CATHOLIC SECONDARY SCHOOLS ASSOCIATION OF NEW SOUTH WALES

YEAR TWELVE FINAL TESTS 2000

MATHEMATICS
3/4 UNIT

(i.e. 3 UNIT COURSE – ADDITIONAL PAPER:
4 UNIT COURSE – FIRST PAPER)

Afternoon session

Thursday 10 August 2000

Time allowed – two hours

EXAMINERS
Graham Arnold, Terra Sancta College, Nirimba
Sandra Hayes, Aquinas College, Menai
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DIRECTIONS TO CANDIDATES:

- **ALL** questions may be attempted.
- **ALL** questions are of equal value.
- All necessary working should be shown in every question.
- Full marks may not be awarded for careless or badly arranged work.
- Approved calculators may be used.
- Standard integrals are printed at the end of the exam paper.

Question 1 **Begin a new page** **Marks**

(a) Solve the inequality $x^2 \geq 2x$. **2**

(b) Find the number of ways in which 4 girls and 3 boys can be seated in a row so that no two girls are next to each other. **2**

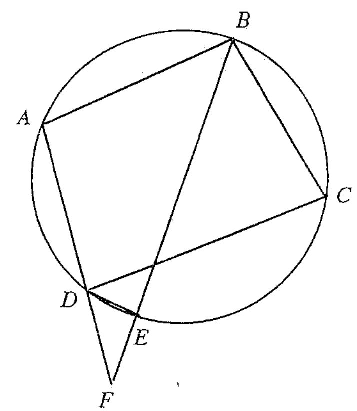
(c) The curves $y = \ln x$ and $xy = e$ intersect at the point $P(e, 1)$. **3**

(i) Find the gradients m_1 and m_2 of the tangents to each of the curves at P.

(ii) If θ is the acute angle between the tangents to the curves at P, show that

$$\tan\theta = \frac{2e}{e^2 - 1}.$$

(d) **5**



In the diagram $ABCD$ is a cyclic quadrilateral. The bisector of $A\hat{B}C$ cuts the circle at E, and meets AD produced at F.

(i) Copy the diagram showing the above information.

(ii) Give a reason why $C\hat{D}E = C\hat{B}E$.

(iii) Show that DE bisects $C\hat{D}F$.

Marks

Question 2 — Begin a new page

(a) $A(-1,5)$ and $B(5,-4)$ are two points. Find the coordinates of the point P which divides the interval AB internally in the ratio $2:1$. **2**

(b) The equation $2x^3-5x-1=0$ has roots α, β and γ. **2**

Find the value of $\dfrac{1}{\alpha}+\dfrac{1}{\beta}+\dfrac{1}{\gamma}$.

(c) Consider the series $\tan x + \tan^3 x + \tan^5 x + \ldots$, where $0 \le x < \frac{\pi}{4}$ **3**

(i) Show that the limiting sum S of the series exists.

(ii) Show that $S=\frac{1}{2}\tan 2x$.

(d) $P\left(2at, at^2\right)$, $t>0$ is a point on the parabola $x^2=4ay$. The normal to the parabola at P cuts the x axis at X and the y axis at Y. **5**

(i) Show that the normal at P has equation $x+ty-2at-at^3=0$.

(ii) Find the coordinates of the points X and Y.

(iii) Find the value of t such that P is the midpoint of XY.

Question 3 — Begin a new page

(a) Consider the function $f(x)=\dfrac{3x-4}{x-1}$. **8**

(i) Show that the function $f(x)$ is an increasing function for all values of x in its domain.

(ii) Sketch the graph $y=f(x)$ showing clearly the coordinates of the points of intersection with the axes and the equations of the horizontal and vertical asymptotes.

(iii) Show that the x coordinates of the points of intersection of the line $y=mx$ and the curve $y=f(x)$ satisfy the equation $mx^2-(m+3)x+4=0$.

(iv) Find the equations of the tangent lines to the curve $y=f(x)$ which pass through the origin.

(b) (i) The equation $x^3-kx+1=0$ has exactly one root between $x=0$ and $x=1$. Show that $k>2$. **4**

(ii) The equation $x^3-3x+1=0$ has a root α where $0<\alpha<1$. Starting with an initial approximation $\alpha \approx 0\cdot 3$, use one application of Newton's method to find a second approximation to the value of α, giving the answer correct to two decimal places.

