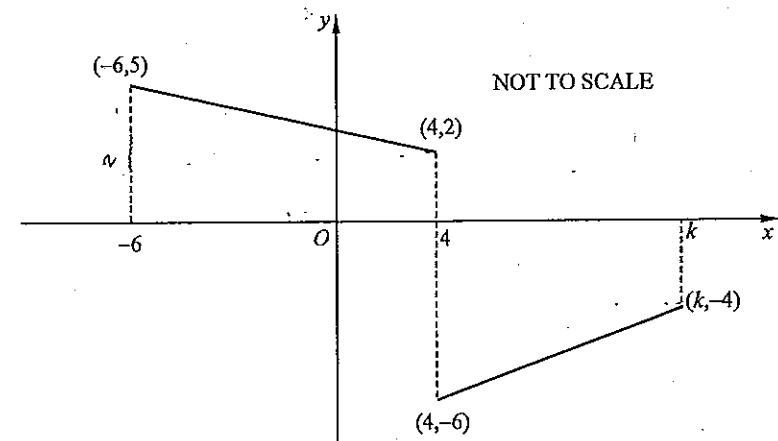
At which point is  $\frac{dy}{dx} = 0$  and  $\frac{d^2y}{dx^2} < 0$ ?

- (A)  $A$
- (B)  $B$
- (C)  $C$
- (D)  $D$

7 Evaluate  $\sum_{k=3}^{15} 2^k$

- (A)  $2^{15} - 2$
- (B)  $2^{13} - 8$
- (C)  $2^{15} - 8$
- (D)  $2^{16} - 8$

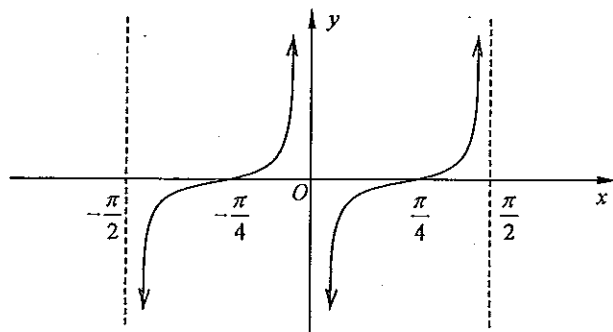
8



Use the graph above to find the value of  $k$  which satisfies  $\int_{-6}^k f(x) dx = 0$ .

- (A) 6
- (B) 10
- (C) 11
- (D) 12

9 Part of the graph  $y = f(x)$  is shown below.



The equation of  $f(x)$  could be:

- (A)  $f(x) = \tan\left(2x - \frac{\pi}{2}\right)$       (B)  $f(x) = \tan x$   
 (C)  $f(x) = \tan\left(2x - \frac{\pi}{4}\right)$       (D)  $f(x) = \tan\left(x + \frac{\pi}{4}\right)$

10 There are 10 green marbles and  $W$  white marbles in a bag. The probability of selecting a white marble is  $\frac{4}{9}$ . How many more white marbles need to be added to the bag so that the probability of selecting a white marble from the bag is  $\frac{3}{5}$ ?

- (A) 7  
 (B) 8  
 (C) 14  
 (D) 21

## Section II

90 marks

Attempt Questions 11 - 16

Allow about 2 hours and 45 minutes for this section

Answer each question in a separate writing booklet. Extra writing booklets are available.

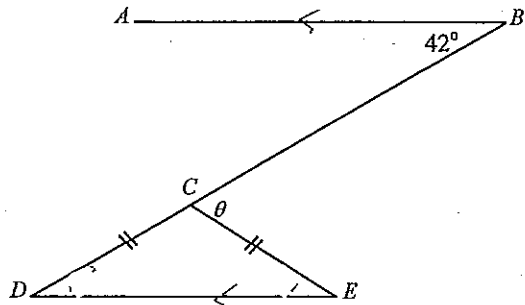
In Questions 11 - 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

- (a) Evaluate  $\log_3 32 - \log_3 5$  correct to 3 significant figures. 2
- (b) Write  $\frac{6}{2\sqrt{5}-4}$  with a rational denominator. 2
- (c) Find the focus of the parabola  $(x-3)^2 = 8y+16$ . 2
- (d) If  $f(x) = ax^3 - 2x + 1$ , find  $a$  if  $f'(2) = 0$ . 2
- (e) Simplify  $\frac{m-n}{m^2-n^2} + \frac{1}{m+n}$  2
- (f) Solve simultaneously:  
 $y = \frac{2}{x}$   
 $2x + 5y = 12$  3
- (g) Find  $\frac{d}{dx}\left(\frac{1}{\sqrt{x}} + \frac{7x^2}{2}\right)$ . 2

Question 12 (15 marks) Use a SEPARATE writing booklet.

