Centre Number

Student Number

CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NEW SOUTH WALES

2005
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

Morning Session
Monday 8 August 2005

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided separately
- All necessary working should be shown in every question
- Write your Centre Number and Student Number at the top of this page

Total marks – 120

- Attempt Questions 1-10
- All questions are of equal value

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Answer each question in a SEPARATE writing booklet.

Question 1 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Write down the value of $|-6|-|-12|$. **2**

(b) If $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$, find the value of f when $u=-5$ and $v=7.5$. **2**

(c) Solve the equation $(x-3)^2=9$. **2**

(d) Differentiate x^5+4x^{-2}. **2**

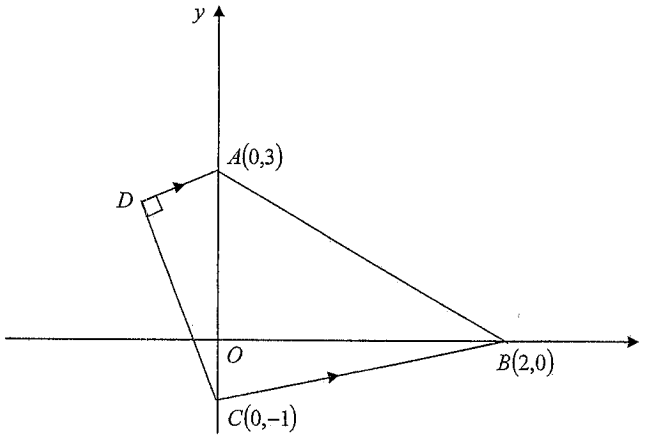
(e) Sketch the curve $y=e^x$. State its range. **2**

(f) If $\frac{1}{a}=\sqrt{10}-3$, show that $a=\sqrt{10}+3$. **2**

Question 2 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) The definition of an odd function $f(x)$ is given by the rule $f(-x) = -f(x)$. **2**
Show that the function $f(x) = x^5 - x^3$ is an odd function.

(b)



NOT TO SCALE

In the diagram above, points A, B and C have coordinates $(0,3)$, $(2,0)$ and $(0,-1)$ respectively. Also $AD \parallel BC$ and $AD \perp CD$.

Copy this diagram into your answer sheet.

(i) Show that the gradient of the line BC is equal to $\frac{1}{2}$. **1**

(ii) Show that the equation of the line AD is $x - 2y + 6 = 0$. **2**

(iii) Find the equation of line CD. **2**

(iv) By solving simultaneously the equations from (ii) and (iii), find the coordinates of point D. **2**

(v) Find the area of the quadrilateral $ABCD$. **3**

Question 3 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) In a right angled triangle $\tan\theta = \frac{3}{4}$. Find $\sin\theta$, for $0 \le \theta \le \frac{\pi}{2}$. **1**

(b) Differentiate the following functions:

(i) $\sin x \log_e x$ **2**

(ii) $3 \tan\frac{\pi x}{3}$ **2**

(c) Find: **2**

(i) $\int \sin(e - x)dx$ **2**

(ii) $\int_0^1 \frac{2x}{x^2+1}dx$, leaving answer in exact form. **2**

(d) Find the equation of the normal to the curve $y = e^{4x} - 1$ at the point on the curve where $x = 0$. **3**

Question 4 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) A quadratic equation with roots α and β has the form: **2**

$$x^2-(\alpha+\beta)x+\alpha\beta=0.$$

Hence, or otherwise, form a quadratic equation whose roots are $2+\sqrt{3}$ and $2-\sqrt{3}$.

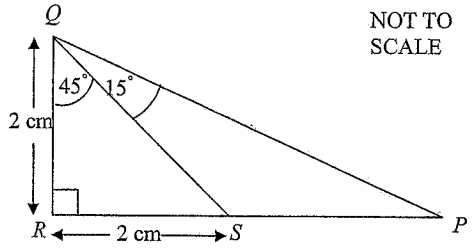
(b) The first and the thirteenth terms of an arithmetic progression are 7 and 1 respectively. Calculate:

(i) the common difference, **2**

(ii) the number of terms which have a sum of zero. **2**

(c) In the diagram below triangle QRP has a right angle at R. Also $\angle RQS=45^\circ$, $\angle SQP=15^\circ$ and $QR=RS=2$ cm.

Copy the diagram in your writing booklet.



(i) Using triangle QRS find the exact length of QS. **1**

(ii) Using triangle QRP find the exact length of PR and hence the exact length of PS. **2**

(iii) Use the Sine Rule in triangle QPS to prove that $\sin 15^0=\dfrac{\sqrt{3}-1}{2\sqrt{2}}$ **3**

Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Consider the curve given by $y=x^3-6x^2+9x+4$.

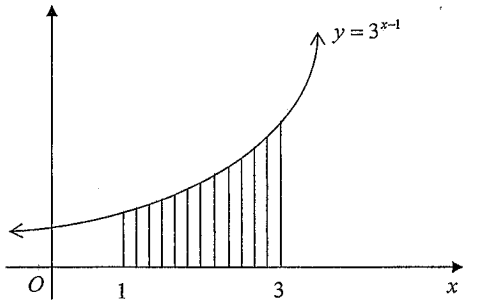
(i) Find the coordinates of the stationary points and determine their nature. **4**

(ii) Find the coordinates of any point of inflexion. **2**

(iii) Sketch the curve, showing all of the above information. **2**

(iv) Determine the values of x for which $\dfrac{dy}{dx}<0$ **1**

(b) The diagram below shows the shading of a region bounded by the graph $y=3^{x-1}$ and the lines $x=1$ and $x=3$.



