Centre Number

Student Number

CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NEW SOUTH WALES

2006
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

Morning Session
Monday 7 August 2006

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided separately
- All necessary working should be shown in every question
- Write your Centre Number and Student Number at the top of this page

Total marks – 120

- Attempt Questions 1-10
- All questions are of equal value

2602-1

Total marks – 120
Attempt Questions 1-10
All questions are of equal value.

Answer each question in a SEPARATE writing booklet.

Question 1 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Evaluate $\sqrt[3]{\dfrac{|-38.67\times 7.2|}{(11.7)^2-(1.83)^2}}$ correct to 3 decimal places. **2**

(b) By rationalising the denominator, simplify $\dfrac{1-\sqrt{2}}{2-\sqrt{8}}$. **2**

(c) Solve $\cos\theta = -\dfrac{1}{2}$, for $0 \le \theta \le 2\pi$. **2**

(d) Completely factorise $4xy + xb + 8ay + 2ab$. **2**

(e) For what values of k does the quadratic equation $4x^2 + kx + 9 = 0$ have equal roots? **2**

(f) Solve the equation $|x-2| = 3$. **2**

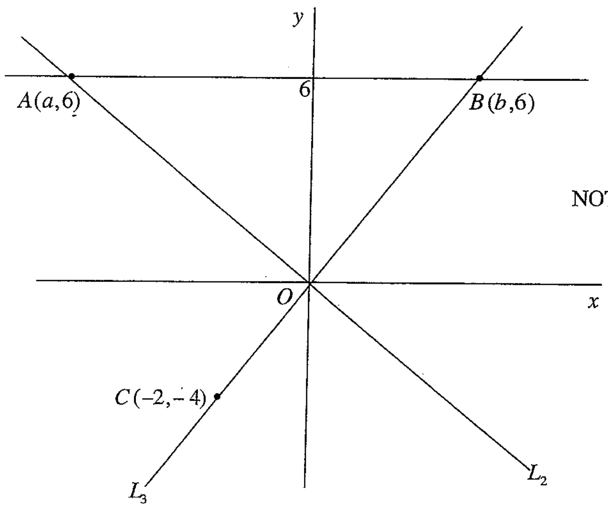
Question 2 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Sketch the region represented by $x^2 + y^2 < 4$. **2**

(b) The function $f(x)$ is given by $f(x) = \begin{cases} x+1 & \text{for } x \le 3 \\ x^2 - 9 & \text{for } x > 3 \end{cases}$ **2**

Find $f(3)$ - $f(6)$.

(c)



On the diagram above line L_1 is parallel with the x-axis and crosses the y-axis at 6. Lines L_2 and L_3 each pass through the origin, O, and intersect with the line L_1 at the points $A(a,6)$ and $B(b,6)$ respectively. The point $C(-2,-4)$ lies on the line L_3.

(i) Show that the equation of the line L_3 is $2x - y = 0$. **2**

(ii) Show that the x-ordinate of point B is 3. **1**

(iii) Find the coordinates of point A such that $\angle AOB$ is a right angle. **3**

(iv) Write down the distance between points A and B. Hence, or other calculate the area of ΔAOB. **2**

Question 3 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) For the parabola $(x-2)^2 = 16y$, find the coordinates of the:

(i) Vertex. **1**

(ii) Focus. **1**

(b) Differentiate with respect to x the following expressions:

(i) $3x \log_e x$. **2**

(ii) $\sin^2 x$. **2**

(c) Find:

(i) $\int \cos 2006x \, dx$. **2**

(ii) $\int_0^1 e^{2x} \, dx$. (Leave your answer in exact form). **2**

(d) Show that the equation of the normal to the curve $y = x^3 - 5x$ at the point $(1,-4)$ is given by $x - 2y - 9 = 0$. **2**

Question 4 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Consider the curve $f(x) = -\frac{1}{3}x^3 - x^2 + 3x + 1$.

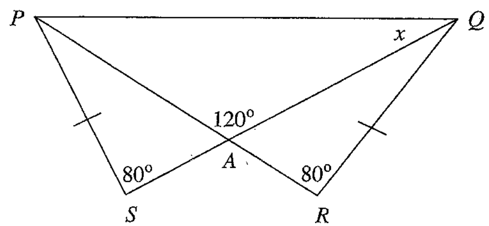
(i) Find the coordinates of any stationary points and determine their nature. **3**

(ii) Find any point(s) of inflexion. **2**

(iii) Sketch the curve in the domain, $-6 \le x \le 3$. **2**

(iv) What is the maximum value of $f(x)$ in the given domain? **1**

(b)



NOT TO SCALE

PR and QS are straight lines intersecting at point A. Also $PS = QR$, $\angle PSA = \angle QRA = 80°$, $\angle PAQ = 120°$ and $\angle PQA = x$.

(i) Copy the diagram into your writing booklet.

(ii) Prove that ΔPSA is congruent to ΔQRA. **2**

(iii) Hence, show that $x = 30°$. **2**

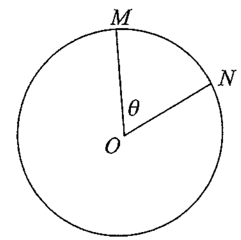
Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) Write $0.7\dot{5}$ as the sum of an infinite geometric series. **1**

(ii) Hence, express $0.7\dot{5}$ as a fraction. **2**

(b) Simplify $\frac{1-\sin^2 x}{\cot x}$. **2**

(c) The circle below has centre O, radius $\frac{6}{\pi}$ cm and arc length $MN = 1$ cm.



NOT TO SCALE

(i) Find the size of θ in radians. **1**

(ii) Hence, or otherwise, find the exact area of the sector MON. **2**

(d) During qualification for the 2006 World Cup, the Socceroos goalkeeper, Mark, defended many penalty shots at goal. In fact, the probability that he can stop a penalty shot at goal is $\frac{3}{5}$. During a particular match, the opposing team had three penalty shots at goal.

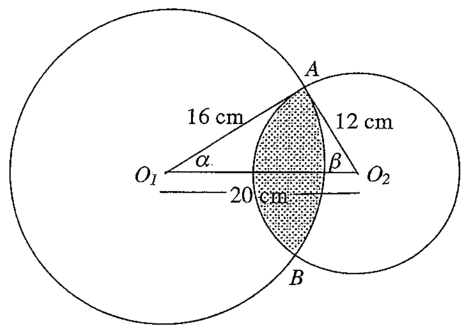
Using a tree diagram, find the probability that:

(i) the goalkeeper will stop all shots at goal. **2**

(ii) the goalkeeper will stop at least 1 shot at goal. **2**

Question 6 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a)



A circle with centre at O_1 and radius 16 cm intersects with another circle with centre at O_2 and radius 12 cm. Their points of intersection are A and B and the distance between their centres, O_1O_2, is 20 cm. $\angle AO_1O_2 = \alpha$ and $\angle AO_2O_1 = \beta$.

