Centre Number

Student Number

CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NEW SOUTH WALES

2002

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

Morning Session
Monday 12 August 2002

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided on page 15
- All necessary working should be shown in every question

Total marks – 120

- Attempt Questions 1–10
- All questions are of equal value

2602 – 1

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Total marks (120)
Attempt Questions 1 – 10
All questions are of equal value

Answer each question in a SEPARATE writing booklet.

Question 1 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Evaluate $\dfrac{x^3+y^4}{y^2}$ if $x=\left(\dfrac{2}{5}\right)^{\frac{1}{3}}$ and $y=\left(\dfrac{3}{5}\right)^{\frac{1}{2}}$. **2**
Give your answer in fractional form.

(b) Express $0.2\dot{3}$ as a fraction in simplest form. **2**

(c) Factorise $40-5y^3$ **2**

(d) Solve $x^2+4x-1=0$ leaving your answer in simplest surd form. **3**

(e) (i) Solve $4^x=32$. **2**

(ii) Hence, or otherwise, write down the value of $\log_4 32$. **1**

Question 2 (12 marks) Use a SEPARATE writing booklet. **Marks**

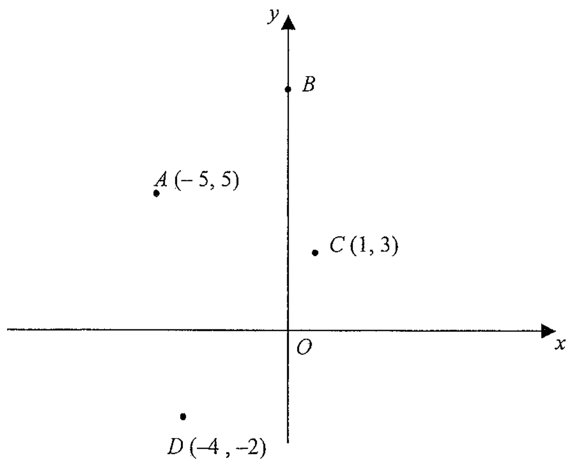
(a) Differentiate with respect to x:

(i) $5x+\dfrac{3}{x^2}$ **1**

(ii) e^{2x^2+3} **1**

(iii) $\dfrac{3x}{\sin x}$ **2**

(b)



NOT TO SCALE

The diagram shows the points A (–5, 5) and C (1, 3) and D (–4, –2).
B is a point on the y axis.

(i) Find the gradient of AC. **1**

(ii) Find the midpoint of AC. **1**

(iii) Show that the equation of the perpendicular bisector of AC is $3x-y+10=0$.

(iv) Find the coordinates of B given that B lies on $3x-y+10=0$. **1**

(v) Show that the point D (– 4, –2) lies on $3x-y+10=0$. **1**

(vi) Show that $ABCD$ is a rhombus. **2**

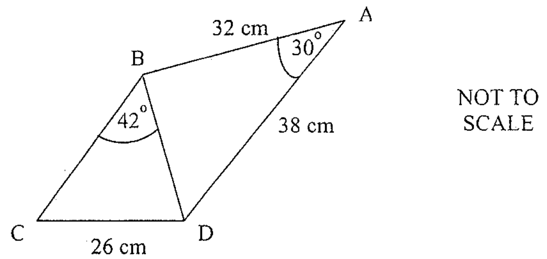
Question 3 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find $\int \frac{x}{x^2+5}\,dx$ **2**

(b) Evaluate $\int_0^{\frac{\pi}{8}} \sec^2 2x\,dx$ **2**

(c) Find the equation of the normal to the curve $y = x\log_e x$ at the point (e, e). **4**

(d)



In the diagram AB is 32 cm, AD is 38 cm and CD is 26 cm
$\angle BAD$ is 30° and $\angle CBD$ is 42°.

(i) Use the cosine rule to find the length of BD. **2**

(ii) Hence, find the size of $\angle BCD$ to the nearest degree. **2**

Question 4 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) The second term of a geometric series is 120 and the fifth term is 50.625.