(i) Copy and complete the following table giving your answer correct to three decimal places: **1**

x	1	1.5	2	2.5	3
$y=3^{x-1}$	1	1.732			

(ii) Use Simpson's Rule with five function values to approximate the shaded area to three decimal places. **2**

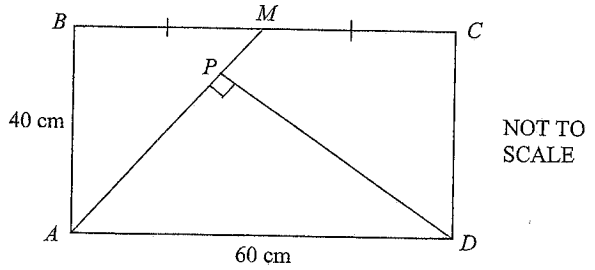
Question 6 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) Factorise the expression $2a^2 - 7a + 3$. **1**

(ii) Hence, solve the following equation for x: **3**

$$2(\log_2 x)^2 - 7(\log_2 x) + 3 = 0$$

(b)



NOT TO SCALE

$ABCD$ is a rectangle in which $AB = 40$ cm and $AD = 60$ cm. M is the midpoint of BC and DP is perpendicular to AM.

Draw a neat sketch on your answer sheet. Hence:

(i) Prove that triangles ABM and APD are similar. **2**

(ii) Calculate the length of PD. **2**

(iii) Using Pythagoras' Theorem in triangle APD show that $AP = 36$ cm. **1**

(iv) By finding the two areas of the triangles ABM and APD, prove that the area of the quadrilateral $PMCD$ is 936 cm^2. **3**

Question 7 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Nicole and Mariana play against each other, in the third round of the Australian Open. In this tournament, the first player to win 2 sets wins the match. The probability that Nicole wins any set is 70%.

(i) Find the probability that the game will last two sets only. **2**

(ii) Find the probability that Nicole wins the match. **2**

(iii) Find the probability that Mariana wins the match. **1**

(b) The number N of bacteria in a culture at time t seconds is given by the equation $N = 20000e^{0.003t}$.

(i) What is the number of bacteria initially? **1**

(ii) Determine the number of bacteria after 20 seconds. **2**

(iii) After what period of time will the number of bacteria have doubled? **2**

(iv) At what rate is the number of bacteria increasing when t = 20 seconds? **2**

Question 8 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) Sketch the graph of $y = \cos x$, for $0 \le x \le 2\pi$. **1**

(ii) Solve the trigonometric equation $\cos x = \frac{1}{2}$, for $0 \le x \le 2\pi$. **2**

(iii) Hence, find the values of x for which $\frac{1}{2} > \cos x$. **2**

(b) At time t seconds, the position x cm of a point moving in the straight line $X'OX$ is given by $x = at^2 + bt$ cm, where a and b are constants.

The particle passes through the origin O with velocity 16 cm / s in the positive direction at time $t = 0$ seconds, and after 8 seconds, it is again at O.

(i) Find the velocity of the particle at any time, in terms of a and b. **1**

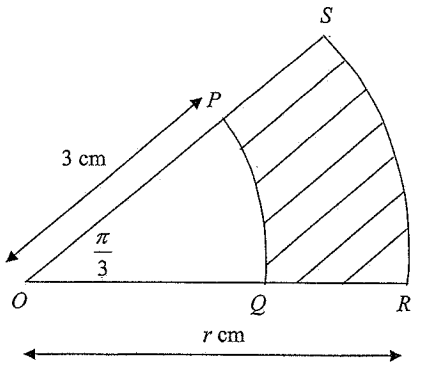
(ii) Find the values of the constants a and b. **3**

(iii) Find the time when the object is at rest. **1**

(iv) Find the position of the particle when it is at rest. **2**

Question 9 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a)



NOT TO SCALE

In the diagram above PQ and RS are arcs of concentric circles with centre O. $\angle POQ = \frac{\pi}{3}$ radians and $OP = 3$ cm.

(i) Find the area of the sector OPQ. **1**

(ii) If OR is r cm, find the area of the sector OSR in terms of r. **2**

(iii) If the shaded area is $\frac{27\pi}{6}$ cm^2, find the length of PS. **2**

(b) On 1 July 2005, Nadia invested \$12 000 in a bank account that paid interest at a rate of 6% p.a., compounded annually.

(i) How much would be in the account after the payment of interest on 1 July 2015 if no additional deposits were made? **2**

(ii) In fact Nadia added \$1 000 to her account on 1 July each year, beginning on 1 July 2006. After the payment of interest and her deposit on 1 July 2015, how much was in her account? **4**

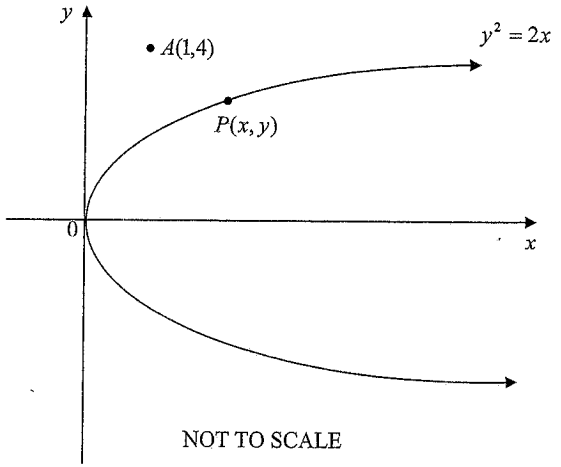
(iii) Nadia's friend Ana deposited \$12 000 in an account at another bank on 1 July 2005 and made no further deposit. On 1 July 2015, the balance of her account was \$35 639.36. What was the annual rate of compound interest paid on Ana's account? **2**