Marks

Question 4 — Begin a new page

(a) (i) Find the domain and range of the function $y=3\cos^{-1}\frac{x}{2}$. **4**

(ii) Find the equation of the tangent to the curve $y=3\cos^{-1}\frac{x}{2}$ at the point on the curve where $x=0$.

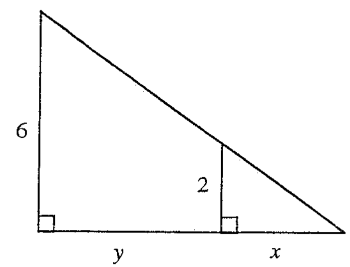
(b) Evaluate $\displaystyle\int_0^3 \frac{3x}{\sqrt{1+x}}\,dx$ using the substitution $u=1+x$. **4**

(c) The region bounded by the curve $y=\dfrac{1}{\sqrt{3+x^2}}$ and the x axis between the lines $x=1$ and $x=3$ is rotated through one complete revolution about the x axis. Find the exact volume of the solid formed. **4**

Question 5 — Begin a new page

(a) Use the method of Mathematical Induction to show that $n!>2^n$ for all positive integers $n\ge 4$. **4**

(b) **4**



A 6 metre high vertical street lamp stands on horizontal ground. A 2 metre tall man runs away from the street lamp at a constant speed of $2\cdot 5\ \text{ms}^{-1}$. When he is y metres from the street lamp his shadow has length x metres.

(i) Show that $\dfrac{x+y}{6}=\dfrac{x}{2}$.

(ii) Find the rate at which his shadow lengthens.

(c) In the Binomial expansion of $(1+x)^n$ the coefficient of x^4 is 6 times the coefficient of x^2. **4**

(i) Show that $n^2-5n-66=0$.

(ii) Find the value of n.

Marks

Question 6 **Begin a new page**

(a) After t years, $t \geq 0$, the number N of individuals in a population is given by $N = 1000 - Ae^{-kt}$ for some constants $A > 0$, $k > 0$. The initial population size is 200 individuals and the initial rate of increase of the population size is 80 individuals per year. **4**

(i) Find the values of A and k.

(ii) Sketch the graph of N against t.

(b) A particle is moving with Simple Harmonic Motion in a straight line about a fixed point O. At time t seconds, $t \geq 0$, it has displacement x metres from O given by $x = a\cos(2t + \alpha)$ for some constants $a > 0$, $0 < \alpha < \frac{\pi}{2}$. Initially it is 4 metres to the right of O. After $\frac{\pi}{4}$ seconds it is 3 metres to the left of O. **4**

(i) Show that $a\cos\alpha = 4$ and $a\sin\alpha = 3$.

(ii) Find the exact value of a, and the value of α correct to two decimal places.

(c) Each time that Bill and Vlad play a set of tennis there is a probability of $\frac{2}{3}$ that Bill wins and a probability of $\frac{1}{3}$ that Vlad wins the set. **4**

(i) If Bill and Vlad play 4 sets of tennis, find the probability that Bill wins 2 sets and Vlad wins 2 sets.

(ii) If Bill and Vlad play sets of tennis until one of them wins 3 sets, find the probability that Bill wins 3 sets and Vlad wins 2 sets.

Marks

Question 7 **Begin a new page**

(a) A garden sprinkler is positioned at the centre of a large, flat lawn. Water droplets are projected from the sprinkler at a fixed speed of 20 ms^{-1} and at an angle θ above the horizontal. The acceleration due to gravity is 10 ms^{-2}. **6**

(i) Use integration to show that the horizontal displacement x metres and the vertical displacement y metres of the water droplets after time t seconds are given by $x = 20t\cos\theta$ and $y = 20t\sin\theta - 5t^2$.

(ii) Show that the horizontal range R of the water droplets is given by $R = 40\sin 2\theta$.

(iii) If the angle of projection varies between $15°$ and $45°$ above the horizontal, find the exact area of that part of the lawn which can be watered in this way.