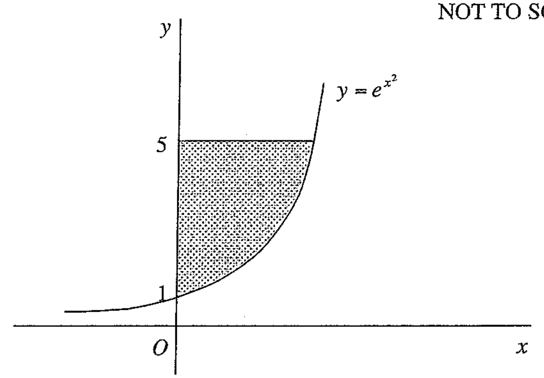
(i) Show that ΔO_1AO_2 is a right angled triangle. **1**

(ii) Find the size of the angles α and β. **2**

(iii) Find the shaded area enclosed by these circles. (Give your answer correct to 2 decimal places). **3**

Question 6 continues on page 8

(b)



The shaded region bounded by the graph $y = e^{x^2}$, the line $y = 5$ and the y-axis is rotated about the y-axis to form a solid of revolution.

(i) Show that the volume of the solid is given by $V_y = \pi\int_1^5 \log_e y\,dy$. **2**

(ii) Copy and complete the following table into your writing booklet. Give all answers correct to three decimal places. **1**

y	1	2	3	4	5
$\log_e y$	0	0.693	1.099		1.609

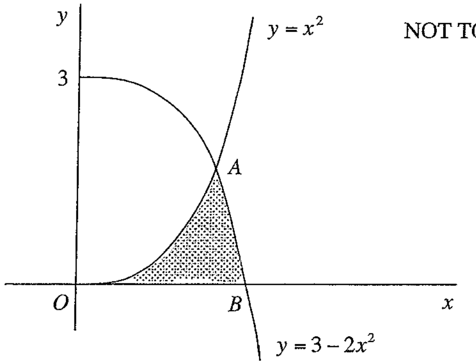
(iii) Use Simpson's Rule with five function values to approximate the volume of the solid of revolution V_y, correct to three decimal places. **3**

End of Question 6

Question 7 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Solve the following equation: $\log_2 x + \log_2(x+7) = 3$, for $x > 0$. **3**

(b)



NOT TO SCALE

The shaded region OAB is bounded by the parabolas $y = x^2$, $y = 3 - 2x^2$ and the x-axis. Point A is the intersection of the two parabolas and point B is the x-intercept of the parabola $y = 3 - 2x^2$.

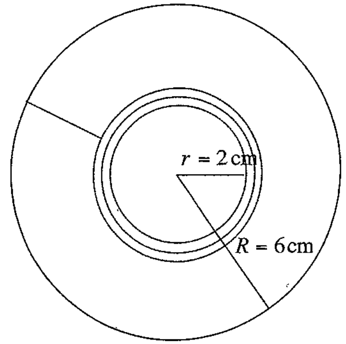
(i) Find the x-ordinates of points A and B. **2**

(ii) By considering the sum of two areas, show that the exact area of the shaded region OAB is given by $2\sqrt{\frac{3}{2}} - 2$ square units. **3**

Question 7 continues on page 10

Question 7 (continued) **Marks**

(c)



NOT TO SCALE

The playing track of a CD is made out of a number of concentric circles with the inner circle having a radius of 2 cm and the outer circle having a radius of 6 cm. The CD is rotating at 5 revolutions per second and takes 25 minutes to completely play.

(i) Find the total number of revolutions for this CD. **1**

(ii) Find the total length of the playing track in km, correct to one decimal place. **3**

End of Question 7

Question 8 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) If $A^m = 3$, find the value of $A^{4m} - 5$. **2**

(b) A 100 mg tablet is dissolved in a glass of water. After t minutes the amount of *undissolved* tablet, U in mg, is given by the formula:

$$U = 100\,e^{-kt}\text{, where } k \text{ is a constant.}$$

(i) Calculate the value of k, correct to 4 decimal places, given that 2 mg of the tablet remain after 10 minutes. **2**

(ii) Find the rate at which the tablet is dissolving in the glass of water after 12 minutes. Give your answer correct to two decimal places. **2**

(c) Mr. Egan borrows $\$P$ from a bank to fund his house extensions. The term of the loan is 20 years with an annual interest rate of 9%. Each month, interest is calculated on the balance at the beginning of the month and added to the balance owing. Mr. Egan repays the loan in equal monthly instalments of $1 050.

(i) Write an expression for the amount, A_1, Mr. Egan owes immediately at the end of the first month. **1**

(ii) Show that at the end of n months the amount owing, A_n, is given by: **3**

$$A_n = P(1.0075)^n - 140000(1.0075)^n + 140000$$

(iii) If at the end of 20 years the loan has been repaid, calculate the amount Mr. Egan originally borrowed, correct to the nearest dollar. **2**

Question 9 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Two particles, A and B, move along a straight line so that their displacements, x_A and x_B, in metres, from the origin at time t seconds are given by the following equations respectively:

$$x_A = 12t + 5 \qquad x_B = 6t^2 - t^3$$

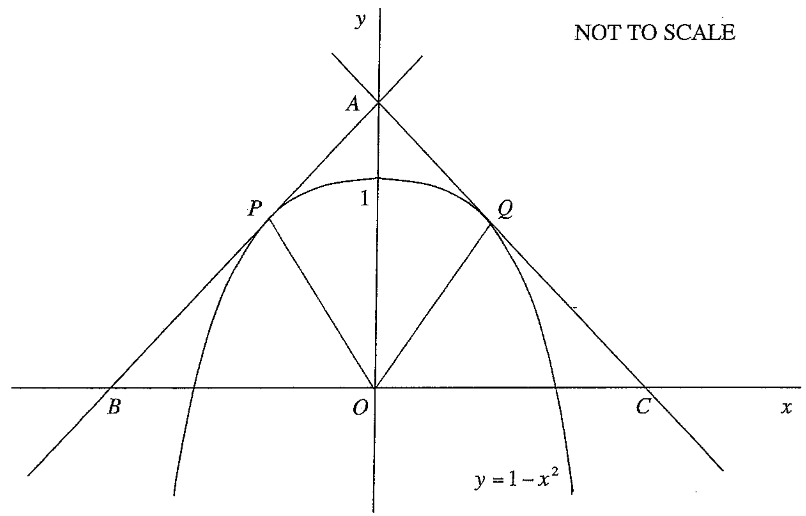
(i) Find two expressions for the velocities of particles A and B. **2**

(ii) Which of the two particles is travelling faster at $t = 1$ second? **1**

(iii) At what time does particle B come to rest? **1**

(iv) Find the maximum positive displacement of particle B. **2**

(b)



In the diagram above P and Q are two points on the parabola $y = 1 - x^2$. The tangents to the parabola at P and Q intersect each other on the y-axis at A and the x-axis at B and C respectively. Triangle ABC is an equilateral triangle.