Question 10 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) Simplify $\log_e e^{2ax}$. **1**

(ii) Hence evaluate $\int_0^a \log_e e^{2ax}\,dx$. **2**

(b)



The diagram above shows the graph of the parabola $y^2 = 2x$. The point $A(1,4)$ is outside the parabola while the point $P(x,y)$ is on the parabola as shown in the above diagram.

(i) If D is the distance between the two points A and P, show that **3**
$D^2 = \left(\frac{1}{2}y^2 - 1\right)^2 + (y-4)^2$.

(ii) Show that the value of D in the equation in part (i) is a minimum when $y = 2$. **4**

(iii) Show that the minimum distance between A and P is $\sqrt{5}$ units. **2**

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CATHOLIC SECONDARY SCHOOLS ASSOCIATION

2005 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

MATHEMATICS – MARKING GUIDELINES/SOLUTIONS

These marking guidelines show the criteria to be applied to responses along with the marks to be awarded in line with the quality of responses. These guidelines are suggested and not prescriptive. This is not intended to be an exhaustive list but rather an indication of the considerations that students could include in their responses.

Question 1 (12 marks)

(a) (2 marks)

Outcomes Assessed: P3

Targeted Performance Bands: 2-3

Criteria	Marks
• Calculates correctly the absolute values	1
• Gives correct answer of -6	1

Sample answer

$|-6|-|-12| = 6-12 = -6$

(b) (2 marks)

Outcomes Assessed: P3, P4

Targeted Performance Bands: 2-3

Criteria	Marks
• Calculates correctly $\frac{1}{f}$	1
• Gives correct answer $f = -15$	1

Sample answer

$\frac{1}{f} = \frac{1}{-5} + \frac{1}{7.5} = -\frac{1}{15} \therefore f = -15$

(c) (2 marks)

Outcomes Assessed: P3

Targeted Performance Bands: 2 - 3

Criteria	Marks
• Calculates correctly ONE solution	1
• Calculates correctly the other solution	1

Sample answer

$(x-3)^2 = 9 \therefore x-3 = 3$ or $x-3 = -3 \therefore x = 6$ or $x = 0$

(d) (2 marks)
Outcomes Assessed: P7
Targeted Performance Bands: 2-3

Criteria	Marks
• Uses correctly the rules of differentiation	1
• Gives correct answer	1.

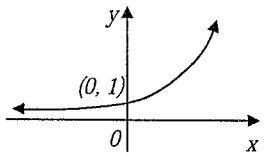
Sample answer

$\frac{d}{dx}\left(x^5+4x^{-2}\right)=5x^4+4\times(-2)x^{-3}$, i.e. $\frac{d}{dx}\left(x^5+4x^{-2}\right)=5x^4-\frac{8}{x^3}$

(e) (2 marks)
Outcomes Assessed: P5, H3
Targeted Performance Bands: 2-3

Criteria	Marks
• Sketches the correct graph	1
• Gives correct range: $y>0$	1

Sample answer



Range is: $y>0$

(f) (2 marks)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 2-3

Criteria	Marks
• Shows multiplication / division with conjugate	1
• Gives correct range answer	1

Sample answer

$\frac{1}{a}=\sqrt{10}-3 \therefore a=\frac{1}{\sqrt{10}-3}\times\frac{\sqrt{10}+3}{\sqrt{10}+3} \therefore a=\frac{\sqrt{10}+3}{10-9} \therefore a=\frac{\sqrt{10}+3}{10-9}$ (q.e.d.)

Question 2 (12 marks)
(a) (2 marks)
Outcomes Assessed: P4, P5
Targeted Performance Bands: 2-4

Criteria	Marks
• Calculates correctly $f(-x)$	1
• Shows correctly the required relationship $f(-x)=-f(x)$	1

Sample answer

$f(-x)=(-x)^5-(-x)^3$
$=-x^5+x^3$
$=-\left(x^5-x^3\right)$
$=-f(x) \therefore f(x)$ is an odd function (conform definition)

(b) (i) (1 mark)
Outcomes Assessed: P4
Targeted Performance Bands: 2-3

Criteria	Mark
• Correctly calculates the gradient of line BC	1

Sample answer

Gradient of BC is $m_{BC}=\frac{0+1}{2-0}=\frac{1}{2}$

(b) (ii) (2 marks)
Outcomes Assessed: P4
Targeted Performance Bands: 2-3

Criteria	Marks
• Realises that gradient of line AD is equal to $\frac{1}{2}$	1
• Substitutes the correct values to find the required equation	1

Sample answer

Since $AD // BC \therefore m_{AD}=m_{BC}=\frac{1}{2}$

Equation of AD is: $y-3=\frac{1}{2}(x-0) \therefore 2y-6=x$

Equation of AD is: $x-2y+6=0$

(b) (iii) (2 marks)
Outcomes Assessed: P4
Targeted Performance Bands: 2-3

Criteria	Marks
• Correctly calculates the gradient of line CD	1
• Substitutes the correct values to find the required equation	1