(i) Find the common ratio and the first term of the series. **2**

(ii) Find the limiting sum of the series. **1**

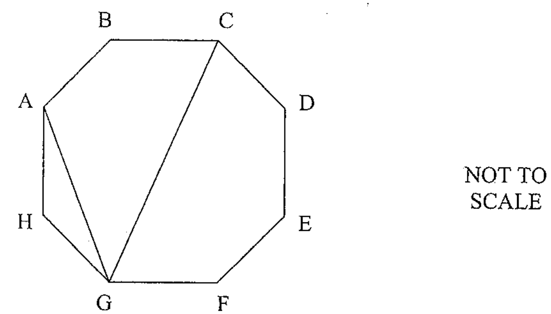
(iii) Hence, find the difference between the limiting sum and the sum of the first 40 terms giving your answer in scientific notation correct to 2 significant figures. **2**

(b) For the quadratic equation $x^2 + kx - 3x + 2 - k = 0$,

(i) find the value of the discriminant in terms of k, **1**

(ii) explain why the roots of this quadratic equation are real for all values of k. **2**

(c)



ABCDEFGH is a regular octagon.

(i) Explain clearly why $\angle ABC$ is 135°. **1**

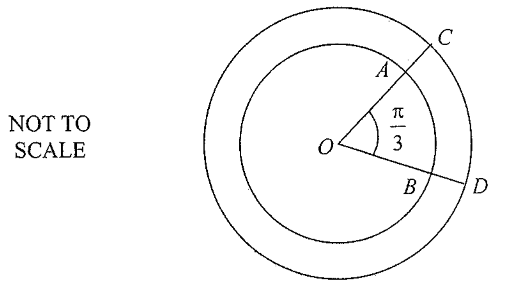
(ii) Calculate the size of $\angle GAH$. **1**

(iii) Using (i), or otherwise, calculate the size of $\angle CGF$. **1**

(iv) Hence, calculate the size of $\angle AGC$. **1**

Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a)



The diagram shows two concentric circles with centre O.
The radius of the larger circle is 8.2 cm.

(i) Calculate the area of sector COD. **1**

(ii) The area of the sector AOB is 18.4 cm^2. Calculate the radius of this sector AOB. **2**

(iii) Calculate the area of triangle COB. **2**

(b) Let $f(x) = 3x^2 + 1$.

(i) Copy the following table and supply the missing values. **1**

x	0	0.2	0.4	0.6	0.8	1
$f(x)$	1					4

(ii) Use these six values of the function and the trapezoidal rule to find the approximate value of **2**

$$\int_0^1 (3x^2 + 1)\, dx .$$

Question 5 continues on page 8

Question 5 (continued) **Marks**

(c) The population P of a town is growing at a rate proportional to the town's current population. The population at time t years is given by $P = A\,e^{kt}$, where A and k are constants.

The population 20 years ago was 100 000 people and today the population of the town is 150 000 people.

(i) Find the value of A. **1**

(ii) Find the value of k. **1**

(iii) Find the population that will be present 20 years from now. **2**

End of Question 5

Question 6 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Consider the curve given by $y = x^3 + 3x^2 - 9x - 5$.

(i) Find $\frac{dy}{dx}$. **1**

(ii) Find the coordinates of the two stationary points. **2**

(iii) Determine the nature of the stationary points. **2**

(iv) Sketch the curve for the domain $-5 \le x \le 3$. **2**

(v) By drawing an appropriate line on your graph, or otherwise, solve **2**

$$x^3 + 3x^2 - 9x + 5 = 0 .$$

(b) Calculate the exact volume generated when the region enclosed by the curve **3**

$$y = 1 + 2e^{-x} \text{ for } 0 \le x \le 1,$$

is rotated about the x axis.