(b) A particle is moving in a straight line. After time t seconds it has displacement x metres from a fixed point O on the line, velocity v ms^{-1} given by $v = \frac{1-x^2}{2}$ and acceleration a ms^{-2}. Initially the particle is at O. **6**

(i) Find an expression for a in terms of x.

(ii) Show that $\frac{2}{1-x^2} = \frac{1}{1+x} + \frac{1}{1-x}$ and hence find an expression for x in terms of t.

(iii) Describe the motion of the particle, explaining whether it moves to the left or right of O, whether it slows down or speeds up, and where its limiting position is.

STANDARD INTEGRALS

$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$

$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$

$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$

$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$

$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$

$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$

$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$

$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$

$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$

$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$

$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$

NOTE: $\ln x = \log_e x, \quad x > 0$

Answers to 3 Unit Mathematics CSSA Trial 2000

Question 1.

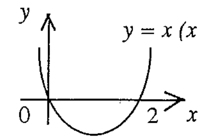
(a)

$x^2 \geq 2x$

$x^2 - 2x \geq 0$

$x(x-2) \geq 0$

$x \leq 0$ or $x \geq 2$



(b) The only possible pattern is

G B G B G B G

Arrange G's in 4! ways then arrange B's in 3! ways.

Number of ways is $4! \times 3! = 24 \times 6 = 144$

(c) (i)

$y = \ln x$ $y = \frac{e}{x}$

$\frac{dy}{dx} = \frac{1}{x}$ $\frac{dy}{dx} = \frac{-e}{x^2}$

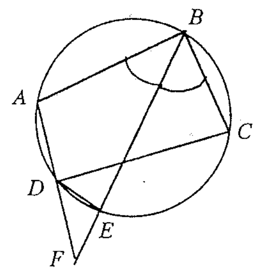
$m_1 = \frac{1}{e}$ $m_2 = \frac{-1}{e}$

(c) (ii)

$\tan\theta = \left|\frac{\frac{1}{e} - \left(\frac{-1}{e}\right)}{1 + \frac{1}{e}\left(\frac{-1}{e}\right)}\right|$

$= \frac{2e}{e^2 - 1}$ (since $e > 1$)

(d) (i)



(d)(ii)

$C\hat{D}E = C\hat{B}E$ (Angles in the same segment standing on arc CE are equal)

(iii)

$C\hat{B}E = A\hat{B}E$ (given BE bisects $A\hat{B}C$)

$A\hat{B}E = F\hat{D}E$ (In cyclic quad. $ABED$, exterior angle FDE is equal to opposite interior angle $A\hat{B}E$)

$\therefore\ F\hat{D}E = C\hat{D}E$ ($C\hat{D}E = C\hat{B}E = A\hat{B}E = F\hat{D}E$)

and hence DE bisects $C\hat{D}F$.

Question 2

(a)

A   B

$(-1,\ 5)$ $(5,\ -4)$

2 : 1

$\frac{10-1}{2+1}$, $\frac{-8+5}{2+1}$ $P(3, -1)$

(b) $2x^3 - 5x - 1 = 0$ has roots α, β and γ.

$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \gamma\alpha + \alpha\beta}{\alpha\beta\gamma}$

$= \frac{-\left(\frac{5}{2}\right)}{\left(\frac{1}{2}\right)}$

$= -5$

(c) (i) Series is geometric, $a = \tan x$, $r = \tan^2 x$.

$0 \leq x < \frac{\pi}{4} \Rightarrow 0 \leq \tan x < 1$

Hence $0 \leq r < 1$ and since $|r| < 1$, limiting sum S exists.

(c) (ii)

$S = \frac{\tan x}{1 - \tan^2 x} = \frac{1}{2} \cdot \frac{2\tan x}{1 - \tan^2 x}$

$\therefore\ S = \frac{1}{2}\tan 2x$

Question 2 (cont)

(d) (i)

$y = at^2 \Rightarrow \frac{dy}{dt} = 2at$

$x = 2at \Rightarrow \frac{dx}{dt} = 2a$

$\therefore \frac{dy}{dx} = \frac{2at}{2a} = t$

Normal at P has gradient $\frac{-1}{t}$ and equation

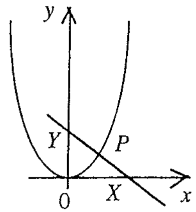
$x + ty = k$, for some constant k.