Sample answer

$CD \perp BC \ \therefore \ m_{CD} = -\dfrac{1}{m_{BC}} \ \therefore \ m_{CD} = -\dfrac{1}{\frac{1}{2}} = -2$

$\therefore$ Equation of line CD is: $(y--1) = -2(x-0)$

$\therefore$ Equation of line CD is: $y+1 = -2x$ $\therefore$ Equation of line CD is: $2x+y+1=0$

(b) (iv) (2 marks)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 2-4

Criteria	Marks
• Finding the x co – ordinate	1
• Finding the y co – ordinate (through any method possible)	1

Sample answer

Solving simultaneously the equations $x-2y+6=0$ and $2x+y+1=0$

$\therefore$ point D has the coordinates

$\left(-\dfrac{8}{5}, \dfrac{11}{5}\right)$

(b) (v) (3 marks)
Outcomes Assessed: P4
Targeted Performance Bands: 2- 4

Criteria	Mark
• Finds the area of ONE of the triangle	1
• Find areas of BOTH triangles	1
• Correctly calculates the answer	1

Sample answer

Area of trapezium $ABCD$ = Area of ΔABC + Area of ΔACD

$$= \frac{1}{2}\times 4\times 2 + \frac{1}{2}\times 4\times \frac{8}{5}$$

$$= 7\frac{1}{5} \text{ sq. units}$$

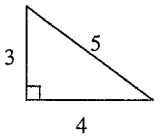
Question 3 (12 marks)
(a) (1 mark)
Outcomes Assessed: P3, P4, H5
Targeted Performance Bands: 2-3

Criteria	Mark
• Gives correct answer	1

Sample answer

Since $\tan\theta = \dfrac{3}{4}$

$\therefore \sin\theta = \dfrac{3}{5}$



(b) (i) (2 marks)
Outcomes Assessed: P7, H5
Targeted Performance Bands: 2-3

Criteria	Marks
• Correctly uses the product rule of differentiation but has ONE mistake in calculation	1
• Correctly works out the answer	1

Sample answer

$\dfrac{d}{dx}(\sin x \log_e x) = \cos x \log_e x + \dfrac{\sin x}{x}$

(b) (ii) (2 marks)
Outcomes Assessed: H5
Targeted Performance Bands: 2-4

Criteria	Mark
• Correctly uses the rule of differentiation but has ONE mistake in calculation	1
• Correctly works out the answer	1

Sample answer

$\dfrac{d}{dx}\left(3\tan\dfrac{\pi x}{3}\right) = 3\times\dfrac{\pi}{3}\sec^2\dfrac{\pi x}{3} = \pi\sec^2\dfrac{\pi x}{3}$

(c) (i) (2 marks)
Outcomes Assessed: H5
Targeted Performance Bands: 2-4

Criteria	Mark
• Gives as answer $-\cos(e-x)+c$	1
• Correctly works out the answer	1

Sample answer

$\int \sin(e-x)dx = \cos(e-x)+c$

(c) (ii) (2 marks)
Outcomes Assessed: H3, H5
Targeted Performance Bands: 3-4

Criteria	Marks
• Finds the primitive $\ln(x^2+1)$ but has an error in calculating the integral	1
• Correctly applies Newton – Leibnitz formula to obtain the answer in exact form	1

Sample answer

$$\int_0^1 \frac{2x}{x^2+1}dx = \left[\ln(x^2+1)\right]_0^1 = \ln 2 - \ln 1 = \ln 2$$

(d) (2 marks)
Outcomes Assessed: P6, H3, H5
Targeted Performance Bands: 2-4

Criteria	Marks
• Finds the gradient of the tangent	1
• Finds the gradient of the normal	1
• Correctly substitutes the values for x and y to find the equation of the normal	1

Sample answer

$y = e^{4x} - 1 \quad \therefore \frac{dy}{dx} = 4e^{4x}$ and when $x=0,\ y=0 \ \therefore \frac{dy}{dx} = 4$ (=gradient of tangent)

$\therefore m_{\perp} = -\frac{1}{4} \quad \therefore$ equation of the normal is given by : $y - 0 = -\frac{1}{4}(x-0)$

equation of the normal: $y = -\frac{1}{4}x$ (or $x+4y=0$, in general form)

Question 4 (12 marks)
(a) (2 marks)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 2-4

Criteria	Marks
• Correctly calculates $\alpha+\beta$ and $\alpha\beta$	1
• Correctly constructs the required equation	1

Sample answer

$\alpha+\beta = 2+\sqrt{3}+2-\sqrt{3} = 4$ and $\alpha\beta = (2+\sqrt{3})(2-\sqrt{3}) = 4-3 = 1$

$\therefore$ The quadratic equation is given by: $x^2 - 4x + 1 = 0$

(b) (i) (2 marks)
Outcomes Assessed: P3, H5
Targeted Performance Bands: 3-4

Criteria	Marks
• Correctly uses the general term formula to obtain a linear equation in d	1
• Correctly calculates the value of d	1

Sample answer

$a = 7$ and $T_{13} = 1 \ \therefore \ 7 + 12d = 1$ (Since $T_n = a + (n-1)d$)