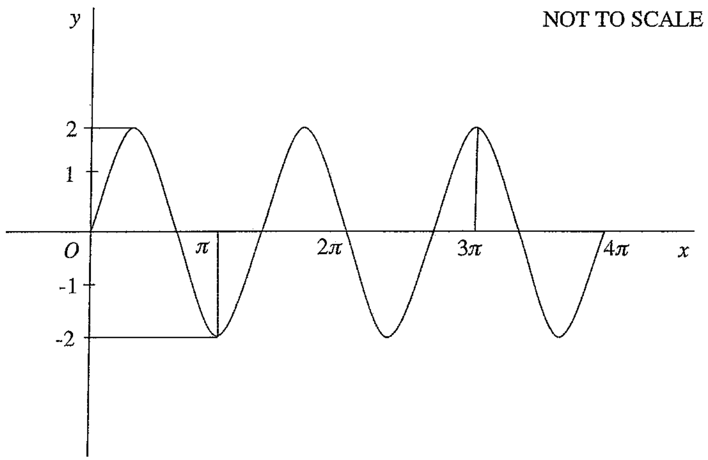
(i) Show that the gradient of the tangent at point P is equal to $\sqrt{3}$. **1**

(ii) Show that the coordinates of point P are $\left(-\frac{\sqrt{3}}{2}, \frac{1}{4}\right)$. **2**

(iii) Find the value of $\angle POQ$, in radians, correct to one decimal place. **3**

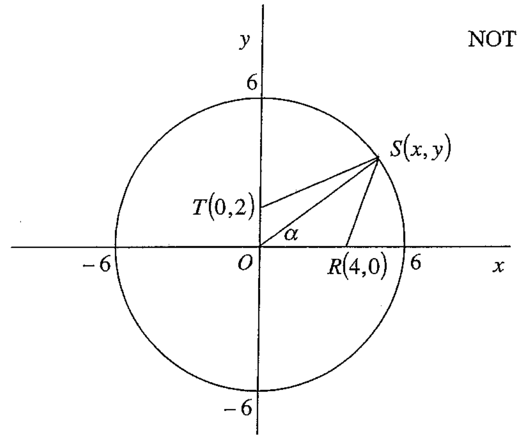
Question 10 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the trigonometric equation for the graph below: **2**



Question 10 continues on page 14

(b)



The diagram above shows the circle $x^2 + y^2 = 36$. The point $S(x, y)$ lies on the circle in the first quadrant. O is the origin, $R(4,0)$ lies on the x-axis and $T(0,2)$ lies on the y-axis. The size of $\angle ROS$ is α radians, where $0 < \alpha < \frac{\pi}{2}$.

(i) Show that the area of triangle SOR is $12\sin\alpha$. **1**

(ii) Hence show that the area, A, of the quadrilateral $ORST$ is given by: **3**

$$A = 6\cos\alpha(2\tan\alpha + 1)$$

(iii) Find the value of $\tan\alpha$ for which the area A is a maximum. **3**

(iv) Hence, show that for this maximum area, the coordinates of point S are $\left(\frac{6}{5}\sqrt{5}, \frac{12}{5}\sqrt{5}\right)$ **3**

End of Paper

CATHOLIC SECONDARY SCHOOLS ASSOCIATION

2006 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

MATHEMATICS – SUGGESTED SOLUTIONS

These marking guidelines show the criteria to be applied to responses along with the marks to be awarded in line with the quality of responses. These guidelines are suggested and not prescriptive. This is not intended to be an exhaustive list but rather an indication of the considerations that students could include in their responses.

Question 1 (12 marks)
(a) (2 marks)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 2-3

Criteria	Mark
• Gives correct answer.	1
• Correctly rounds to THREE decimal places.	1

Sample Answer

$$\sqrt[3]{\frac{|-38.67\times 7.2|}{(11.7)^2-(1.83)^2}}=1.277508818$$

$$=1.278 \quad \text{(3 decimal places)}$$

(b) (2 marks)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 2-3

Criteria	Mark
• Correctly write the conjugate and expands to get the denominator OR the numerator correct	1
• Gives the correct answer	1

Sample Answer

$$\frac{1-\sqrt{2}}{2-\sqrt{8}}=\frac{1-\sqrt{2}}{2-\sqrt{8}}\times\frac{2+\sqrt{8}}{2+\sqrt{8}}$$

$$=\frac{2-2\sqrt{2}+\sqrt{8}-\sqrt{16}}{4-8}$$

$$=\frac{-2}{-4}$$

$$=\frac{1}{2}$$

2602-2

(c) (2 marks)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 2-3

Criteria	Mark
• Gives ONE correct answer in radians OR TWO correct answers in degrees.	1
• Gives TWO correct answers in radians.	1

Sample Answer

$$\cos\theta=-\frac{1}{2}$$

Basic angle is $\frac{\pi}{3}$ (First Quadrant).

$$\therefore\ \theta=\frac{2\pi}{3},\frac{4\pi}{3} \qquad 0\le\theta\le 2\pi$$

(d) (2 marks)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 2-3

Criteria	Mark
• Factorises in pairs, e.g. $x(4y+b)+2a(4y+b)$.	1
• Completes the factorisation into TWO brackets.	1

Sample Answer

$$4xy+xb+8ay+2ab=x(4y+b)+2a(4y+b)$$
$$=(x+2a)(4y+b)$$

(e) (2 marks)
Outcomes Assessed: P2, P4
Targeted Performance Bands: 2-3

Criteria	Mark
• Correctly writes down the discriminant and equates to zero.	1
• Solves the equation to give correct values of k.	1

Sample Answer

$4x^2+kx+9=0$ equal roots when $\Delta=0$
$\Delta=k^2-144$
$0=k^2-144$
$k=\pm 12$

DISCLAIMER
The information contained in this document is intended for the professional assistance of teaching staff. It does not constitute advice to students. Further it is not the intention of CSSA to provide specific marking outcomes for all possible Trial HSC answers. Rather the purpose is to provide teachers with information so that they can better explore, understand and apply HSC marking requirements, as established by the NSW Board of Studies.
No guarantee or warranty is made or implied with respect to the application or use of CSSA Marking Guidelines in relation to any specific trial exam question or answer. The CSSA assumes no liability or responsibility for the accuracy, completeness or usefulness of any Marking Guidelines provided for the Trial HSC papers.