Question 7 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) A bag contains 5 blue balls, 4 red balls, 2 yellow balls and 1 green ball. Three balls are selected at random without replacement from the bag. Calculate the probability that

(i) the three balls drawn are blue, **1**

(ii) the three balls drawn are of the same colour, **2**

(iii) exactly two of the balls drawn are blue. **2**

(b) A particle is projected vertically upwards from a point 2 metres above horizontal ground. The displacement at time t seconds is given by

$$x = 24.5t - 4.9t^2, \qquad t \ge 0.$$

(i) Find an expression for the velocity of the particle. **1**

(ii) Find when the particle comes to rest. **2**

(iii) Hence, find the greatest height of the particle above the ground. **2**

(iv) Find the length of time for which the particle is at least 21.6 metres above the ground. **2**

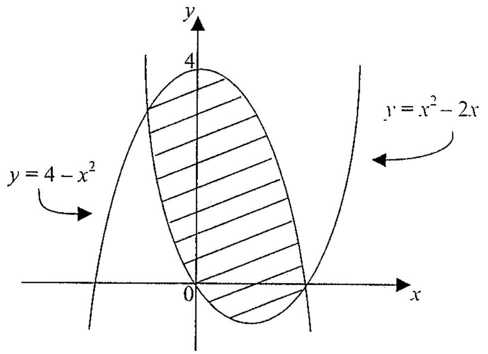
Question 8 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) A function $y = f(x)$ is continuous for all values. After finding the first and second derivatives a student discovers the following, for all values of x. **2**

When	$x < 2$,	$f'(x) < 0$	and	$f''(x) > 0$
When	$x = 2$,	$f'(x) = 0$	and	$f''(x) = 0$
When	$x > 2$,	$f'(x) < 0$	and	$f''(x) < 0$.

Draw a neat sketch of $y = f(x)$, showing all the important characteristics of the function given that $f(2) = 0$.

(b) The graphs of the functions $y = 4 - x^2$ and $y = x^2 - 2x$.



NOT TO SCALE

(i) Describe, using inequalities, the shaded region. **1**

(ii) By solving simultaneously, show that the points of intersection are at $x = -1$ and $x = 2$. **2**

(iii) Calculate the area of the shaded region. **2**

Question 8 continues on page 12

Question 8 (continued) **Marks**

(c) On a factory production line a tap opens and closes to fill containers with liquid. As the tap opens, the rate of flow increases for the first 10 seconds according to the relation $R = \frac{6t}{50}$, where R is measured in L/sec. The rate of flow then remains constant until the tap begins to close. As the tap closes, the rate of flow decreases at a constant rate for 10 seconds, after which time the tap is fully closed.

(i) Show that, while the tap is fully open, the volume in the container at any time is given by **3**

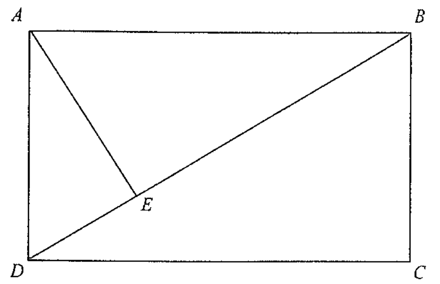
$$V = \frac{6}{5}(t - 5).$$

(ii) For how many seconds must the tap remain fully open in order to exactly fill a 120L container with no spillage.

End of Question 8

Question 9 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) $ABCD$ is a rectangle and $AE \perp BD$. $AE = 5$ cm and $DE = 2$ cm.



(i) Copy the diagram and prove that triangles AED and BCD are similar. 2

(ii) Hence, show that $AD^2 = BD.DE$. 1

(iii) Find the area of $ABCD$. 3

(b) A closed water tank in the shape of a right cylinder is to be constructed with a surface area of 54π cm^2. The height of the cylinder is h cm and the base radius is r cm.

(i) Show that the height of the water tank in terms of r is given by 2

$$h = \frac{27}{r} - r$$

(ii) Show that the volume V that can be contained in the tank is given by 1

$$V = 27\pi r - \pi r^3$$

(iii) Find the radius r cm which will give the cylinder its greatest possible volume. Justify your answer. 3

Question 10 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) (i) Show that $x = \frac{\pi}{8}$ is a solution of $\sin 2x = \cos 2x$. 1

(ii) On the same set of axes, sketch the graphs of the functions $y = \sin 2x$ and $y = \cos 2x$ for $-\pi \le x \le \pi$. 2

(iii) Hence, find graphically the number of solutions of $\tan 2x = 1$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$. 1

(iv) Use your graphs to solve $\tan 2x \le 1$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$. 1

(b) Mr and Mrs Matthews decide to borrow \$250 000 to buy a house. Interest is calculated monthly on the balance still owing, at a rate of 6.06% per annum. The loan is to be repaid at the end of 15 years with equal monthly repayments of \$$M$.