$\therefore \ 12d = -6 \ \therefore \ d = -\frac{1}{2}$

(b) (ii) (2 marks)
Outcomes Assessed: P3, H5
Targeted Performance Bands: 2-4

Criteria	Marks
• Correctly uses the sum of n terms formula to obtain a linear equation in n	1
• Correctly calculates the value of n	1

Sample answer

Since $S_n = \frac{n}{2}[2a+(n-1)d] \ \therefore \ 0 = \frac{n}{2}\left[2\times 7 + (n-1)\times\left(-\frac{1}{2}\right)\right] \ \therefore \ n = 29$

(c) (i) (1 mark)
Outcomes Assessed: P4, H5
Targeted Performance Bands: 2-3

Criteria	Marks
• Correctly uses ONE of the methods to obtain the correct answer	1

Sample answer

By using Pythagoras or a trigonometric ratio in ΔQRS $\therefore$ $QS = 2\sqrt{2}$

(c) (ii) (2 marks)
Outcomes Assessed: P4, H5
Targeted Performance Bands: 2-3

Criteria	Marks
• Correctly uses tan ratio	1
• Correctly obtains the length of PS in exact form	1

Sample answer

In ΔQPR: $\tan 60^\circ = \dfrac{PR}{2}$ $\therefore$ $PR = 2\sqrt{3}$ $\therefore$ $PS = 2\sqrt{3} - 2$

(c) (iii) (3 marks)
Outcomes Assessed: P4, H5
Targeted Performance Bands: 3-5

Criteria	Marks
• Correctly uses the Sine Rule realising that $\angle QPS = 30^\circ$	1
• Correctly makes $\sin 15^\circ$ the subject	1
• Correctly finds the required value	1

Sample answer

In ΔQPS: $\dfrac{PS}{\sin 15^\circ} = \dfrac{QS}{\sin 30^\circ}$ $(\angle QPS = 90^\circ - 60^\circ = 30^\circ)$

$\therefore$ $\sin 15^\circ = \dfrac{PS \times \sin 30^\circ}{QS}$ $\therefore$ $\sin 15^\circ = \dfrac{\left(2\sqrt{3} - 2\right) \times \sin 30^\circ}{2\sqrt{2}}$

$\therefore$ $\sin 15^\circ = \dfrac{2\left(\sqrt{3} - 1\right) \times \dfrac{1}{2}}{2\sqrt{2}}$ $\therefore$ $\sin 15^\circ = \dfrac{\sqrt{3} - 1}{2\sqrt{2}}$ (q.e.d.)

Question 5 (12 marks)
(a) (i) (4 marks)
Outcomes Assessed: P7, H6,
Targeted Performance Bands: 3-5

Criteria	Marks
• Finds the first derivative	1
• Finds the stationary points	1
• Finds the nature of ONE stationary point	1
• Finds the nature of the other stationary point	1

Sample answer

$y = x^3 - 6x^2 + 9x + 4$ $\therefore$ $\dfrac{dy}{dx} = 3x^2 - 12x + 9$ and at stationary points, $\dfrac{dy}{dx} = 0$

$\therefore$ $3x^2 - 12x + 9 = 0$, i.e. $x^2 - 4x + 3 = 0$ $\therefore$ $(x-1)(x-3) = 0$ $\therefore$ $x = 1$ $x = 3$

$\therefore$ the stationary points are $(1,8)$ and $(3,4)$

For the nature of stationary points: $\dfrac{d^2y}{dx^2} = 6x - 12$

at $x = 1$ $\therefore$ $\dfrac{d^2y}{dx^2} = -6 < 0$ $\therefore$ $(1,8)$ is a maximum turning point

at $x = 3$ $\therefore$ $\dfrac{d^2y}{dx^2} = 6 > 0$ $\therefore$ $(3,4)$ is a minimum turning point

(a) (ii) (2 marks)
Outcomes Assessed: P7, H6
Targeted Performance Bands: 2-4

Criteria	Mark
• Correctly solves the equation $\dfrac{d^2y}{dx^2}$	1
• Analyse the sign of the second derivative and gives correct answer	1

Sample answer

$\dfrac{d^2y}{dx^2} = 6x - 12$ $\therefore$ $6x - 12 = 0$ $\therefore$ $x = 2$ (and $y = 6$)

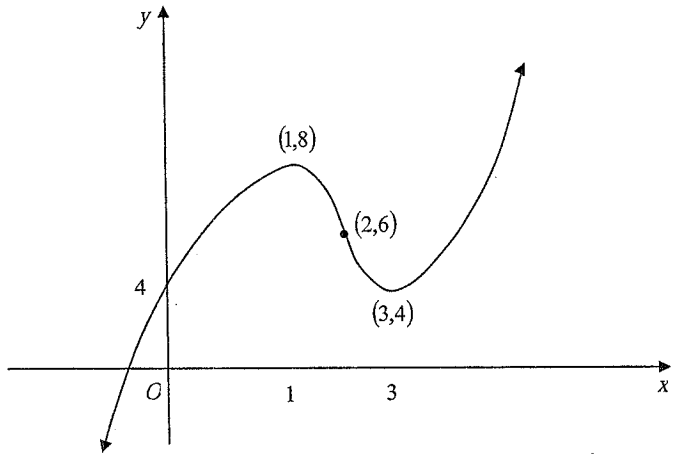
x	< 2	2	> 2
$\dfrac{d^2y}{dx^2} = 6x - 12$	-	0	+

$\therefore$ change in sign of the second derivative $\therefore$ $(2,6)$ is a point of inflexion