(f) (2 marks)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 2-3

Criteria	Mark
• Gives ONE correct answer.	1
• Gives the second correct answer.	1

Sample Answer

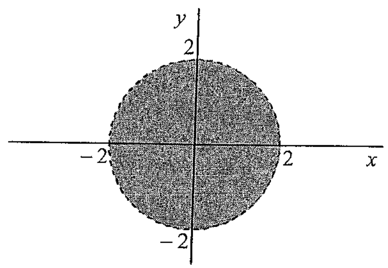
$|x-2|=3$
$x-2=3$ or $-(x-2)=3$

$\therefore x=5$ or $x=-1$

Question 2 (12 marks)
(a) (2 marks)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 2-3

Criteria	Mark
• Correctly draws the circle using a dotted line.	1
• Correctly shades the inside of the circle.	1

Sample Answer



Question 2
(b) (2 marks)
Outcomes Assessed: P3, P4, P5
Targeted Performance Bands: 2-4

Criteria	Mark
• Gives correct answer ONE partial answer (e.g. $f(3)=4$ or $f(6)=27$)	1
• Gives correct answer	1

Sample Answer

$$f(x)=\begin{cases} x+1, & x\le 3 \\ x^2-9, & x>3 \end{cases}$$

$f(3)=(3)+1=4$, $f(6)=6^2-9=27$

$\therefore f(3)-f(6)=4-27=-23$

(c) (i) (2 marks)
Outcomes Assessed: P4, P5
Targeted Performance Bands: 2-3

Criteria	Mark
• Correctly finds the gradient of line L_3.	1
• Correctly finds the equation of line L_3.	1

Sample Answer

$$m_{L_3}=\frac{0+4}{0+2}=\frac{4}{2}=2$$

$y-0=2(x-0)$

$\therefore 2x-y=0$ (as required)

(c) (ii) (1 mark)
Outcomes Assessed: P4, P5
Targeted Performance Bands: 2-3

Criteria	Mark
• Correctly shows that the x-coordinate of point B is 3.	1

Sample Answer

B lies on the line $y=6$
$y=6\ \therefore\ 6=2x\ \therefore\ x=3$

$\therefore B(3,6)$

DISCLAIMER
The information contained in this document is intended for the professional assistance of teaching staff. It does not constitute advice to students. Further it is not the intention of CSSA to provide specific marking outcomes for all possible Trial HSC answers. Rather the purpose is to provide teachers with information so that they can better explore, understand and apply HSC marking requirements, as established by the NSW Board of Studies.
No guarantee or warranty is made or implied with respect to the application or use of CSSA Marking Guidelines in relation to any specific trial exam question or answer. The CSSA assumes no liability or responsibility for the accuracy, completeness or usefulness of any Marking Guidelines provided for the Trial HSC papers.

(c) (iii) (3 marks)
Outcomes Assessed: P2, P4, P5
Targeted Performance Bands: 2-4

Criteria	Mark
• Correctly finds the gradient of L_2 which gives $\angle AOB = 90°$.	1
• Correctly finds the equation of L_2	1
• Correctly finds the value of the x-ordinate of A.	1

Sample Answer

$$m_{L_2} = \frac{-1}{m_{L_3}} = \frac{-1}{2} = -\frac{1}{2}$$

$$\therefore y - 0 = -\frac{1}{2}(x - 0)$$

$$y = -\frac{1}{2}x$$

For $y = 6 \therefore 6 = -\frac{1}{2}x \therefore x = -12$

$A(-12,6)$

(c) (iv) (2 marks)
Outcomes Assessed: P2, P4, P5
Targeted Performance Bands: 2-3

Criteria	Mark
• Writes down the correct distance between points A and B.	1
• Calculates the correct area of ΔAOB .	1

Sample Answer

Distance between points A and B is 15 units.

$$\text{Area of } \Delta AOB = \frac{1}{2} \times 15 \times 6$$
$$= 45 \text{ units}^2$$

Question 3 (12 marks)
(a) (i) (1 mark)
Outcomes Assessed: P3, P4
Targeted Performance Band: 2-4

Criteria	Mark
• Finds the correct vertex	1

Sample Answer

$Vertex \equiv (2,0)$

(a) (ii) (1 mark)
Outcomes Assessed: P3, P4
Targeted Performance Band: 2-4

Criteria	Mark
• Finds the correct focus	1

Sample Answer

$Focus \equiv (2,4)$

(b) (i) (2 marks)
Outcomes Assessed: P7, H5
Targeted Performance Band: 2-3

Criteria	Mark
• Correctly uses the product rule of differentiation but has ONE mistake in calculation	1
• Correctly finds the answer	1

Sample Answer

$$\frac{d}{dx}(3x\log_e x) = 3 + 3\log_e x$$

(b) (ii) (2 marks)
Outcomes Assessed: P7, H5
Targeted Performance Band: 2-4

Criteria	Mark
• Correctly uses the chain rule of differentiation but has ONE mistake in calculation	1
• Correctly finds the answer	1

Sample Answer

$$\frac{d}{dx}(\sin^2 x) = 2(\sin x)^1.\cos x = 2\sin x\cos x$$

(c) (i) (2 marks)
Outcomes Assessed: H5
Targeted Performance Band: 2-4

Criteria	Mark
• Gives an answer of $\sin 2006x + c$	1
• Correctly finds the answer	1

Sample Answer

$$\int \cos 2006x \, dx = \frac{1}{2006}\sin 2006x + c$$

(c) (ii) (2 marks)
Outcomes Assessed: H3, H5
Targeted Performance Band: 3-4

Criteria	Mark
• Finds the primitive $\frac{1}{2}e^{2x}$ but has an error in calculating the integral	1
• Correctly applies the Newton-Leibnitz formula to obtain the correct answer in exact form	1

Sample Answer

$$\int_0^1 e^{2x} \, dx = \left[\frac{1}{2}e^{2x}\right]_0^1 = \frac{1}{2}e^2 - \frac{1}{2}e^0 = \frac{1}{2}e^2 - \frac{1}{2} \text{ or } \frac{1}{2}\left(e^2 - 1\right)$$

(d) (2 marks)
Outcomes Assessed: P6, H5
Targeted Performance Band: 2-4

Criteria	Mark
• Correctly finds the gradient of the normal	1
• Correctly substitutes the values for x and y into the point/gradient formula to find the equation of the normal	1

Sample Answer

$y = x^3 - 5x \quad \therefore \frac{dy}{dx} = 3x^2 - 5$? At $x = 1$, $m_T = -2 \therefore m_N = \frac{1}{2}$

$\therefore$ equation of normal is given by $y - -4 = \frac{1}{2}(x - 1)$

$\therefore \; x - 2y - 9 = 0$ (or $y = \frac{1}{2}x - \frac{9}{2}$)

Question 4 (12 marks)
(a) (i) (3 marks)
Outcomes Assessed: P7, H6
Targeted Performance Band: 3-5