At P, $\left.\begin{matrix} x = 2at \\ y = at^2 \end{matrix}\right\} \Rightarrow 2at + at^3 = k$

$\therefore$ Equation is $x + ty - 2at - at^3 = 0$

(ii) At X and Y, $x + ty = 2at + at^3$

$X\left(2at + at^3, 0\right)$ $Y\left(0, 2a + at^2\right)$



(iii) Midpt of XY is $M\left(at + \frac{1}{2}at^3, a + \frac{1}{2}at^2\right)$

Hence if P is the midpoint of XY,

$at^2 = a + \frac{1}{2}at^2$

$\frac{1}{2}at^2 = a$

$t^2 = 2$

$t = \sqrt{2}$ (since $t > 0$)

For $t = \sqrt{2}$, $P \equiv M \equiv \left(2\sqrt{2}\,a, 2a\right)$

Question 3.

(a) (i)

$f(x) = \frac{3x-4}{x-1}$

$= \frac{3(x-1)-1}{x-1}$

$= 3 - \frac{1}{x-1}$

Throughout domain $\{x : x \neq 1\}$,

$f'(x) = \frac{1}{(x-1)^2} > 0$

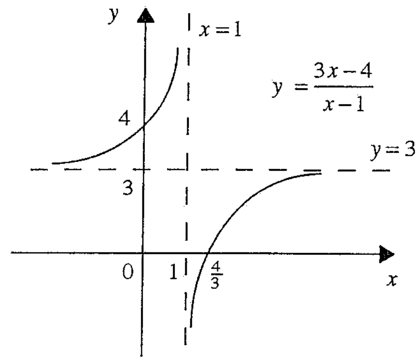
Hence f is increasing.

$y = 3 - \frac{1}{x-1}$

As $x \to \infty$, $y \to 3$ $\therefore$ $y = 3$ is an asymptote

As $x \to 1$, $y \to \infty$ $\therefore$ $x = 1$ is an asymptote

(a)(ii)



(iii)

$mx = \frac{3x-4}{x-1}$

$mx^2 - mx = 3x - 4$

$mx^2 - (m+3)x + 4 = 0$ *

(iv) $y = mx$ is a tangent when * has equal roots and hence discriminant $\Delta = 0$.

$\Delta = (m+3)^2 - 16m$

$= m^2 - 10m + 9$

$= (m-9)(m-1)$

$\Delta = 0 \Rightarrow m = 1$, 9

$\therefore$ tangents are

$y = x$, $y = 9x$

(b)(i) $f(x) = x^3 - kx + 1 \Rightarrow f(0) = 1$, $f(1) = 2 - k$

But if $f(x) = 0$ has exactly one root between 0 and 1, then $f(0)$, $f(1)$ have opposite signs,

$2 - k < 0$. $\therefore$ $k > 2$.

(b)(ii) $f(x) = x^3 - 3x + 1 \Rightarrow f(0\cdot3) = 0\cdot127$

$f'(x) = 3x^2 - 3 \Rightarrow f'(0\cdot3) = -2\cdot73$

$\alpha \approx 0\cdot3 - \frac{f(0\cdot3)}{f'(0\cdot3)} = 0\cdot3 + \frac{0\cdot127}{2\cdot73} = 0\cdot35$

Question 4

(a)(i)

Domain	Range
$-1 \le \frac{x}{2} \le 1$	$0 \le \cos^{-1}\frac{x}{2} \le \pi$
$-2 \le x \le 2$	$0 \le 3\cos^{-1}\frac{x}{2} \le 3\pi$
$\{x : -2 \le x \le 2\}$	$\{y : 0 \le y \le 3\pi\}$

(a) (ii) When $x = 0$,

$y = 3\cos^{-1}\frac{x}{2} \Rightarrow y = \frac{3\pi}{2}$

$\frac{dy}{dx} = \frac{-3}{\sqrt{4-x^2}} \Rightarrow \frac{dy}{dx} = -\frac{3}{2}$

Tangent at $\left(0, \frac{3\pi}{2}\right)$ has gradient $-\frac{3}{2}$ and equation