Let \$$A_n$ be the amount owing after the nth repayment.

(i) Derive an expression for A_{60}. 1

(ii) Find the value of M. 2

(iii) Hence, calculate the amount still owing after 5 years of payment at this rate. 2

(iv) At the end of five years, the interest rate is increased to 7.2% per annum and Mr and Mrs Matthews change their payments to \$1800 per month. How many more months are needed to pay off the remainder of the loan? 2

End of paper

CSSA 2002

Q1. (a) $x^3 = \left(\frac{2}{5}\right)^{\frac{1}{3}\times 3}$ $y^4 = \left(\frac{3}{5}\right)^{\frac{1}{4}\times 4}$ $y^2 = \left(\frac{3}{5}\right)^{\frac{1}{2}\times 2}$

$x^3 = \frac{2}{5}$ $y^4 = \frac{9}{25}$ $y^2 = \frac{3}{5}$

$\frac{x^3 + y^4}{y^2} = \left(\frac{2}{5} + \frac{9}{25}\right) \div \frac{3}{5}$ ✓

$= 1\frac{4}{15}$ ✓

(b) $0.2\dot{3} = 0.23333\ldots$ ②

$10 \times 0.2\dot{3} = 2.3333\ldots$ ① ✓

①−② $9 \times 0.2\dot{3} = 2.1$

$\therefore 0.2\dot{3} = \frac{2.1}{9}$

$0.23 = \frac{21}{90}$ ✓

(c) $40 - 5y^3 = 5(8 - y^3)$ ✓

$= 5(2 - y)(4 + 2y + y^2)$ ✓

(d) $x^2 + 4x - 1 = 0$

$x = \frac{-4 \pm \sqrt{4^2 - 4\times1\times -1}}{2\times1}$ ✓

$= \frac{-4 \pm \sqrt{20}}{2}$

$= \frac{-4 \pm 2\sqrt{5}}{2}$ ✓

$x = -2 \pm \sqrt{5}$ ✓

(e) (i) $4^x = 32$

$(2^2)^x = 2^5$ ✓

$2^{2x} = 5$

$x = 2\frac{1}{2}$ ✓

(ii) $\log_4 32 = \log_4 4^{2\frac{1}{2}}$

$= 2\frac{1}{2}$ ✓

Q2. (a) (i) $\frac{d}{dx}\left(5x + \frac{3}{x^2}\right) = \frac{d}{dx}(5x + 3x^{-2})$

$= 5 - 6x^{-3}$

$= 5 - \frac{6}{x^3}$ ✓

(ii) $\frac{d}{dx} e^{2x^2+3} = 4xe^{2x^2+3}$ ✓

(iii) $\frac{3x}{\sin x} = \frac{u}{v}$

$u = 3x$ $v = \sin x$ $\Rightarrow \frac{d}{dx}\frac{3x}{\sin x} = \frac{3\sin x - 3x\cos x}{\sin^2 x}$ ✓

$u' = 3$ $v' = \cos x$ ✓

(b) $A(-5,5)$ $C(1,3)$ $D(-4,-2)$

(i) $m_{AC} = \frac{3-5}{1+5}$

$m_{AC} = \frac{-1}{3}$ ✓

(ii) $M_{AC} = \left(\frac{-5+1}{2}, \frac{5+3}{2}\right)$ ✓

$M_{AC} = (-2, 4)$

(iii) $m_\perp = \frac{-1}{m_{AC}}$

$= 3$ ✓

$\Rightarrow$ $y - 4 = 3(x + 2)$ ✓

$y - 4 = 3x + 6$

$3x - y + 10 = 0$

(iv) B lies on y axis $\therefore x = 0$

$3\times0 - y + 10 = 0$

$y = 10$

$\therefore B(0, 10)$ ✓

(v) $D(-4, -2)$ LHS $= 3x - y + 10$

$= 3\times -4 - (-2) + 10$

$= 0$

$=$ RHS ✓

$\therefore$ D lies on line

(vi) $M_{BD} = \left(\frac{0-4}{2}, \frac{10-2}{2}\right)$

$= (-2, 4)$

$= M_{AC}$ ✓

Since Midpoint of BD an Midpoint of AC are the same & both B & D lie on same line ie diagonals bisect each other ✓