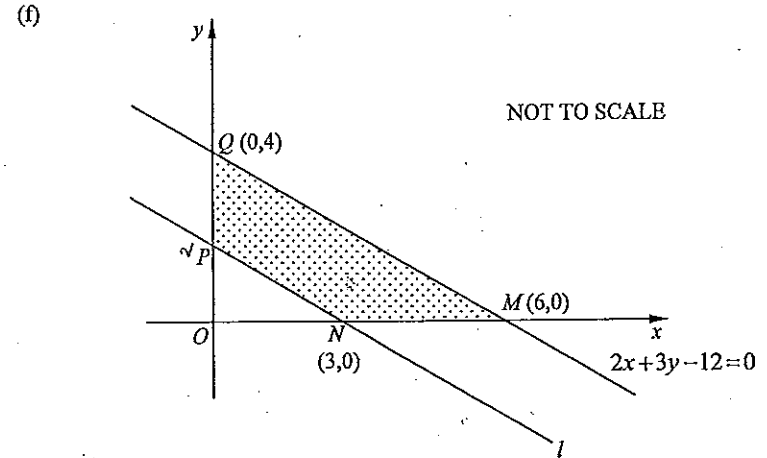
- (a) Evaluate the sum of the series  $2 + 0 - 2 + \dots - 30$ . 2
- (b) Differentiate with respect to  $x$  and express your answer in simplest form:
- (i)  $\frac{x}{\ln x^2}$  2
- (ii)  $(e^x - 1)(3x + 2)^3$  2
- (c) Find  $\int (1 + \cos 2\pi x) dx$ . 2
- (d) Evaluate  $\int_0^{\ln 2} \frac{e^{2x}}{2} dx$ . 2
- (e) In the diagram,  $AB \parallel DE$  and  $CD = CE$ .  
Find  $\theta$  giving reasons. 2



NOT TO SCALE

Question 12 continues on page 8

Question 12 (continued)



The diagram above shows two lines which are parallel.

$M$  and  $Q$  are the intercepts of the line  $2x + 3y - 12 = 0$ .  
 $N$  and  $P$  are the intercepts of line  $l$ .

- (i) Show that the equation of line  $l$  is  $2x + 3y - 6 = 0$ . 1
- (ii) Find the  $y$ -intercept of line  $l$ . 1
- (iii) Find the area of the shaded region  $MNPQ$ . 1

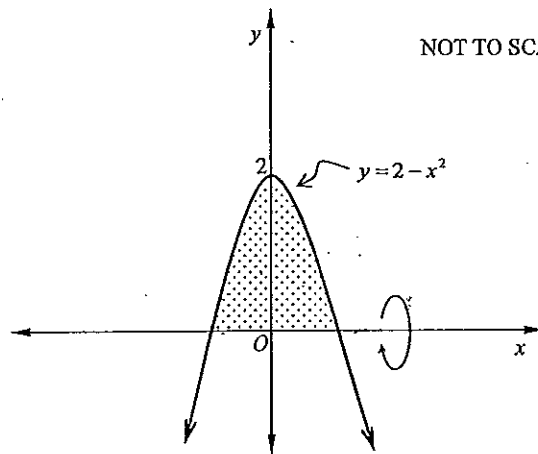
End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

- (a) Consider the function  $f(x) = x^3 - 3x^2 - 1$ .
- (i) Find the stationary points and determine their nature. 3
- (ii) Find the point of inflexion. 2
- (iii) Sketch the graph of  $y = f(x)$ , showing the stationary points and point of inflexion. 2

- (b) Adelina and Tumaini are taking turns in a game with a single die, where the first person to roll a 6 is the winner. Adelina starts the game.
- (i) What is the probability that Adelina wins on her first turn? 1
- (ii) Draw a tree diagram with probabilities to indicate the outcomes of the first three turns of this game. 2
- (iii) What is the probability that Adelina eventually wins the game? 2

- (c) NOT TO SCALE 3



In the diagram the region bounded by the curve  $y = 2 - x^2$  and the  $x$ -axis is shaded.

Find the volume of the solid that is formed if this region is rotated about the  $x$ -axis.

Question 14 (15 marks) Use a SEPARATE writing booklet.

- (a) The half-life of radium-226 is 1390 years. This means that the rate of decay is proportional to the amount present, and half of any given quantity will decay in 1390 years.

The rate at which the radium decays is given by  $M = M_0 e^{-kt}$ , where  $M$  is measured in milligrams and  $t$  in years.

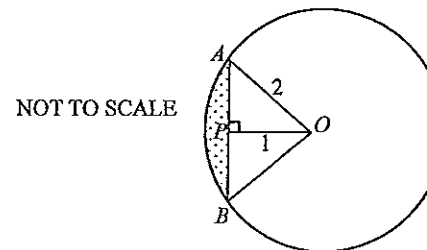
- (i) Use the information above to find the value of  $k$ . 2
- (ii) After how many years will 20% of the initial quantity remain? 2

- (b) Joly recently opened a savings policy at the Bank of Gauss. The bank is offering an interest rate of 4.8% per annum on all investments. Joly contributes \$250 into her savings account at the beginning of each month. The savings policy is compounded monthly with interest paid just before each contribution.

- (i) Show that the value of the first \$250 contribution amounts to \$275.14 after 2 years. 2
- (ii) Joly has a savings goal of \$15 000. If she continues to invest \$250 each month, after how many years and months will it take her to reach this goal? 3

- (c) Use Simpson's rule with five function values to approximate  $\int_0^2 3^x dx$ , correct to 3 decimal places. 2

- (d)

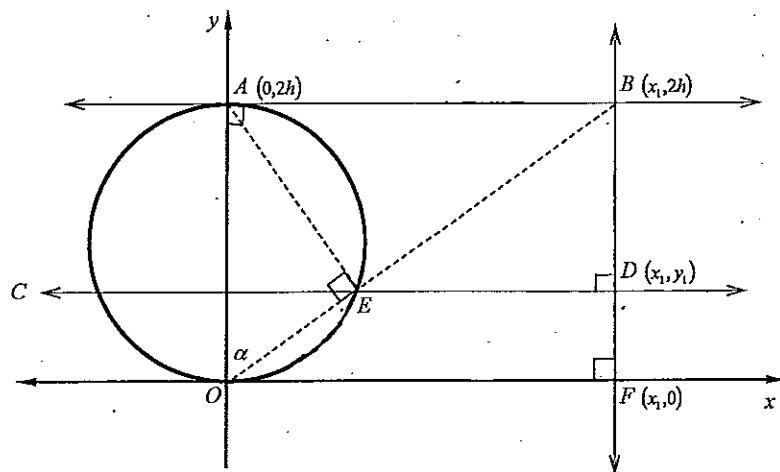


The diagram shows a circle with centre  $O$ , radius 2 cm and  $OP = 1$  cm.