$3x + 2y = k$, k constant

$x = 0$, $y = \frac{3\pi}{2} \Rightarrow 3\pi = k$

$\therefore$ Tangent is $3x + 2y - 3\pi = 0$

(b)

$u = 1 + x$

$du = dx$

$x = 0 \Rightarrow u = 1$

$x = 3 \Rightarrow u = 4$

$\frac{x}{\sqrt{x+1}} = \frac{u-1}{\sqrt{u}}$

$= u^{\frac{1}{2}} - u^{-\frac{1}{2}}$

$\int_0^3 \frac{3x}{\sqrt{1+x}}\,dx$

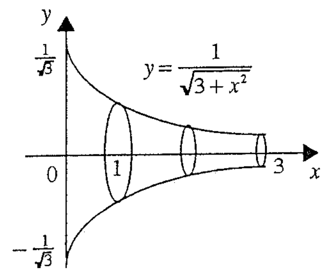
$= 3\int_1^4 u^{\frac{1}{2}} - u^{-\frac{1}{2}}\,du$

$= 3\left[\frac{2}{3}u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\right]_1^4$

$= 2(8-1) - 6(2-1)$

$= 8$

(c)



$V = \pi\int_1^3 \frac{1}{3+x^2}\,dx$

$= \frac{\pi}{\sqrt{3}}\left[\tan^{-1}\frac{x}{\sqrt{3}}\right]_1^3$

$= \frac{\pi}{\sqrt{3}}\left\{\tan^{-1}\sqrt{3} - \tan^{-1}\frac{1}{\sqrt{3}}\right\}$

$= \frac{\pi}{\sqrt{3}}\left\{\frac{\pi}{3} - \frac{\pi}{6}\right\}$

$= \frac{\pi^2\sqrt{3}}{18}$

Volume is $\frac{\pi^2\sqrt{3}}{18}$ cu. units.

Question 5

(a) Let $S(n)$ be the statement $n! > 2^n$, n=4, 5, 6 ...

Consider $S(4)$:

$4! = 24 > 16 = 2^4 \Rightarrow S(4)$ is true

If $S(k)$ is true, then $k! > 2^k$ **

Consider $S(k+1)$, $k \ge 4$:

$(k+1)! = (k+1)\,k!$

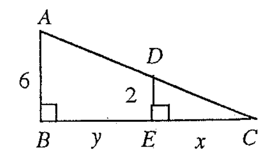
$> (k+1)\,2^k$ if $S(k)$ is true, using **

$> 2.2^k$ since $k \ge 4 \Rightarrow k+1 > 2$

$= 2^{k+1}$

Hence if $S(k)$ is true for some $k \ge 4$, then $S(k+1)$ is true. But $S(4)$ is true, hence $S(5)$ is true and then $S(6)$ is true, and so on. Hence by Mathematical induction, $n! > 2^n$ for all integers $n \ge 4$.

(b) (i)



$\Delta ABC \,|||\, \Delta DEC$ (equiangular)

$\therefore \frac{x+y}{x} = \frac{6}{2}$

$\frac{x+y}{6} = \frac{x}{2}$

(b)(ii)

$x + y = 3x$

$2x = y$

$2\frac{dx}{dt} = \frac{dy}{dt}$

$\therefore \frac{dx}{dt} = \frac{2\cdot5}{2} = 1\cdot25$

Shadow lengthens at rate $1\cdot25$ ms^{-1}

Question 5 (cont)

(c) (i)

$(1+x)^n$ Coeff. of x^4 is ${}^nC_4$
Coeff. of x^2 is ${}^nC_2$

$${}^nC_4 = 6\,{}^nC_2 \Rightarrow \frac{n(n-1)(n-2)(n-3)}{4!} = \frac{6n(n-1)}{2!}$$

$\therefore\ n \ge 4$ and $(n-2)(n-3) = 72$

$n \ge 4$ and $n^2 - 5n - 66 = 0$

(c) (ii)

$n \ge 4$ and $(n-11)(n+6) = 0$

$\therefore\ n = 11$

Question 6

(a) (i)

$N = 1000 - Ae^{-kt}$

$\left.\begin{matrix} t=0 \\ N=200 \end{matrix}\right\} \Rightarrow \begin{matrix} 200 = 1000 - A.1 \\ A = 800 \end{matrix}$