(a) (iii) (2 marks)
Outcomes Assessed: P6, H6, H7, H9
Targeted Performance Bands: 3-5

Criteria	Marks
• Draws the correct curve	1
• Plots all important points	1

Sample answer



(a) (iv) (1 marks)
Outcomes Assessed: P6, H7
Targeted Performance Bands: 3-4

Criteria	Marks
• Gives correct answer	1

Sample answer

$\frac{dy}{dx} < 0$ is where the graph is decreasing $\therefore\ 1 < x < 3$

(b) (i) (1 marks)
Outcomes Assessed: P3, H3
Targeted Performance Bands: 2-3

Criteria	Mark
• Gives correct answer	1

Sample answer

x	1	1.5	2	2.5	3
$y = 3^{x-1}$	1	1.732	3.000	5.196	9.000

(b) (ii) (2 marks)
Outcomes Assessed: P3, H5
Targeted Performance Bands: 2-4

Criteria	Marks
• Correctly applies Simpson's Rule	1
• Gives correct answer	1

Sample answer

$$A = \frac{\frac{1}{2}}{3}\left[(1+9)+4(1.732+5.196)+2(3)\right] = 7.285$$

Question 6 (12 marks)

(a) (i) (1 mark)

Outcomes Assessed: P3

Targeted Performance Bands: 2-3

Criteria	Mark
• Factorises correctly	1

Sample answer

$2a^2-7a+3=(2a-1)(a-3)$

(a) (ii) (3 marks)

Outcomes Assessed: P3, H3,

Targeted Performance Bands: 3-5

Criteria	Marks
• Realises to let $\log_2 x = a$ and solves the equation for a	1
• Finds one solution for x	1
• Finds the other solution for x	1

Sample answer

let $\log_2 x = a$ $\therefore 2a^2-7a+3=0$ $\therefore (2a-1)(a-3)=0$ $\therefore a=\frac{1}{2}$ or $a=3$ $\quad\therefore \log_2 x=\frac{1}{2}$ $\therefore$

$x=2^{\frac{1}{2}}$ $\therefore$ $x=\sqrt{2}$ or $\log_2 x=3$ $\quad\therefore x=2^3$ $\therefore$ $x=8$

(b) (i) (2 marks)

Outcomes Assessed: P2, H2

Targeted Performance Bands: 2-4

Criteria	Marks
• Realises that $\angle MBA=\angle APD(=90^\circ)$	1
• Gives the second reason	1

Sample answer

In Δ's ABM and APD: $\angle MBA=\angle APD(=90^\circ)$

$\angle BMA=\angle PAD$ (alternate angles; BC // AD with AM transverse)

$\therefore$ $\Delta ABM \,|||\, \Delta APD$

(b) (ii) (2 marks)

Outcomes Assessed: P4, H5,

Targeted Performance Bands: 2-4

Criteria	Marks
• Writes the sides ratios	1
• Finds the answer	1

Sample answer

$\frac{AB}{PD}=\frac{AM}{AD}=\frac{BM}{AP}$ (Sides of similar triangles are proportion)

$\therefore$ $\frac{40}{PD}=\frac{50}{60}$ (Since $AM=50$ cm using Pythagoras in right angled ΔABM)

$\therefore$ $PD=48$ cm

(b) (iii) (1 mark)

Outcomes Assessed: P4, H5,

Targeted Performance Bands: 2-3

Criteria	Mark
• Finds AP	1

Sample answer

In the right angled ΔAPD: $AP^2=60^2-48^2$ $\therefore$ $AP=36$ cm

(b) (iv) (3 marks)

Outcomes Assessed: P4, H5,

Targeted Performance Bands: 2-4

Criteria	Marks
• Finds Area of ΔABM	1
• Finds Area of ΔAPD	1
• Finds Area of the quadrilateral $PMCD$	1

Sample answer

Area of $\Delta ABM=\frac{1}{2}\times 30\times 40=600cm^2$ (M is a midpoint of BC)

Area of $\Delta APD=\frac{1}{2}\times 36\times 48=864cm^2$

$\therefore$ Area of the quadrilateral $PMCD=(40\times 60)-(600+864)=936$ cm^2.

Question 7 (12 marks)

(a) (i) (2 marks)

Outcomes Assessed: H1, H5

Targeted Performance Bands: 3-5

Criteria	Marks
• Finds at least ONE partial probability	1
• Finds the correct answer	1

Sample answer

$P(N)=\frac{7}{10}\qquad P(M)=\frac{3}{10}$

$P(\text{2 sets only})=P(N,N)+P(M,M)$

$=\frac{7}{10}\times\frac{7}{10}+\frac{3}{10}\times\frac{3}{10}=\frac{29}{50}$

(a) (ii) (2 mark)

Outcomes Assessed: H1, H5

Targeted Performance Bands: 3-5

Criteria	Marks
• Finds at least TWO partial probability	1
• Finds the correct answer	1

Sample answer

$P(\text{Nicole wins the match})=P(N,N)+P(N,M,N)+P(M,N,N)$

$=\frac{7}{10}\times\frac{7}{10}+\frac{7}{10}\times\frac{3}{10}\times\frac{7}{10}+\frac{3}{10}\times\frac{7}{10}\times\frac{7}{10}$

$=\frac{784}{1000}=\frac{98}{125}$

(a) (iii) (1 mark)

Outcomes Assessed: H1, H5

Targeted Performance Bands: 2-3

Criteria	Mark
• Uses the complementary events formula to find the answer (or any other method)	1