- (i) Show that the area of the sector  $AOB$  is  $\frac{4\pi}{3} \text{ cm}^2$ . 2
- (ii) Hence, or otherwise, find the shaded area. 2

Question 15 (15 marks) Use a SEPARATE writing booklet.

(a)



The diagram above is a circle with equation  $x^2 + (y-h)^2 = h^2$ .  
The lines  $AB$  and  $CD$  are parallel to the  $x$ -axis and  $BF$  is parallel to the  $y$ -axis. The line  $OB$  cuts the circle at  $E$  and  $\angle AEO$  is right angle.

Let  $\angle AOE = \alpha$ , where  $0 < \alpha < \frac{\pi}{2}$ .

The coordinates of the points  $A$ ,  $B$ ,  $D$  and  $F$  are given and shown in the diagram.

- (i) Explain why  $\angle OBF = \alpha$ . 1
- (ii) Hence show that  $x_1 = 2h \tan \alpha$ . 2
- (iii) Using  $\triangle OAE$ , show  $OE = 2h \cos \alpha$ . 1
- (iv) Hence show that  $y_1 = 2h \cos^2 \alpha$ . 2
- (v) It is known that  $\tan^2 \theta + 1 = \sec^2 \theta$ . 3

Use this result, or otherwise, to show that  $x_1 = 2h \sqrt{\frac{2h}{y_1} - 1}$ .

Question 15 continues on page 12

Question 15 (continued)

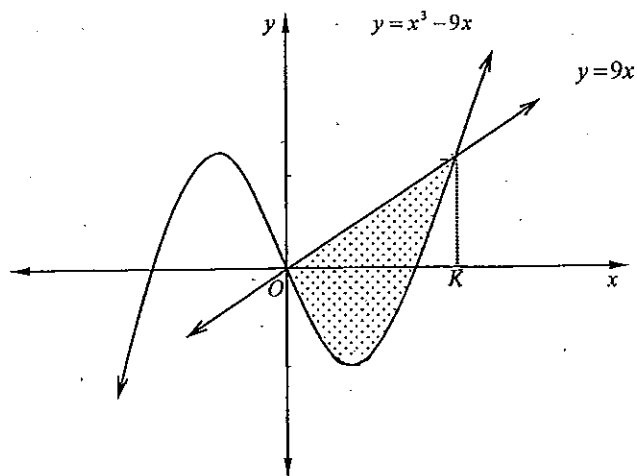
(b) A particle moves in a straight line. At time  $t$  seconds its displacement  $x$  metres from the origin is given by  $x = 1 - 2 \sin 2t$  for  $0 \leq t \leq \pi$ .

- (i) Sketch the graph of  $x$  as a function of  $t$ . 2
- (ii) Using your graph, or otherwise, write the first time the particle is at rest. 1
- (iii) Hence, find the position of the particle at this time. 1
- (iv) What is the maximum speed of the particle? 2

End of Question 15

Question 16 (15 marks) Use a SEPARATE writing booklet.

(a)



The graphs of the functions  $y = x^3 - 9x$  and  $y = 9x$  are shown above.

The graphs intersect at  $x = 0$  and  $x = K$  for  $x \geq 0$ .

- (i) By solving the two equations simultaneously, show that  $K = 3\sqrt{2}$ . 2
- (ii) Hence find the shaded area in the diagram above. Write your answer in exact form. 3

(b) The gradient function of a curve,  $g(x)$ , is given by  $g'(x) = \frac{6}{3x-2}$ .

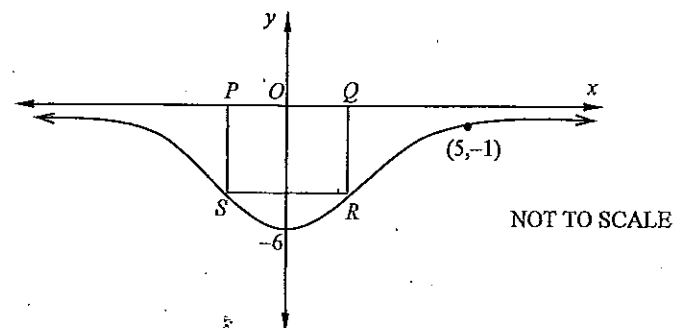
The line  $y = 6x - 2$  is tangent to the curve  $g(x)$ .

Find the equation of the curve,  $g(x)$ . 3

Question 16 continues on page 14

Question 16 (continued)

(c)



In the diagram above, the curve  $y = \frac{h}{x^2 + k}$ , where  $h$  and  $k$  are constants,

has a minimum turning point at  $(0, -6)$  and passes through the point  $(5, -1)$ .

A rectangle  $PQRS$  is inscribed within the curve as shown with its axis of symmetry  $x = 0$ .