Criteria	Mark
• Finds the stationary points	1
• Finds the nature of ONE stationary point	1
• Finds the nature of the other stationary point	1

Sample Answer

$f(x) = -\frac{1}{3}x^3 - x^2 + 3x + 1 \quad \therefore f'(x) = -x^2 - 2x + 3$

For stationary points $f'(x) = 0 \quad \therefore (1 - x)(x + 3) = 0 \quad \therefore x = 1$ or $x = -3$

$\therefore$ the stationary points are $\left(1, 2\frac{2}{3}\right)$ & $(-3, -8)$

Also for the nature of the stationary points, $f''(x) = -2x - 2$

At $x = 1$, $f''(1) = -4 < 0 \quad \therefore \left(1, 2\frac{2}{3}\right)$ is a MAXIMUM turning point

At $x = -3$, $f''(-3) = 4 > 0 \quad \therefore (-3, -8)$ is a MINIMUM turning point

(a) (ii) (2 marks)
Outcomes Assessed: P7, H6
Targeted Performance Band: 2-4

Criteria	Mark
• Correctly solves the equation $f''(x) = 0$	1
• Analyse the sign of the second derivative and gives correct answer	1

Sample Answer

$f''(x) = -2x - 2 \quad \therefore -2x - 2 = 0 \rightarrow x = -1 \; (y = -2\frac{2}{3})$

x	-1^-	-1	-1^+
$f''(x)$	+	0	-

$\therefore$ a change in the sign of the second derivative has occurred

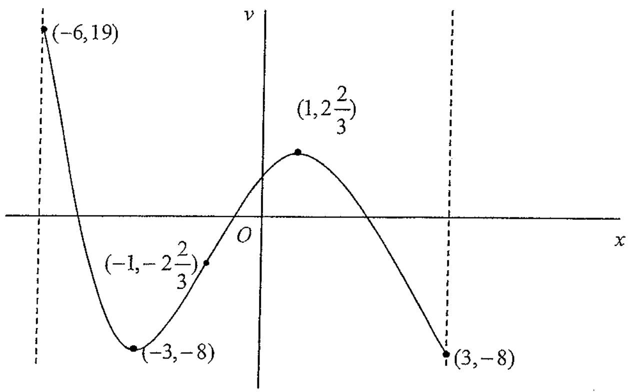
$\therefore \left(-1, -2\frac{2}{3}\right)$ is a point of inflection

(a) (iii) (2 marks)
Outcomes Assessed: P6, H6, H7, H9
Targeted Performance Band: 3-5

Criteria	Mark
• Draws the correct cubic curve	1
• Plots all important points	1

Sample Answer

$f(x) = -\frac{1}{3}x^3 - x^2 + 3x + 1$



(a) (iv) (1 mark)
Outcomes Assessed: H7
Targeted Performance Band: 3-4

Criteria	Mark
• Gives the correct answer	1

Sample Answer

$y = 19$

(b) (i) (0 marks)
(b) (ii) (2 marks)
Outcomes Assessed: P2, H2
Targeted Performance Band: 2-4

Criteria	Mark
• Shows that $\angle PAS = \angle QAR$ (vertically opposite $\angle$'s) or equivalent	1
• Correctly completes proof using (AAS)	1

Sample Answer

$\angle PSA = \angle QRA$ (given)
$\angle PAS = \angle QAR$ (vertically opposite $\angle$'s)
$PS = QR$ (given)
$\therefore\ \Delta PSA \equiv \Delta QRA$ (AAS)

(b) (iii) (2 marks)
Outcomes Assessed: P2, H2
Targeted Performance Band: 3-4

Criteria	Mark
• Realises that ΔPAQ is isosceles & base $\angle$'s are equal	1
• Gives the correct answer	1

Sample Answer

$PA = QA$ (corresponding sides are equal in congruent ?'s)
$\therefore \Delta PAQ$ is isosceles $\therefore \angle PQA = \angle QPA = x$

$2x + 120 = 180°$ ($\angle$ sum of a ? is 180°)

$x = 30°$

Question 5 (12 marks)
(a) (i) (1 mark)
Outcomes Assessed: H5
Targeted Performance Band: 2-3

Criteria	Mark
• Writes the correct sum of an infinite geometric series	1

Sample Answer

$$0.7\dot{5} = \frac{7}{10} + \frac{5}{100} + \frac{5}{1000} + \frac{5}{10000} \ldots$$

(a) (ii) (2 marks)
Outcomes Assessed: H5
Targeted Performance Band: 3-4

Criteria	Mark
• Writes the limiting sum formula with correct a and r	1
• Gives the correct answer	1

Sample Answer

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{7}{10} + \frac{5/100}{1-1/10}$$

$$= \frac{7}{10} + \frac{1}{18}$$

$$= \frac{34}{45}$$

(b) (2 marks)
Outcomes Assessed: P3, H5
Targeted Performance Band: 2-4

Criteria	Mark
• Realises that $\cos^2\theta$ is a substitute or equivalent	1
• Simplifies to give the correct answer	1

Sample Answer

$$\frac{1-\sin^2 x}{\cot x}$$

$$= \frac{\cos^2\theta}{\frac{\cos\theta}{\sin\theta}} = \cos^2\theta \times \frac{\sin\theta}{\cos\theta}$$

$$= \sin\theta\cos\theta$$

(c) (i) (1 mark)
Outcomes Assessed: H5
Targeted Performance Band: 2-4

Criteria	Mark
• Gives the correct answer	1

Sample Answer

$$l = r\theta \quad \therefore \quad \theta = \frac{l}{r}$$

$$\theta = \frac{1}{6/\pi} = \frac{\pi}{6}$$

(c) (ii) (2 marks)
Outcomes Assessed: H5
Targeted Performance Band: 2-4

Criteria	Mark
• Writes the correct formula with necessary substitutions	1
• Gives the correct answer in exact form	1

Sample Answer

$$A = \frac{1}{2}r^2\theta$$

$$A = \frac{1}{2}\times\left(\frac{6}{\pi}\right)^2\times\frac{\pi}{6}$$

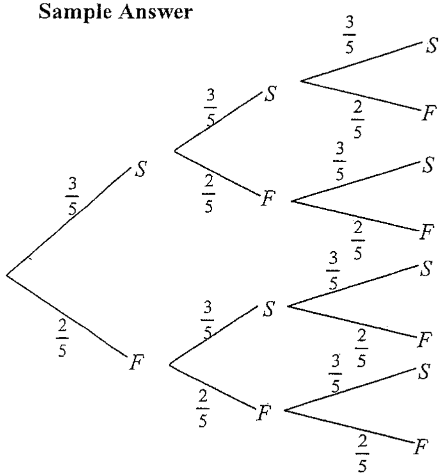
$\therefore$ Area of sector $MON \Rightarrow A = \frac{3}{\pi}\text{cm}^2$

DISCLAIMER
The information contained in this document is intended for the professional assistance of teaching staff. It does not constitute advice to students. Further it is not the intention of CSSA to provide specific marking outcomes for all possible Trial HSC answers. Rather the purpose is to provide teachers with information so that they can better explore, understand and apply HSC marking requirements, as established by the NSW Board of Studies.
No guarantee or warranty is made or implied with respect to the application or use of CSSA Marking Guidelines in relation to any specific trial exam question or answer The CSSA assumes no liability or responsibility for the accuracy, completeness or usefulness of any Marking Guidelines provided for the Trial HSC papers.