Sample answer

$P(\text{Mariana wins the match})=1-P(\text{Nicole wins the match})$

$=1-\frac{98}{125}$

$=\frac{27}{125}$

(b) (i) (1 mark)

Outcomes Assessed: H1, H3, H5, H9

Targeted Performance Bands: 2-3

Criteria	Mark
• Finds the correct answer	1

Sample answer

at $t=0\ \therefore\ N=20000$

(b) (ii) (2 marks)

Outcomes Assessed: H1, H3, H5, H9

Targeted Performance Bands: 2-4

Criteria	Marks
• Correctly substitutes $t=20$	1
• Finds the correct answer	1

Sample answer

at $t=20\ \therefore N=20000e^{0.003\times20}=21237$

(b) (iii) (2 marks)

Outcomes Assessed: H1, H3, H5, H9

Targeted Performance Bands: 3-5

Criteria	Marks
• Correctly works out t as subject in the formula	1
• Finds the correct answer	1

Sample answer

When $N=40000\ \therefore 40000=20000e^{0.003t}$

$\therefore 2=e^{0.003t}\ \therefore \ln 2=0.003t\ \therefore t=\frac{\ln 2}{0.003}=231$ seconds

(b) (iv) (2 mark)

Outcomes Assessed: H1, H3, H5, H9

Targeted Performance Bands: 3-5

Criteria	Mark
• Correctly works out $\frac{dN}{dt}$	1
• Finds the correct answer	1

Sample answer

$\frac{dN}{dt}=60e^{0.003t}$ and when $t=20\ \therefore\ \frac{dN}{dt}=63.7$ bacteria / second

Question 8 (12 marks)

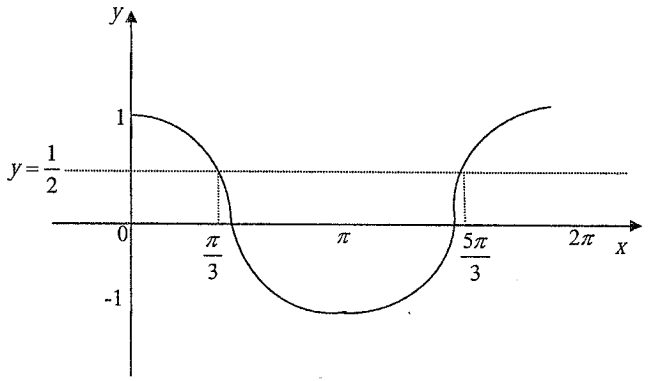
(a) (i) (1 mark)

Outcomes Assessed: H1, H5, H9

Targeted Performance Bands: 3-4

Criteria	Mark
• Correctly sketches the graph including important points	1

Sample answer



(a) (ii) (2 marks)

Outcomes Assessed: H1, H5

Targeted Performance Bands: 3-4

Criteria	Marks
• Finds ONE solution (in the first quadrant)	1
• Finds the second solution	1

Sample answer

$\cos x = \frac{1}{2} \therefore x = \frac{\pi}{3}, \frac{5\pi}{3}$

(a) (iii) (2 marks)

Outcomes Assessed: H1, H5, H9

Targeted Performance Bands: 3-5

Criteria	Mark
• Finds ONE solution	1
• Finds the other solution	1

Sample answer

$\frac{\pi}{3} < x < \frac{5\pi}{3}$ (see diagram)

(b) (i) (1 mark)

Outcomes Assessed: P7

Targeted Performance Bands: 2-3

Criteria	Mark
• Finds the correct answer	1

Sample answer

$v = \frac{dx}{dt} = 2at + b$

(b) (ii) (3 marks)

Outcomes Assessed: H1, H4, H5

Targeted Performance Bands: 3-4

Criteria	Marks
• Correctly constructs an equation to find a	1
• Correctly finds a	1
• Correctly finds b	1

Sample answer

At $t = 8$, $x = 0 \therefore 0 = 64a + 8 \therefore a = -2$

At $t = 0$, $v = 16 \therefore b = 16$

(b) (iii) (1 mark)

Outcomes Assessed: H1, H4, H5

Targeted Performance Bands: 3-4

Criteria	Mark
• Correctly finds t	1

Sample answer

When $v = 0 \therefore -4t + 16 = 0$ (since $v = -4t + 16$)

$\therefore t = 4$ seconds

(b) (iv) (2 marks)

Outcomes Assessed: H1, H4, H5, H9

Targeted Performance Bands: 3-4

Criteria	Marks
• Substitutes correct value of t in the equation of x	1
• Correctly finds displacement	1

Sample answer

Since $x = -2t^2 + 16t$ and when $v = 0$, $t = 4$

$\therefore x = -2 \times 4^2 + 16 \times 4 = 48$ cm from origin

Question 9 (12 marks)

(a) (i) (1 mark)

Outcomes Assessed: H5

Targeted Performance Bands: 2-4

Criteria	Mark
• Finds the area of the sector OPQ	1

Sample answer

Area of the sector $OPQ = \frac{1}{2}r^2\theta = \frac{1}{2}\times 3^2 \times \frac{\pi}{3} = \frac{3\pi}{2}\ cm^2$

(a) (ii) (1 mark)

Outcomes Assessed: H5

Targeted Performance Bands: 2-4

Criteria	Marks
• Finds ONE solution (in the first quadrant)	1
• Finds the second solution	1