- (i) Find the value of  $h$  and  $k$ . 2
- (ii) If  $Q$  has coordinates  $(\alpha, 0)$ , find the coordinates of  $R$  in terms of  $\alpha$ . 1
- (iii) Show that the area,  $A$ , of the rectangle  $PQRS$  is  $A = \frac{60\alpha}{\alpha^2 + 5}$ . 1
- (iv) Hence show that the maximum area of the rectangle  $PQRS$  is  $6\sqrt{5}$  square units. 3

End of Paper



**CATHOLIC SECONDARY SCHOOLS ASSOCIATION  
2016 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION  
MATHEMATICS – MARKING GUIDELINES**

These marking guidelines show the criteria to be applied to responses along with the marks to be awarded in line with the quality of responses. These guidelines are suggested and not prescriptive. This is not intended to be an exhaustive list but rather an indication of the considerations that students could include in their responses.

**Section I  
10 Marks**

Questions 1-10 (1 mark each)

Question	Answer	Outcomes Assessed	Targeted Performance Bands <sup>1</sup>
1	A	P2	2-3
2	C	P2	2-3
3	D	P1,P3	2-3
4	C	P4	3-4
5	C	H5	3-4
6	B	H7	4-5
7	D	H5	4-5
8	C	H7,H8	4-5
9	A	H5,H7	4-5
10	A	H5	5-6

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**Section II  
90 Marks  
Question 11 (15 marks)  
(a) (2 marks)  
Outcomes Assessed: H3  
Targeted Performance Bands: 2-3**

Criteria	Marks
• Correctly determines the value to 3 significant figures	2
• Manipulates using log laws	1

Sample answer:

$$\frac{\log\left(\frac{32}{5}\right)}{\log 4} = 1.34$$

**(b) (2 marks)  
Outcomes Assessed: P3  
Targeted Performance Bands: 2**

Criteria	Marks
• Correctly writes fraction with rational denominator in simplest form	2
• Multiplies fraction by the conjugate of the denominator	1

Sample answer:

$$\frac{6}{2\sqrt{5}-4} \times \frac{2\sqrt{5}+4}{2\sqrt{5}+4} = \frac{6(2\sqrt{5}+4)}{4} = \frac{3(2\sqrt{5}+4)}{2}$$

**(c) (2 marks)  
Outcomes Assessed: P4, P5  
Targeted Performance Bands: 3**

Criteria	Marks
• Correctly finds the focus	2
• Substantial progress towards a solution	1

Sample answer:

$$(x-3)^2 = 8y+16$$

$$(x-3)^2 = 8(y+2)$$

$$\therefore a=2 \text{ \& } V(3,-2)$$

Hence Focus (3,0)

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(d) (2 marks)

Outcomes Assessed: P7

Targeted Performance Bands: 2-3

Criteria	Marks
• Correctly finds the value of $a$	2
• Finds the derivative	1

Sample answer:

$$f'(x) = 3ax^2 - 2$$

$$0 = 12a - 2$$

$$a = \frac{1}{6}$$

(e) (2 marks)

Outcomes Assessed: P3

Targeted Performance Bands: 2-3

Criteria	Marks
• Correctly simplifies expression	2
• Progress towards factorising expression	1

Sample answer:

$$\frac{(m-n)}{(m-n)(m+n)} \times \frac{(m+n)}{1} = 1$$

(f) (3 marks)

Outcomes Assessed: P4

Targeted Performance Bands: 3

Criteria	Marks
• Finds the correct solutions	3
• Progress towards finding the value of $x$ or $y$	2
• Uses substitution method to solve simultaneously	1

Sample answer:

$$2x + \frac{10}{x} = 12$$

$$2x^2 - 12x + 10 = 0$$

$$(x-5)(x-1) = 0$$

$$x = 5, y = \frac{2}{5}$$

$$x = 1, y = 2$$

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(g) (2 marks)

Outcomes Assessed: P7

Targeted Performance Bands: 3-4

Criteria	Marks
• Correctly finds the derivative	2
• Progress towards differentiating the expression	1

Sample answer:

$$\begin{aligned} \frac{d}{dx} \left( x^{-\frac{1}{2}} + \frac{7}{2}x^2 \right) &= -\frac{1}{2}x^{-\frac{3}{2}} + 7x \\ &= -\frac{1}{2\sqrt{x^3}} + 7x \end{aligned}$$

Question 12 (15 marks)

(a) (2 marks)

Outcomes Assessed: H5

Targeted Performance Bands: 3

Criteria	Marks
• Evaluates the sum of the series	2
• Correctly finds $n$	1

Sample answer:

$$a = 2, d = -2, \text{ also } -30 = 2 - 2(n-1) \therefore n = 17$$

$$S_{17} = \frac{17}{2}(2 \times 2 + 16 \times -2)$$

$$= -238$$

(b) (i) (2 marks)

Outcomes Assessed: H5

Targeted Performance Bands: 3-4

Criteria	Marks
• Correctly finds the derivative	2
• Applies quotient rule	1

Sample answer:

$$\begin{aligned} \frac{d}{dx} \left( \frac{x}{\ln x^2} \right) &= \frac{\ln x^2 \cdot 1 - x \cdot \frac{2}{x}}{(\ln x^2)^2} \\ &= \frac{\ln x^2 - 2}{(\ln x^2)^2} \end{aligned}$$

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(b) (ii) (2 marks)

Outcomes Assessed: H5

Targeted Performance Bands: 3-4

Criteria	Marks
• Correctly finds the derivative	2
• Applies product rule	1

Sample answer:

$$\begin{aligned}\frac{d}{dx}(e^x - 1)(3x + 2)^3 &= (e^x - 1) \cdot 9(3x + 2)^2 + (3x + 2)^3 \cdot e^x \\ &= (3x + 2)^2(11e^x + 3xe^x - 9)\end{aligned}$$