$\therefore$ RHOMBUS

Q3. (a) $\int \frac{x}{x^2+5}\,dx = \frac{1}{2}\int \frac{2x}{x^2+5}\,dx$

$= \frac{1}{2}\ln(x^2+5) + C$ ✓✓

(b) $\int_0^{\pi/8} \sec^2 2x\,dx = \frac{1}{2}\int_0^{\pi/8} 2\sec^2 2x\,dx$

$= \frac{1}{2}\tan 2x \Big]_0^{\pi/8}$ ✓

$= \frac{1}{2}\left(\tan\frac{\pi}{4} - \tan 0\right)$

$= \frac{1}{2}$ ✓

(c) $y = x\ln x$ (e, e)

$m_T = y'$

$y' = 1.\ln x + x.\frac{1}{x}$

$= \ln x + 1$ ✓

at (e, e) $m_T = \ln e + 1$

$= 2$ ✓

$\therefore m_N = \frac{-1}{m_T}$

$= \frac{-1}{2}$ ✓

Eqⁿ normal: $y - e = \frac{-1}{2}(x - e)$

$2y - 2e = -x + e$

$x + 2y - 3e = 0$ ✓

(d) (i) $BD^2 = 38^2 + 32^2 - 2\times38\times32\cos 30^\circ$ ✓

$BD = 19\text{cm (2 sf)}$ ✓

(ii) $\frac{\sin\angle BCD}{19} = \frac{\sin 42^\circ}{26}$ ✓

$\sin\angle BCD = \frac{19\sin 42^\circ}{26}$

$\angle BCD = 29^\circ$ (NDEG) ✓

Q4 (a) GP: $t_2 = 120$, $t_5 = 50.625$

(i) $ar = 120$ $ar^4 = 50.625$

$\frac{ar^4}{ar} = \frac{50.625}{120}$

$r^3 = 0.421875$

$= \frac{27}{64}$

$r = \frac{3}{4}$ $(= 0.75)$

$\Rightarrow a = 120 \div \frac{3}{4}$

$a = 160$ ✓, $r = \frac{3}{4}$ ✓

(ii) $\lim S = \frac{a}{1-r}$

$= \frac{160}{1 - 3/4}$

$\lim S = 640$ ✓

(iii) $S_{40} = \frac{160(1 - 0.75^{40})}{1 - 0.75}$

$= 639.9935\ldots$ ✓

$S_\infty - S_{40} = 640 - 639.9935\ldots$

$= 0.0064$ (2SF)

$= 6.4 \times 10^{-3}$(2SF) ✓

(b) $x^2 + kx - 3x + 2 - k = 0$

$a = 1$ $b = (k - 3)$ $c = 2 - k$

(i) $\Delta = (k-3)^2 - 4\times1\times(2-k)$

$= k^2 - 6k + 9 - 8 + 4k$

$= k^2 - 2k + 1$

(ii) $k^2 - 2k + 1 = (k-1)^2$

≥ 0

$\Delta \geq 0$

Means that all roots are real.

(c)

(i) Angle Sum $= 180^\circ \times 6$

$= 1080^\circ$

$\angle ABC = 1080^\circ \div 8$

$\angle ABC = 135^\circ$ ✓

(ii) $\angle GAH = \frac{180^\circ - 135^\circ}{2}$

$= 22.5^\circ$ ✓

(iii) $\angle CGF = \frac{135}{2}$

$= 67.5^\circ$ ✓

(iv) $\angle AGC = 135^\circ - (67.5^\circ + 22.5^\circ)$

$= 45^\circ$ ✓

Q5. (a) $\theta = \frac{\pi}{3}$ $OC = OD = 8.2\text{cm}$

(i) $A_{COD} = \frac{1}{2}r^2\theta$

$= \frac{1}{2}\times 8.2^2 \times \frac{\pi}{3}$ ✓

$= 35.21\text{cm}^2$ (2DP)