(d) (i) (2 marks)
Outcomes Assessed: H5
Targeted Performance Band: 3-4

Criteria	Mark
• Correctly draws a tree diagram	1
• Gives the correct answer	1

Sample Answer



$$\therefore P(S) = \frac{3}{5}\times\frac{3}{5}\times\frac{3}{5} = \left(\frac{3}{5}\right)^3 = \frac{27}{125}$$

(d) (ii) (2 marks)
Outcomes Assessed: H5
Targeted Performance Band: 3-4

Criteria	Mark
• Uses the complementary events method (or otherwise)	1
• Gives the correct answer with required working	1

Sample Answer

$$P(S) = 1 - \left[\frac{2}{5}\times\frac{2}{5}\times\frac{2}{5}\right] = 1 - \left(\frac{2}{5}\right)^3 = \frac{117}{125}$$

Question 6 (12 marks)
(a) (i) (1 mark)
Outcomes Assessed: P2
Targeted Performance Bands: 2-3

Criteria	Mark
• Correctly verifies Pythagoras' Theorem	1

Sample Answer

Triangle with sides 12 cm, 16 cm and 20 cm is right angled at A ($12^2 + 16^2 = 20^2$, converse of Pythagoras' Theorem)

(a) (ii) (2 marks)
Outcomes Assessed: P4
Targeted Performance Bands: 2-3

Criteria	Marks
• Correctly gives answer for α in either degrees or radians	1
• Correctly gives answer for β in either degrees or radians	1

Sample Answer

$\sin\alpha = \frac{12}{20}$ $\therefore$ $\alpha = 36^\circ 52'$ or $\alpha = 0.644^c$

$\beta = 90^\circ - 36^\circ 52' = 53^\circ 8'$ or $\beta = 0.927^c$ (using complementary angles in ΔAO_1O_2)

(a) (iii) (3 marks)
Outcomes Assessed: H5
Targeted Performance Bands: 3-5

Criteria	Marks
• Correctly substitutes in the segment formula for the circle O_1	1
• Correctly substitutes in the segment formula for the circle O_2	1
• Correctly gives answer (to 2 d.p.)	1

Sample Answer

Area enclosed between the two circles is the sum of the two segments, with angles 2α and 2β subtended respectively at the centre.

$$\text{Area} = \frac{1}{2}\times 16^2(2\times 0.644 - \sin 2\times 0.644) + \frac{1}{2}\times 12^2(2\times 0.927 - \sin 2\times 0.927)$$

$\therefore$ Area = 106.2667952 cm^2 $\therefore$ Area = 106.27 cm^2 (2 d.p.)

(b) (i) (2 marks)
Outcomes Assessed: H3, H8
Targeted Performance Bands: 3-4

Criteria	Mark
• Correctly works out x^2 as a subject ($x^2 = \log_e y$)	1
• Correctly substitutes $x^2 = \log_e y$ in the volume formula and deduce the answer	1

Sample Answer

$V_y = \pi \int_1^5 x^2\,dy$; Since $y = e^{x^2}$ $\therefore \log_e y = x^2$ $\therefore V_y = \pi \int_1^5 \log_e y\,dy$ (as required)

(b) (ii) (1 mark)
Outcomes Assessed: H3
Targeted Performance Bands: 2-3

Criteria	Mark
• Correctly completes the required value ($\log_e y = 1.386$)	1

Sample Answer

y	1	2	3	4	5
$\log_e y$	0	0.693	1.099	1.386	1.609

(b) (iii) (3 marks)
Outcomes Assessed: H3, H5
Targeted Performance Bands: 2-4

Criteria	Mark
• Substitutes the correct values in the correct Simpson's formula	1
• Correctly calculates the answer in decimal form (e.g. 4.041)	1
• Gives the correct answer	1

Sample Answer

Using Simpson's Formula: $\int_a^b y\,dx \approx \frac{h}{3}[y_0 + 4(y_1 + y_3 + \ldots) + 2(y_2 + y_4 + \ldots) + y_n]$

$V_y = \pi \times \frac{1}{3}[0 + 4(0.693 + 1.386) + 2 \times 1.099 + 1.609]$ $\therefore V_y = 4.041\pi$

$\therefore V_y = 12.695$ (3 d.p.)

Question 7 (12 marks)
(a) (3 marks)
Outcomes Assessed: H3
Targeted Performance Bands: 3-4

Criteria	Mark
• Correctly uses the properties of logarithms	1
• Correctly uses the definition of logarithms to obtain a quadratic equation	1
• Solves the quadratic equation, indicating the correct answer	1

Sample Answer

$\log_2 x + \log_2(x+7) = 3$ $\therefore \log_2 x(x+7) = 3$ $\therefore x(x+7) = 2^3$ $\therefore x^2 + 7x - 8 = 0$

$\therefore (x+8)(x-1) = 0$ $\therefore x = -8$ or $x = 1$ $\therefore$ solution is $x = 1$ $(x > 0)$ or checking the solution by substitution.

(b) (i) (2 marks)
Outcomes Assessed: P4
Targeted Performance Bands: 2-3

Criteria	Mark
• Correctly finds x-ordinate of point A	1
• Correctly finds x-ordinate of point B	1

Sample Answer

At point A: $x^2 = 3 - 2x^2$ $\therefore 3x^2 = 3$ $\therefore x = \pm 1$ $\therefore x_A = 1$ (first quadrant)

At point B: $3 - 2x^2 = 0$ $\therefore x = \pm\sqrt{\frac{3}{2}}$ $\therefore x_B = \sqrt{\frac{3}{2}}$ (first quadrant)

(b) (ii) (3 marks)
Outcomes Assessed: H8
Targeted Performance Bands: 2-4

Criteria	Mark
• Correctly uses the sum of integrals to find area under curves	1
• Correctly finds the primitives	1
• Correctly finds the area using Leibnitz – Newton Formula	1

Sample Answer

$A_{AOB} = \int_0^1 x^2\,dx + \int_1^{\sqrt{\frac{3}{2}}} (3 - 2x^2)\,dx = \frac{1}{3}[x^3]_0^1 + \left[3x - \frac{2}{3}x^3\right]_1^{\sqrt{\frac{3}{2}}}$