(c) (2 marks)

Outcomes Assessed: H5

Targeted Performance Bands: 3-4

Criteria	Marks
• Correctly finds the primitive	2
• Progress towards finding the primitive	1

Sample answer:

$$\int (1 + \cos 2\pi x) dx = x + \frac{1}{2\pi} \sin 2\pi x + C$$

(d) (2 marks)

Outcomes Assessed: H5

Targeted Performance Bands: 3-4

Criteria	Marks
• Evaluates definite integral	2
• Finds correct primitive	1

Sample answer:

$$\begin{aligned}\int_0^{\ln 2} \frac{e^{2x}}{2} dx &= \left[ \frac{e^{2x}}{4} \right]_0^{\ln 2} \\ &= \frac{e^{\ln 4}}{4} - \frac{e^0}{4} \\ &= \frac{3}{4}\end{aligned}$$

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(e) (2 marks)

Outcomes Assessed: H2

Targeted Performance Bands: 3

Criteria	Marks
• Finds $\theta$ giving reasons	2
• Progress towards finding the value of $\theta$	1

Sample answer:

$$\angle CDE = \angle CED = 42^\circ \text{ (alternate angles; } \triangle CDE \text{ isosceles)}$$

$$\theta = 42 + 42 = 84^\circ \text{ (exterior angle of a triangle equal sum of the two opposite interior angles)}$$

(f) (i) (1 mark)

Outcomes Assessed: P4

Targeted Performance Bands: 3

Criteria	Mark
• Finds gradient and shows equation of line is $2x + 3y - 6 = 0$	1

Sample answer:

$$2x + 3y - 12 = 0$$

$$m = \frac{-2}{3}$$

$$N(3, 0)$$

$$y = \frac{-2}{3}(x - 3)$$

$$3y = -2x + 6$$

$$\therefore 2x + 3y - 6 = 0$$

(f) (ii) (1 mark)

Outcomes Assessed: P4

Targeted Performance Bands: 3

Criteria	Mark
• Finds y-intercept	1

Sample answer:

$$\text{sub } x = 0 \text{ into } 2x + 3y - 6 = 0$$

$$3y - 6 = 0$$

$$y = 2$$

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(f) (iii) (1 mark)

Outcomes Assessed: P4

Targeted Performance Bands: 3

Criteria	Mark
• Finds area of quadrilateral	1

Sample answer:

Area = Area triangle  $OQM$  - Area triangle  $OPN$

$$\text{Area} = \left(\frac{1}{2} \times 6 \times 4\right) - \left(\frac{1}{2} \times 3 \times 2\right)$$

$$= 9 \text{ units}^2$$

Question 13 (15 marks)

(a) (i) (3 marks)

Outcomes Assessed: H5, H6

Targeted Performance Bands: 3-4

Criteria	Marks
• Correctly determines both stationary points and their nature	3
• Some progress – eg determines the $x$ values where the curve is stationary and their nature	2
• Correctly differentiates $f(x) = x^3 - 3x^2 - 1$	1

Sample answer

$$f(x) = x^3 - 3x^2 - 1$$

$$f'(x) = 3x^2 - 6x$$

stationary when  $f'(x) = 0$

$$3x^2 - 6x = 0$$

$$3x(x - 2) = 0$$

$$x = 0 \text{ or } x = 2$$

$f(0) = -1$  and  $f(2) = -5$ , so stationary points occur at  $(0, -1)$  and  $(2, -5)$ .

$$f''(x) = 6x - 6$$

$f''(0) = -6$ ,  $< 0$  so  $\cap$  is a MAXIMUM turning point at  $(0, -1)$

$f''(2) = 6$ ,  $> 0$  so  $\cup$  is a MINIMUM turning point at  $(2, -5)$

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(a)(ii) (2 marks)

Outcomes Assessed: H5, H6

Targeted Performance Bands: 3-4

Criteria	Marks
• Correctly determines point of inflexion with concavity change	2
• Correctly determines the point of inflexion	1

Sample answer

Possible point of inflexion at  $f''(x) = 0$

$$6x - 6 = 0$$

$$x = 1$$

$$f(1) = -3$$

So the point of inflexion is  $(1, -3)$ .

Check concavity

$x$	0	1	2
$f''(x)$	-6	0	+6

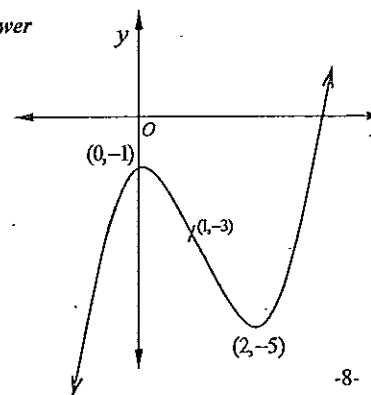
(a)(iii) (2 marks)

Outcomes Assessed: H5, H6

Targeted Performance Bands: 3-4

Criteria	Marks
• Correctly draws graph, including all critical points	2
• Some attempt, but does not include all points	1

Sample answer



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(b)(i) (1 mark)

Outcomes Assessed: H4, H5

Targeted Performance Bands: 3

Criteria	Mark
• Correct answer	1

Sample answer

$$P(\text{Adelina wins on her first turn}) = \frac{1}{6}$$

(b)(ii) (2 marks)