Sample answer

Area of the sector $OSR = \frac{1}{2}\times r^2 \times \frac{\pi}{3} = \frac{\pi}{6}r^2\ \ cm^2$

(a) (iii) (2 marks)

Outcomes Assessed: H5

Targeted Performance Bands: 3-4

Criteria	Mark
• Correctly finds r	1
• Finds the answer	1

Sample answer

Shaded area = Area of sector OSR – Area of sector OPQ

$\therefore \frac{27\pi}{6} = \frac{\pi}{6}r^2 - \frac{3\pi}{2}\ \therefore r^2 = 36\ \therefore r = 6$ cm

$\therefore PS = 6 - 3 = 3$ cm

(b) (i) (2 marks)

Outcomes Assessed: H1, H4, H5, H9

Targeted Performance Bands: 2-4

Criteria	Marks
• Correctly substitutes in the compound interest formula	1
• Finds the correct answer	1

Sample answer

$A_{10} = 12000\left(1+\frac{6}{100}\right)^{10} = 12000(1.06)^{10} = \$21490.17$

(b) (ii) (4 marks)

Outcomes Assessed: H1, H4, H5, H9

Targeted Performance Bands: 4-6

Criteria	Marks
• Correctly finds $A_1, A_2, \ldots, A_9$	1
• Correctly constructs the geometric series	1
• Correctly finds the sum of the geometric series with 9 terms	1
• Finds the correct answer	1

Sample answer

$A_1 = 1000(1.06)^9,\ A_2 = 1000(1.06)^8, \ldots,\ A_9 = 1000(1.06)^1$

$\therefore A = 1000(1.06)^1 + 1000(1.06)^2 + \ldots + 1000(1.06)^9$

$\therefore A = \frac{1000(1.06)\left[1.06^9 - 1\right]}{1.06 - 1} = \$12180.79$

Total Amount = \$ 12180.79 + \$ 21490.17 = \$ 33670.96

(b) (iii) (2 marks)

Outcomes Assessed: H1, H4, H5, H9

Targeted Performance Bands: 3-5

Criteria	Marks
• Correctly uses the compound interest formula	1
• Make r the subject and provides the answer	1

Sample answer

$\$35639.36 = \$12000\left(1+\frac{r}{100}\right)^{10}\ \therefore 2.969946 = \left(1+\frac{r}{100}\right)^{10}$

$\therefore \left(1+\frac{r}{100}\right) = (2.969946)^{\frac{1}{10}} = 1.1149$

$\therefore \frac{r}{100} = 0.1149\ \therefore r = 11.5\%$

Question 10 (12 marks)
(a) (i) (1mark)
Outcomes Assessed: H5
Targeted Performance Bands: 3-4

Criteria	Mark
• Correctly finds answer	1

Sample answer

$\log_e e^{2ax} = 2ax\log_e e = 2ax \times 1 = 2ax$

(a) (ii) (2 marks)
Outcomes Assessed: H5
Targeted Performance Bands: 3-4

Criteria	Marks
• Correctly finds the primitive	1
• Correctly uses Newton – Leibnitz Formula to find the answer	1

Sample answer

$\int_0^a \log_e e^{2ax} = \int 2ax.dx = \left[ax^2\right]_0^a = a^3 - 0 = a^3$

(b) (i) (3 marks)
Outcomes Assessed: P1, P2, H5
Targeted Performance Bands: 3-4

Criteria	Marks
• Correctly uses the distance formula	1
• Realises that $x = \frac{y^2}{2}$	1
• Correctly substitutes $x = \frac{y^2}{2}$ to find answer	1

Sample answer

$D^2 = (x-1)^2 + (y-4)^2$, and since $y^2 = 2x \therefore x = \frac{y^2}{2}$

$\therefore D^2 = \left(\frac{y^2}{2} - 1\right)^2 + (y-4)^2$

(b) (ii) (4 marks)
Outcomes Assessed: P1, P2, H5
Targeted Performance Bands: 3-4

Criteria	Marks
• Finds the first derivative	1
• Correctly solves the equation $\frac{d}{dy}(D^2) = 0$	1
• Finds the second derivative	1
• Substitutes $y = 2$ in the second derivative to show point is a minimum	1

Sample answer

$D^2 = \left(\frac{y^2}{2} - 1\right)^2 + (y-4)^2 \therefore \frac{d}{dx}(D^2) = 2\left(\frac{1}{2}y^2 - 1\right) \times \frac{1}{2} \times 2y + 2(y-4)$ $\therefore$

For minimum distance $\frac{d}{dy}(D^2) = 0$

$\therefore y^3 - 2y + 2y - 8 = 0 \therefore y^3 - 8 = 0 \therefore y = 2$

Second derivative is $\frac{d^2}{dx}(D^2) = 3y^2$, and for $y = 2 \therefore \frac{d^2}{dx}(D^2) = 3y^2 = 12 > 0$

$\therefore$ minimum distance is when $y = 2$

(b) (iii) (2 marks)
Outcomes Assessed: P1, P2, H5
Targeted Performance Bands: 2-4

Criteria	Marks
• Substitutes $y = 2$ into the distance formula	1
• Finds correct answer with argument	1

Sample answer

Substitute $y = 2$ into the distance formula

$\therefore D^2 = (2-1)^2 + (2-4)^2 \therefore D^2 = 1 + 4 \therefore D = \pm\sqrt{5}$

$\therefore D = \sqrt{5}$ (as distance can't be negative)