$\therefore A_{AOB} = \frac{1}{3}(1-0) + \left[\left(3\sqrt{\frac{3}{2}} - \frac{2}{3}\sqrt{\frac{3}{2}}^3\right) - \left(3 - \frac{2}{3}\right)\right] = 2\sqrt{\frac{3}{2}} - 2$ square units

(c) (i) (1 mark)
Outcomes Assessed: P4
Targeted Performance Bands: 2

Criteria	Mark
• Correctly shows the number of revolutions the track will rotate	1

Sample Answer

Record is played at 5 revolutions per second for 25 minutes therefore will rotate

$5\times 60\times 25=7500$ revolutions

(c) (ii) (3 marks)
Outcomes Assessed: H1, H5
Targeted Performance Bands: 3-5

Criteria	Mark
• Correctly realises that circles' radii are in a arithmetic sequence	1
• Correctly uses the formula for arithmetic series	1
• Finds the correct answer	1

Therefore there will be 7500 concentric circles whose radii increases in an arithmetic sequence from 2 cm to 6 cm. The length of the track record can be found using the sum of the arithmetic sequence with 7500 terms:

$S_{7500}=\frac{7500}{2}(2\pi\times 2+2\pi\times 6)=188495.5595$ cm =1.9 km (1 d.p.)

Question 8 (12 marks)
(a) (2 marks)
Outcomes Assessed*: *P3, P4
Targeted Performance Bands*: *2 – 4

Criteria	Mark
• Correctly uses index law	1
• Find the correct answer	1

Sample Answer

If $A^m=3\ \therefore (A^m)^4=3^4=81\ \therefore A^{4m}-5=81-5=76$

(b) (i) (2 marks)
Outcomes Assessed*: *H3, H4, H5
Targeted Performance Bands*: *3 – 4

Criteria	Mark
• Correctly substitutes in the formula and writes $-10k$ as a subject	1
• Gives the correct answer	1

Sample Answer

$U=100e^{-kt}\ \therefore 2=100e^{-10k}\ \therefore \frac{1}{50}=e^{-10k}\ \therefore \ln\frac{1}{50}=\ln e^{-10k}\ \therefore k=\frac{-\ln 50}{-10}$

$\therefore k=0.3912$

(b) (ii) (2 marks)
Outcomes Assessed*: *H3, H4, H5
Targeted Performance Bands*: *3-5

Criteria	Mark
• Correctly differentiates U	1
• Gives the correct answer	1

Sample Answer

$\frac{dU}{dt}=-k(100e^{-kt})$

For $t=12$ and $k=0.3912$: $\frac{dU}{dt}=-0.36$ mg/minute

(c) (i) (1 mark)
Outcomes Assessed*: *H4, H5
Targeted Performance Bands*: *2-3

Criteria	Mark
• Correctly gives answer for A_1	1

Sample Answer

$$A_1 = P\left(1+\frac{0.75}{100}\right) - 1050 \quad \therefore A_1 = P(1.0075) - 1050$$

(c) (ii) (3 marks)
Outcomes Assessed*: *H4, H5
Targeted Performance Bands*: *3-6

Criteria	Mark
• Correctly gives answer for A_2	1
• Write the correct sum of the geometric series	1
• Gives the correct answer	1

Sample Answer

$$A_2 = [P(1.0075) - 1050](1.0075) - 1050$$

$$= P(1.0075)^2 - 1050(10075) - 1050$$

$$= P(1.0075)^2 - 1050(1 + 1.0075)$$

$$A_3 = P(1.0075)^3 - 1050(1 + 1.0075 + 1.0075^2)$$

$$\vdots$$

$$A_n = P(1.0075)^n - 1050(1 + 1.0075 + \ldots + 1.0075^{n-1})$$

$$= P(1.0075)^n - 1050\left[\frac{1(1.0075^n - 1)}{0.0075}\right]$$

$$= P(1.0075)^n - 140\,000\,[1.0075^n - 1]$$

$$= P(1.0075)^n - 140\,000\,(1.0075)^n + 140000$$

(c) (iii) (2 marks)
Outcomes Assessed*: *H4, H5
Targeted Performance Bands*: *3-4

Criteria	Mark
• Correctly substitutes n=240 and makes $A_n = 0$	1
• Gives the correct answer	1

Sample Answer

When $n = 240$

$$\therefore 0 = P(1.0075)^{240} - 140000(1.0075)^{240} + 140\,000$$

$$\therefore P(1.0075)^{240} = 140\,000(1.0075)^{240} - 140\,000$$

$$\therefore P = \frac{140\,000(1.0075)^{240} - 140\,000}{(1.0075)^{240}}$$

$\therefore P = \$116\ 702$ (to the nearest dollar)

Question 9 (12 marks)

(a) (i) (2 marks)

Outcomes Assessed*: *H4, H5

Targeted Performance Bands*: *2-3

Criteria	Mark
• Correctly differentiate to find the velocity v_A	1
• Correctly differentiate to find the velocity v_B	1

Sample Answer

$x_A = 12t + 5 \quad \therefore v_A = 12$

$x_B = 6t^2 - t^3 \quad \therefore v_B = 12t - 3t^2$

(a) (ii) (1 mark)

Outcomes Assessed*: *H4, H5

Targeted Performance Bands*: *2-3

Criteria	Mark
• Gives the correct answer	1

Sample Answer

At $t = 1$ second $\therefore v_A = 12$ m/s, $v_B = 12 - 3 = 9$ m/s

$\therefore$ particle A is faster

(a) (iii) (1 mark)

Outcomes Assessed*: *H4, H5

Targeted Performance Bands*: *3 – 4

Criteria	Mark
• Gives the correct answer	1

Sample Answer

$v_B = 0 \therefore 12t - 3t^2 = 0 \therefore 3t(4 - t) = 0 \therefore t = 0$ or $t = 4$ seconds

$\therefore$ Particle comes at rest at $t = 4$ seconds

(a) (iv) (2 marks)

Outcomes Assessed*: *H4, H5

Targeted Performance Bands*: *3 – 5

Criteria	Mark
• Correctly substitutes the value of $t = 4$ into the x_B equation	1
• Gives the correct answer	1

Sample Answer

For maximum displacement, $v_B = 0 \therefore t(12 - 3t) = 0 \therefore t = 4$ seconds

$\therefore x_B = 6 \times 4^2 - 4^3 = 32$ metres

(b) (i) (1 mark)

Outcomes Assessed*: *P6

Targeted Performance Bands*: *2 – 4

Criteria	Mark
• Gives the correct answer	1

Sample Answer

The gradient of the tangent at point P is $m_P = \tan 60^\circ = \sqrt{3}$

(b) (ii) (2 marks)