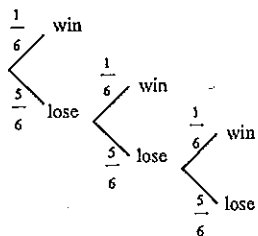
Outcomes Assessed: H4, H5

Targeted Performance Bands: 3-4

Criteria	Marks
• Correctly draws tree diagram	2
• Some progress towards an answer	1

Sample answer

Adelina Tumaini Adelina



(b)(iii) (2 marks)

Outcomes Assessed: H4, H5

Targeted Performance Bands: 5-6

Criteria	Marks
• Correctly applies limiting sum to achieve correct result	2
• Some progress towards answer	1

Sample answer

$$P(\text{Adelina eventually wins}) = \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \dots$$

This is a limiting sum:  $a = \frac{1}{6}$ ,  $r = \frac{25}{36}$ . Using  $S_{\infty} = \frac{a}{1-r}$ :

$$P(\text{Adelina eventually wins}) = \frac{\frac{1}{6}}{1 - \frac{25}{36}} = \frac{6}{11}$$

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(c) (3 marks)

Outcomes Assessed: H8, H9

Targeted Performance Bands: 4-5

Criteria	Marks
• Evaluates the volume correctly	3
• Correctly integrates the expression	2
• Some progress towards the solution	1

Sample answer

$y = 2 - x^2$ , cuts the  $x$ -axis when  $y = 0$ .

$$0 = 2 - x^2$$

$$x = \pm\sqrt{2}$$

$$V = 2\pi \int_0^{\sqrt{2}} (4 - 4x^2 + x^4) dx = 2\pi \left[ 4x - \frac{4x^3}{3} + \frac{x^5}{5} \right]_0^{\sqrt{2}} = 2\pi \left[ 4\sqrt{2} - \frac{8\sqrt{2}}{3} + \frac{4\sqrt{2}}{5} \right]$$

$$= \frac{64\pi\sqrt{2}}{15} \approx 18.96(2 \text{ d.p.})$$

Question 14 (15 marks)

(a) (i) (2 marks)

Outcomes Assessed: H3, H4

Targeted Performance Bands: 4-5

Criteria	Marks
• Gives the correct answer	2
• Progress towards an answer	1

Sample answer:

$$M = M_0 e^{-kt} \quad \therefore \frac{1}{2} M_0 = M_0 e^{-1390t}$$

$$\ln \frac{1}{2} = -1390t$$

$$\therefore k = 0.00049867 \text{ or } 4.98667 \times 10^{-4}$$

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(a) (ii) (2 marks)

Outcomes Assessed: H3

Targeted Performance Bands: 3-5

Criteria	Marks
• Gives the correct answer	2
• Correctly substitutes $M = \frac{1}{5} M_0$	1

Sample answer:

$$\frac{1}{5} M_0 = M_0 e^{-4.98766 \times 10^{-4} t} \quad \ln \frac{1}{5} = -4.98766 \times 10^{-4} t$$

$$\therefore t = 3227.46 \text{ years.}$$

(b) (i) (2 marks)

Outcomes Assessed: H5

Targeted Performance Bands: 2-4

Criteria	Marks
• Gives the correct answer	2
• Progress towards an answer	1

Sample answer:

$r = 4.8\% \text{ p.a.} = 0.4\% \text{ per month and } 2 \text{ years} = 24 \text{ months.}$

$$A = 250 \left( 1 + \frac{0.4}{100} \right)^{24} = 250 (1.004)^{24} = \$275.14$$

(b) (ii) (3 marks)

Outcomes Assessed: H5

Targeted Performance Bands: 3-5

Criteria	Marks
• Gives the correct answer	3
• Progress towards finding $n$	2
• Equate $A_n = 15\,000$	1

Sample answer:

Consider the series that is formed.

$$A_1 = 250(1.004)^n, \quad A_2 = 250(1.004)^{n-1}, \dots, \quad A_{n-1} = 250(1.004)^2, \quad A_n = 250(1.004)$$

Hence,

$$15000 = 250(1.004) + 250(1.004)^2 + 250(1.004)^3 + \dots + 250(1.004)^n$$

$$\therefore 15000 = \frac{250(1.004)[1.004^n - 1]}{1.004 - 1}$$

$$15000 = \frac{251[1.004^n - 1]}{0.004}$$

$$\frac{15000 \times 0.004}{251} = 1.004^n - 1$$

$$\therefore 1.239043 = 1.004^n$$

$$\therefore n = \frac{\ln 1.239043}{\ln 1.004} = 53.69 \text{ months} \quad \therefore n = 4.5 \text{ years}$$

$\therefore$  Joly will reach her savings goal after 4 years and 6 months.

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(c) (2 marks)

Outcomes Assessed: H8, H9

Targeted Performance Bands: 4-5

Criteria	Marks
• Gives the correct answer	2
• Progress towards an answer	1

Sample answer:

$$h = \frac{2-0}{4} = \frac{1}{2}$$

x	0	0.5	1	1.5	2
y	1	1.732	3	5.196	9

$$A \approx \frac{0.5}{3} [(1+9) + 4(1.732 + 5.196) + 2(3)] = 7.285$$

(d) (i) (2 marks)

Outcomes Assessed: H5

Targeted Performance Bands: 4-5

Criteria	Marks
• Gives the correct answer	2
• Gives the angle in radians	1

Sample answer:

Let  $\angle AOP = \theta$

$$\text{In } \triangle AOP \quad \cos \theta = \frac{1}{2} \quad \therefore \theta = \frac{\pi}{3}$$

$$\therefore \angle AOB = \frac{2\pi}{3}$$

$$\therefore \text{area of the sector } AOB = \frac{1}{2} \times 2^2 \times \frac{2\pi}{3} = \frac{4\pi}{3}$$