(ii) $18.4 = \frac{1}{2}r^2\frac{\pi}{3}$ ✓

$r^2 = 18.4 \div \frac{\pi}{6}$

$= 35.1414\ldots$

$r = 5.92$

$r = 5.9\text{cm}$ (1DP) ✓

(iii) Area $\Delta COB = \frac{1}{2}\times 5.9\times 8.2\sin\frac{\pi}{3}$ ✓

$= 21.048\ldots\text{cm}^2$ ✓

$A_{\Delta COB} = 21\text{cm}^2$ (nearest cm)

(b)

(i)

x	0	0.2	0.4	0.6	0.8	1
$f(x)$	1	1.12	1.48	2.08	2.92	4

✓

(ii) $\int_0^1 (3x^2+1)\,dx \doteqdot \frac{0.2}{2}(1 + 1.12) + \frac{0.2}{2}(1.12 + 1.48)$ ✓

$+ \frac{0.2}{2}(1.48 + 2.08) + \frac{0.2}{2}(2.08 + 2.92)$

$+ \frac{0.2}{2}(2.92 + 4)$

$= 2.02$ ✓

(c) $P = Ae^{kt}$

(i) $A = 100\,000$ $(t = 0 \Rightarrow e^{kt} = 1)$ ✓

(ii) $150000 = 100\,000\,e^{20k}$

$e^{20k} = 1.5$

$20k = \ln 1.5$

$k = \frac{1}{20}\ln 1.5$ ✓

(iii) $t = 40$

$P = 100000\,e^{40\times\frac{1}{20}\ln 1.5}$ ✓

$= 100000\,e^{2\ln 1.5}$

$= 225000$

$\therefore$ Popⁿ in 20 years time will be 225000 ✓

Q6. (a) $y = x^3 + 3x^2 - 9x - 5$

(i) $y' = 3x^2 + 6x - 9$ ✓

(ii) S.P occur when $y' = 0$

$3x^2 + 6x - 9 = 0$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x = -3, 1$ ✓

$x = -3 \quad y = (-3)^3 + 3(-3)^2 - 9(-3) - 5 = 22$ ✓

$x = 1 \quad y = -10$

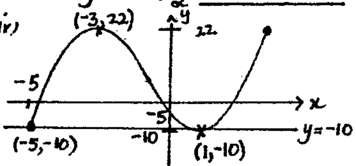
$\Rightarrow$ S.P $(-3, 22)$ $(1, -10)$

(iii) $y'' = 6x + 6$

$x = -3 \quad y'' < 0 \Rightarrow$ Local Max $(-3, 22)$ ✓

$x = 1 \quad y'' > 0 \Rightarrow$ Local Min $(1, -10)$ ✓

(iv)



✓ shape

✓ turning pts.

(v) $x^3 + 3x^2 - 9x - 5 = -10$

ie $y = -10 \Rightarrow x^3 + 3x^2 - 9x + 5 = 0$

$\Rightarrow x = -5, 1$ ✓✓

(b) $y = 1 + 2e^{-x} \quad 0 \le x \le 1$

$V = \pi \int_0^1 (1 + 2e^{-x})^2 dx$ ✓

$= \pi \int_0^1 (1 + 4e^{-x} + 4e^{-2x}) dx$

$= \pi (x - 4e^{-x} - 2e^{-2x})_0^1$ ✓

$V = \pi\left[\left(1 - \frac{4}{e} - \frac{2}{e^2}\right) - (0 - 4 - 2)\right]$

$V = \pi\left(7 - \frac{4}{e} - \frac{2}{e}\right)$ units^3 ✓

Q7 a) 5B, 4R, 2Y, 1G

(i) $P(BBB) = \frac{5}{12} \times \frac{4}{11} \times \frac{3}{10}$

$= \frac{1}{22}$ ✓

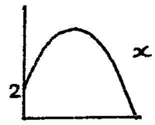
(ii) $P(BBB) + P(RRR) = \frac{1}{22} + \frac{4}{12} \times \frac{3}{11} \times \frac{2}{10}$ ✓