Outcomes Assessed*: *P3, P4

Targeted Performance Bands*: *3 – 5

Criteria	Mark
• Correctly finds the gradient of the tangent to the curve and equates to the gradient obtained from (i)	1
• Correctly finds the coordinates of point P	1

Sample Answer

$y = 1 - x^2 \therefore \frac{dy}{dx} = -2x$ and since the gradient of the tangent is $\sqrt{3}$

$\therefore \sqrt{3} = -2x \therefore x = \frac{-\sqrt{3}}{2} \therefore y = \frac{1}{4} \therefore$ coordinates of point P are $\left(-\frac{\sqrt{3}}{2}, \frac{1}{4}\right)$

(b) (iii) (3 marks)
***Outcomes Assessed*: P4, H5**
***Targeted Performance Bands*: 3-6**

Criteria	Mark
• Correctly find the length of PQ	1
• Correctly find the length of PO^2 (or QO^2)	1
• Gives the correct answer	1

Sample Answer

Coordinates of point Q are $\left(\frac{\sqrt{3}}{2}, \frac{1}{4}\right)$ (By symmetry)

$\therefore PQ = \sqrt{3}$

$PO^2 = \frac{3}{4} + \frac{1}{16} = \frac{13}{16}$

Using the Cosine Rule in ΔPOQ: $\cos \angle POQ = \dfrac{\frac{13}{16} + \frac{13}{16} - (\sqrt{3})^2}{2 \times \sqrt{\frac{13}{16} \times \frac{13}{16}}}$

$\angle POQ = 2.6$ radians

Question 10 (12 marks)
(a) (2 marks)
***Outcomes Assessed*: H5**
***Targeted Performance Bands*: 2-5**

Criteria	Mark
• Writes the correct value of the amplitude A	1
• Find the correct value of n	1

Sample Answer

Since $y = A \sin nx$ $\therefore A = 2$

Period $(T) = \frac{4\pi}{3}$ (from graph) and since Period $(T) = \frac{2\pi}{n}$ $\therefore n = \frac{2\pi}{T}$

$\therefore n = \frac{3}{2}$ $\therefore$ the equation is $y = 2\sin\frac{3}{2}x$

(b) (i) (1 mark)
***Outcomes Assessed*: P4, H5**
***Targeted Performance Bands*: 3 – 4**

Criteria	Mark
• Correctly finds the area of ? SOR	1

Sample Answer

In ? SOR, $OS = 6$ (radius of the circle) and $OR = 4$

$\therefore$ area of ? $SOR = ½ \times 6 \times 4 \times \sin\alpha = 12\sin\alpha$ (as required)

(b) (ii) (3 marks)
***Outcomes Assessed*: P4, H5**
***Targeted Performance Bands*: 3 – 6**

Criteria	Mark
• Correctly finds the area of ? SOT	1
• Correctly finds the area of the quadrilateral $ORST$	1
• Correctly factorise to find the correct answer	1

Sample Answer

$\therefore$ area of ? $SOT = ½ \times 6 \times 2 \times \sin\left(\frac{\pi}{2} - \alpha\right) = 6\cos\alpha$

$\therefore$ area of the quadrilateral $ORST$ is:

$A = 12$ sina + 6 cosa

$A =$ 6cosa(2tana + 1)

(b) (iii) (3marks)

Outcomes Assessed*: *P4, H5

Targeted Performance Bands*: *3 – 6

Criteria	Mark
• Correctly finds the derivative of A	1
• Correctly finds the value of tan a	1
• Correctly shows that the area is a maximum when tan a = 2	1

Sample Answer

$A = 12 \sin a + 6 \cos a \quad \therefore \frac{dA}{d\alpha} = 12\cos\alpha - 6\sin\alpha$

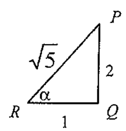
For maximum area $\frac{dA}{d\alpha} = 0 \therefore 0 = 12\cos\alpha - 6\sin\alpha$

$\therefore 6\sin a = 12\cos a \;\therefore \tan\alpha = 2$

To prove the area is maximum when $\tan\alpha = 2$:

$\frac{d^2A}{d\alpha^2} = -12\sin\alpha - 6\cos\alpha$ and since $\tan\alpha = 2 \therefore$ from the triangle PQR

$\therefore \sin\alpha = \frac{2}{\sqrt{5}}$ and $\cos\alpha = \frac{1}{\sqrt{5}}$



$\therefore \frac{d^2A}{d\alpha^2} = -12\times\frac{2}{\sqrt{5}} - 6\times\frac{1}{\sqrt{5}} = -\frac{30}{\sqrt{5}} < 0$

$\therefore$ Maximum Area occurs at $\tan\alpha = 2$

Alternative method: To prove the area is maximum when $\tan\alpha = 2$:

Since $\tan\alpha = 2 \;\therefore \alpha = 1.1$ radians

α	1^c	1.1^c	1.2^c
$\frac{dA}{d\alpha}$	$1.43 > 0$	0	$-1.24 < 0$

Maximum

(b) (iv) (3 marks)

Outcomes Assessed*: *P4, H5

Targeted Performance Bands*: *3 – 6

Criteria	Mark
• Correctly finds the gradient of the line OS and equates to 2 to get the equation $y = 2x$	1
• Correctly substitutes into the equation of the circle to get the x-ordinate of point S	1
• Correctly finds the y-ordinate of point S	1

Sample Answer

The gradient of the line $OS = \tan\alpha = \frac{y}{x}$ and since $\tan\alpha = 2$

$\therefore \frac{y}{x} = 2 \;\therefore y = 2x \quad [1]$

Substitute into $x^2 + y^2 = 36 \;\therefore x^2 + 4x^2 = 36$

$\therefore 5x^2 = 36 \quad \therefore x^2 = \frac{36}{5} \quad \therefore x = \frac{6}{\sqrt{5}} = \frac{6}{5}\sqrt{5}$

$\therefore$ Substitute into $[1] \;\therefore y = \frac{12}{\sqrt{5}} = \frac{12}{5}\sqrt{5} \;\therefore$ point S is $\left(\frac{6}{5}\sqrt{5}, \frac{12}{5}\sqrt{5}\right)$.

DISCLAIMER
The information contained in this document is intended for the professional assistance of teaching staff. It does not constitute advice to students. Further it is not the intention of CSSA to provide specific marking outcomes for all possible Trial HSC answers. Rather the purpose is to provide teachers with information so that they can better explore, understand and apply HSC marking requirements, as established by the NSW Board of Studies.
No guarantee or warranty is made or implied with respect to the application or use of CSSA Marking Guidelines in relation to any specific trial exam question or answer. The CSSA assumes no liability or responsibility for the accuracy, completeness or usefulness of any Marking Guidelines provided for the Trial HSC papers.