(d) (ii) (2 marks)

Outcomes Assessed: H5

Targeted Performance Bands: 4-5

Criteria	Marks
• Gives the correct answer	2
• Progress towards the correct answer	1

Sample answer:

$$\text{Shaded area} = \frac{1}{2} \times 2^2 \left( \frac{2\pi}{3} - \sin \frac{2\pi}{3} \right) = 2 \left( \frac{2\pi}{3} - \frac{\sqrt{3}}{2} \right) = \left( \frac{4\pi}{3} - \sqrt{3} \right) \approx 2.46$$

Question 15 (15 marks)

(a) (i) (1 mark)

Outcomes Assessed: P2

Targeted Performance Bands: 2-3

Criteria	Mark
• Gives the correct answer	1

Sample answer:

$$\angle AOB = \angle OBF = \alpha \quad (\text{alternate angles ; } OA \parallel FB)$$

(a) (ii) (2 marks)

Outcomes Assessed: P3

Targeted Performance Bands: 3-4

Criteria	Marks
• Gives the correct answer	2
• Progress towards the correct answer	1

Sample answer:

$$\text{In } \triangle OBF, OF = x_1 \text{ and } BF = 2h \text{ and } \angle OBF = \alpha$$

$$\therefore \tan \alpha = \frac{x_1}{2h} \quad \therefore x_1 = 2h \tan \alpha \dots \textcircled{1}$$

(a) (iii) (1 mark)

Outcomes Assessed: P3

Targeted Performance Bands: 3-4

Criteria	Mark
• Gives the correct answer	1

Sample answer:

$$\text{In } \triangle OAE, \cos \alpha = \frac{OE}{2h} \therefore OE = 2h \cos \alpha \dots \textcircled{2}$$

(a) (iv) (2 marks)

Outcomes Assessed: H2, H4, H9

Targeted Performance Bands: 5-6

Criteria	Marks
• Gives the correct answer	2
• Finds another expression for $OE$ or equivalent working	1

Sample answer:

Let  $G$  be the foot of the perpendicular from  $E$  to the  $x$ -axis.

In  $\triangle OEG$ ,  $EG = y_1$  and  $\angle OEG = \alpha$

$$\therefore \cos \alpha = \frac{y_1}{OE} \therefore OE = \frac{y_1}{\cos \alpha} \dots \textcircled{3}$$

$$\therefore \text{from } \textcircled{2} \text{ and } \textcircled{3} \quad \frac{y_1}{\cos \alpha} = 2h \cos \alpha$$

$$\therefore y_1 = 2h \cos^2 \alpha \dots \textcircled{4}$$

(a) (v) (3 marks)

Outcomes Assessed: H2, H4, H9

Targeted Performance Bands: 5-6

Criteria	Marks
• Gives the correct answer	3
• Find a correct expression for $\sec^2 \alpha = \frac{2h}{y_1}$	2
• Establishes that $x_1^2 = 4h^2 \tan^2 \alpha$	1

Sample answer:

$$\text{From } \textcircled{1} \quad x_1^2 = 4h^2 \tan^2 \alpha \quad \text{i.e. } x_1^2 = 4h^2 (\sec^2 \alpha - 1) \dots \textcircled{5}$$

$$\text{and from } \textcircled{4} \quad \cos^2 \alpha = \frac{y_1}{2h}$$

$$\text{i.e. } \sec^2 \alpha = \frac{2h}{y_1} \quad \text{and sub into } \textcircled{5} \quad x_1^2 = 4h^2 \left( \frac{2h}{y_1} - 1 \right)$$

$$\therefore x_1 = 2h \sqrt{\frac{2h}{y_1} - 1} \text{ as required.}$$

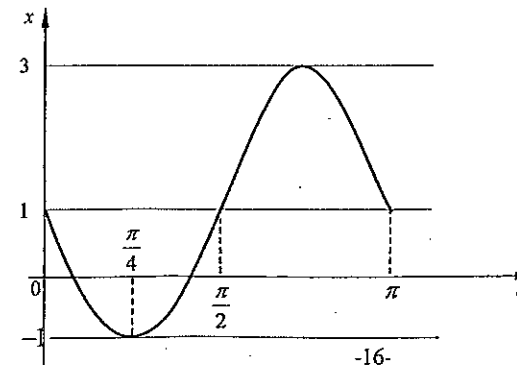
(b) (i) (2 marks)

Outcomes Assessed: H3, H6

Targeted Performance Bands: 3-5

Criteria	Marks
• Gives the correct graph	2
• Shows a sine graph with one correct feature	1

Sample answer:



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(b) (ii) (1 mark)

Outcomes Assessed: H7

Targeted Performance Bands: 4-5

Criteria	Mark
• Gives the correct answer	1

Sample answer:

From the graph, the particle is at rest for the first time when  $t = \frac{\pi}{4}$  sec.