$= \frac{7}{110}$ ✓

(iii) $P(BB\tilde{B}) + P(B\tilde{B}B) + P(\tilde{B}BB) = \frac{5}{12} \times \frac{4}{11} \times \frac{7}{10} + \frac{5}{12} \times \frac{7}{11} \times \frac{4}{10} + \frac{7}{12} \times \frac{5}{11} \times \frac{4}{10}$ ✓

$= \frac{7}{22}$ ✓

(b)



$x = 24.5t - 4.9t^2 \quad t \ge 0$

(i) $\dot{x} = 24.5 - 9.8t$ ✓

(ii) Rest $\dot{x} = 0$

$0 = 24.5 - 9.8t$ ✓

$9.8t = 24.5$

$t = \frac{24.5}{9.8}$

$t = 2\frac{1}{2}$ secs ✓

(iii) Max height $\dot{x} = 0$ ie $t = 2\frac{1}{2}$ secs

$x = 24.5 \times 2.5 - 4.9 \times 2.5^2$

$= 30.625$ ✓

$\therefore$ Max height 32.625m ✓

$(\ddot{x} = -9.8 < 0 \therefore$ Max$)$

(iv) 21.6m above ground $\Rightarrow$ 19.6m from start.

$19.6 \le 24.5t - 4.9t^2$ ✓

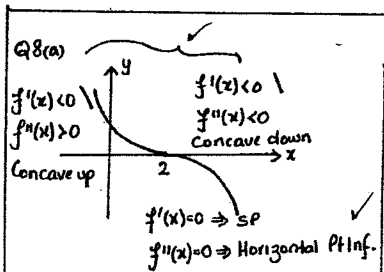
$4.9t^2 - 24.5t + 19.6 \le 0$

$t^2 - 5t + 4 \le 0$

$(t-4)(t-1) \le 0$

$1 \le t \le 4$ seconds. ✓

Q8(a) ✓



✓

(b)

(i) $y \le 4 - x^2$ & $y \ge x^2 - 2x$ ✓

(ii) $y = 4 - x^2 \quad y = x^2 - 2x$

$x^2 - 2x = 4 - x^2$ ✓

$2x^2 - 2x - 4 = 0$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = -1, 2$ ✓

(iii) $A = \int_{-1}^{2} (4 - x^2) - (x^2 - 2x) dx$

$= \int_{-1}^{2} 4 - x^2 - x^2 + 2x \, dx$

$= \int_{-1}^{2} 4 - 2x^2 + 2x \, dx$ ✓

$= \left. 4x - \frac{2}{3}x^3 + x^2 \right)_{-1}^{2}$

$= 8 - \frac{16}{3} + 4 + 4 - \frac{2}{3} - 1$

$= 9$ units^2 ✓

(c) $R = \frac{6t}{50} \quad t = 0 - 10$

$t > 0 \quad R = k$

(i) $\frac{dV}{dt} = \frac{6t}{50}$

$V = \int \frac{6t}{50} dt$

$V = \frac{3t^2}{50} + C$

$t = 0 \quad V = 0 \Rightarrow C = 0$

$V = \frac{3t^2}{50} \quad t \le 10$

$t = 10 \quad V = \frac{3 \times 10^2}{50}$

$= 6L/s$ ✓

$t > 10 \quad \frac{dV}{dt} = \frac{6t}{50}$

$t = 0 \quad \frac{dV}{dt} = \frac{60}{50}$

$= \frac{6}{5}$ L/sec ✓

$\therefore \quad V = \frac{6t}{5} + C$

$t = 10 \quad V = 6 \Rightarrow C = -6$

$V = \frac{6t}{5} - 6 = \frac{6t - 30}{5}$

$V = \frac{6}{5}(t - 5)$ ✓

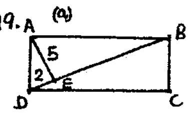
(ii) $V = 120 - 6 = 114L$ ✓

$\frac{6}{5}(t-5) = 114$

$t - 5 = 95$

$t = 100$ sec $\Rightarrow$ Fully open 90sec ✓

Q9. (a)



(i) In $\triangle$s AED & BCD

$\angle AED = \angle BCD = 90°$ ✓

(ABCD rect. & AE⊥BD)