(b) (iii) (1 mark)

Outcomes Assessed: H4

Targeted Performance Bands: 4-5

Criteria	Mark
• Gives the correct answer	1

Sample answer:

As  $x = 1 - 2\sin 2t$  and when  $t = \frac{\pi}{4}$   $\therefore x = -1$  metre. Alternatively, from graph  $x = -1$  metre

(b) (iv) (2 marks)

Outcomes Assessed: H2, H4, H9

Targeted Performance Bands: 3-5

Criteria	Marks
• Gives the correct answer	2
• Finds an expression for velocity	1

Sample answer

$$x = 1 - 2\sin 2t$$

$\therefore v = -4 \cos 2t$ . The maximum speed is the amplitude of  $v$ .  $\therefore$  speed = 4 m/s

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Question 16 (15 marks)

(a) (i) (2 marks)

Outcomes Assessed: P4

Targeted Performance Bands: 3-4

Criteria	Marks
• Correctly shows the value of $K$	2
• Progress towards an answer	1

Sample answer:

$$x^3 - 9x = 9x$$

$$x^3 - 18x = 0 \quad \therefore x(x^2 - 18) = 0$$

$$\therefore x = 0 \quad \text{or} \quad x = \pm\sqrt{18} = \pm 3\sqrt{2} \quad \therefore K = 3\sqrt{2}$$

(a) (ii) (3 marks)

Outcomes Assessed: H8, H9

Targeted Performance Bands: 4-5

Criteria	Marks
• Gives the correct answer	3
• Substantial progress towards an answer	2
• Finds a correct primitive	1

Sample answer:

$$\int_0^{3\sqrt{2}} 9x - (x^3 - 9x) dx \quad \therefore \int_0^{3\sqrt{2}} 18x - x^3 dx$$

$$= \left[ 9x^2 - \frac{x^4}{4} \right]_0^{3\sqrt{2}}$$

$$\therefore \left( 9(3\sqrt{2})^2 - \frac{(3\sqrt{2})^4}{4} \right) - \left( 9(0)^2 - \frac{(0)^4}{4} \right) = 81 \text{ square units.}$$

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(b) (3 marks)

Outcomes Assessed: H8, H9

Targeted Performance Bands: 4-5

Criteria	Marks
• Gives the correct expression for $g(x)$	3
• Substantial progress towards an answer	2
• Finds the point of intersection or equivalent merit	1

Sample answer:

$$g'(x) = \frac{6}{3x-2} \text{ and } m_1 = 6$$

$$\therefore \frac{6}{3x-2} = 6$$

$$\therefore x = 1 \text{ \& } y = 4$$

Hence,

$$g(x) = 2 \log_e(3x-2) + C$$

$$4 = 2 \log_e(3(1)-2) + C$$

$$\therefore C = 4$$

$$\therefore g(x) = 2 \log_e(3x-2) + 4$$

(c) (i) (2 marks)

Outcomes Assessed: P4

Targeted Performance Bands: 3-5

Criteria	Marks
• Correctly determine the values of $h$ and $k$	2
• Correctly substitutes (0,-6) and (5,-1) with progress towards an answer	1

Sample answer:

Substitute (0, -6) and (5, -1) into the equation  $y = \frac{h}{x^2 + k}$

$$\therefore -6 = \frac{h}{0+k} \quad h = -6k \dots \textcircled{1}$$

$$\text{and } -1 = \frac{h}{25+k} \quad \therefore h = -25-k \dots \textcircled{2}$$

$$\therefore \text{from } \textcircled{1} \text{ and } \textcircled{2} \quad \therefore -6k = -25-k \quad \therefore 5k = 25$$

$$\therefore k = 5 \text{ and } h = -30$$

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(c) (ii) (1 mark)

Outcomes Assessed: P3

Targeted Performance Bands: 3-5

Criteria	Mark
• Gives the correct answer	1

Sample answer:

$$\therefore y = \frac{-30}{x^2 + 5} \dots \textcircled{3}$$

Since  $Q$  has coordinates  $(\alpha, 0)$   $\therefore$  the  $x$  coordinate of  $R$  is  $\alpha$

$$\therefore \text{substitute into } \textcircled{3} \quad \therefore y = \frac{-30}{\alpha^2 + 5}$$

$$\text{Hence } R \text{ is } \left( \alpha, \frac{-30}{\alpha^2 + 5} \right)$$

(c) (iii) (1 mark)

Outcomes Assessed: H4

Targeted Performance Bands: 3-5

Criteria	Mark
• Gives the correct answer	1

Sample answer:

$$\text{Area of the rectangle} = 2\alpha \times \frac{30}{\alpha^2 + 25} = \frac{60\alpha}{\alpha^2 + 5}$$

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(c) (iv) (3 marks)

Outcomes Assessed: H2, H5, H9

Targeted Performance Bands: 5-6

Criteria	Marks
• Finds the correct expression for maximum area	3
• Shows that $\alpha = \sqrt{5}$	2
• Correctly differentiates and equates to zero	1

Sample answer:

$$A = \frac{60\alpha}{\alpha^2 + 5} \dots \textcircled{4}$$

$$\therefore \frac{dA}{d\alpha} = \frac{60(\alpha^2 + 5) - 2\alpha(60\alpha)}{(\alpha^2 + 5)^2} = \frac{60\alpha^2 + 300 - 120\alpha^2}{(\alpha^2 + 5)^2}$$

$$\therefore \frac{dA}{d\alpha} = \frac{300 - 60\alpha^2}{(\alpha^2 + 5)^2} \text{ and for maximum area } \frac{dA}{d\alpha} = 0$$

$$\therefore 0 = 300 - 60\alpha^2 \quad \therefore \alpha = \sqrt{5}$$

$\alpha$	$< \sqrt{5}$	$\sqrt{5}$	$> \sqrt{5}$
$\frac{dA}{d\alpha}$	+	0	-

$\therefore$  The area is maximum when  $\alpha = \sqrt{5}$

$$\text{and from } \textcircled{4} \quad A = \frac{60 \times \sqrt{5}}{5 + 5} = 6\sqrt{5}$$

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