$\angle ADE = \angle DBC$ (Alt ∠s = AD ∥ BC) ✓

$\therefore \angle DAE = \angle BDC$ (2 ∠s = Angle Sum △180°)

$\therefore \triangle AED \,|||\, \triangle BCD$ (equiangular)

(ii) $\frac{AD}{BD} = \frac{DE}{BC}$ (Corresp. sides ||| △s)

$AD = \frac{DE}{BC} \cdot BD$ ✓

$BC = AD$ (Opp. Sides Rect)

$\therefore AD^2 = DE.BD$

(iii) $AD^2 = 5^2 + 2^2$ ✓

$DE.BD = 29$

$\therefore 2BD = 29 \quad (DE = 2)$

$BD = 14\frac{1}{2}$ ✓

Area ABCD $= 14\frac{1}{2} \times 5$

$= 72.5 cm^2$ ✓

(b) $SA = 54\pi cm^2$

(i) $SA = 2\pi rh + 2\pi r^2$

$54\pi = 2\pi(rh + r^2)$ ✓

$27 = rh + r^2$

$rh = 27 - r^2$ ✓

$h = \frac{27}{r} - r$

(b)

(ii) $V = \pi r^2 h$

$= \pi r^2 \left(\frac{27}{r} - r\right)$ ✓

$V = 27\pi r - \pi r^3$

(iii) Max V when $V' = 0$

$V' = 27\pi - 3\pi r^2$ ✓

$= 0$

$3\pi r^2 = 27\pi$

$r^2 = 9$

$r = \pm 3$ ✓

$r = 3 \quad r > 0$

$V'' = -6\pi r$

$r = 3 \quad V'' < 0$ ✓

$\therefore$ Max $r = 3$cm

Q10. (a) (i) $x=\frac{\pi}{8}$ $\sin 2x = \sin\frac{\pi}{4}$

$= \frac{1}{\sqrt{2}}$

$\cos 2x = \cos\frac{\pi}{4}$

$= \frac{1}{\sqrt{2}}$

$= \sin 2x \quad x=\frac{\pi}{8}$ ✓

(ii)

(iii) $\tan 2x = \frac{\sin 2x}{\cos 2x}$

$= 1 \Rightarrow \sin 2x = \cos 2x$

Curves have 2 points of intersection between $-\frac{\pi}{2} < x < \frac{\pi}{2}$. ✓

(iv) $\tan 2x < 1$ when $\sin 2x \leq \cos 2x$

ie $-\frac{3\pi}{8} < x < \frac{\pi}{8}$ ✓

(b) \$260000   6.06% p.a = 0.505% p.m

$n = 15 \times 12$

$n = 180$

(i) $A_1 = 250000 \times 1.00505 - M$

$A_2 = 250000 \times 1.00505^2 - M \times 1.00505 - M$

$A_3 = 250000 \times 1.00505^3 - M(1.00505^2 + 1.00505 + 1)$ ✓

$\vdots$

$A_{60} = 250000 \times 1.00505^{60} - M(1 + 1.00505 + 1.00505^2 + \ldots 1.00505^{59})$

(ii) $A_{180} = 250000 \times 1.00505^{180} - M\left(\frac{1.00505^{180} - 1}{1.00505 - 1}\right)$

$= 0$

$M = 250000 \times 1.00505^{180} \times \frac{0.00505}{1.00505^{180} - 1}$ ✓

$= \$2117.75$ ✓

(iii) 5 years   Amount owing is A_{60}

$A_{60} = 250000 \times 1.00505^{60} - 2117.75\left(\frac{1.00505^{60} - 1}{0.00505}\right)$ ✓

$= \$190\ 236.76$ ✓

(iv) 7.2% p.a = 0.6% p.m

$190\ 236.76 \times 1.006^n = 1800 \times \frac{(1.006^n - 1)}{0.006}$ ✓

$= 300\ 000 \times (1.006^n - 1)$

$1.006^n = \frac{300000}{300000 - 190236.76}$

$n = \frac{\ln\left(\frac{300000}{30000 - 190236.76}\right)}{\ln 1.006}$

$n = 168.07\ldots$

ie Approx 169 months. ✓