$\frac{dN}{dt} = kAe^{-kt} = 800ke^{-kt}$

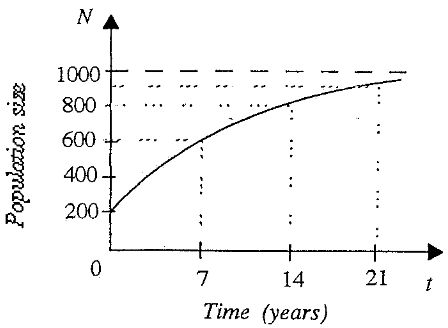
$\left.\begin{matrix} t=0 \\ \frac{dN}{dt} = 80 \end{matrix}\right\} \Rightarrow \begin{matrix} 800k = 80 \\ k = 0\cdot 1 \end{matrix}$

(a) (ii)

$1000 - N = 800e^{-0\cdot 1t} = 800$ when $t = 0$

$1000 - N = 400 \Rightarrow \begin{cases} e^{-0\cdot 1t} = \frac{1}{2} \\ -0\cdot 1\ t = -\ln 2 \\ t = 10\ln 2 \approx 7 \end{cases}$

$\therefore$ $1000 - N$ halves every 7 years and graph has horizontal asymptote at $N = 1000$



(b)(i)

$x = a\cos(2t + \alpha)$

$t = 0,\ x = 4 \Rightarrow a\cos\alpha = 4$

$t = \frac{\pi}{4},\ x = -3 \Rightarrow a\cos\left(\frac{\pi}{2} + \alpha\right) = -3$

$a\sin\alpha = 3$

(ii)

$a^2(\cos^2\alpha + \sin^2\alpha) = 4^2 + 3^2$

$\therefore\ a^2 = 25 \qquad \therefore\ a = 5$

$\frac{a\sin\alpha}{a\cos\alpha} = \frac{3}{4} \Rightarrow \tan\alpha = \frac{3}{4} \qquad \therefore \alpha \approx 0\cdot 64$

(c)(i) The number of arrangements of B, B, V, V is ${}^4C_2 = \frac{4!}{2!\,2!} = 6$. Hence

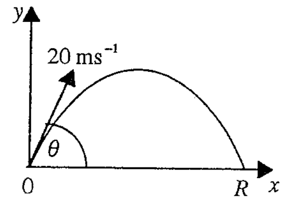
$P(B\ wins\ two\ and\ V\ wins\ two) = 6\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^2 = \frac{8}{27}$

(c) (ii) B and V must each win 2 of the first 4 sets, then B must win the 5'th set. Hence

$P(B\ wins\ three\ and\ V\ wins\ two) = \frac{8}{27} \times \frac{2}{3} = \frac{16}{81}$

Question 7

(a)(i)



Initial conditions

$x = 0 \qquad \dot{x} = 20\cos\theta$

$y = 0 \qquad \dot{y} = 20\sin\theta$

Horizontal motion

$\ddot{x} = 0$

$\dot{x} = c_1$, c_1 constant

when $t = 0$, $\dot{x} = 20\cos\theta$

$\therefore\ c_1 = 20\cos\theta$

$\therefore\ \dot{x} = 20\cos\theta$

$x = 20t\cos\theta + c_2$

when $t = 0$, $x = 0$

$\therefore\ c_2 = 0$

$\therefore\ x = 20t\cos\theta$

Vertical motion

$\ddot{y} = -10$

$\dot{y} = -10t + c_3$, c_3 constant

when $t = 0$, $\dot{y} = 20\sin\theta$

$\therefore\ c_3 = 20\sin\theta$

$\therefore\ \dot{y} = 20\sin\theta - 10t$

$y = 20t\sin\theta - 5t^2 + c_4$

when $t = 0$, $y = 0$

$\therefore\ c_4 = 0$

$\therefore\ y = 20t\sin\theta - 5t^2$

(a)(ii) $x = R$ when $y = 0$

$20t\sin\theta - 5t^2 = 0$

$5t(4\sin\theta - t) = 0$

$\therefore$ for $y = 0$, $t = 4\sin\theta$

and $x = 20t\cos\theta$

$= 40(2\sin\theta\cos\theta)$

$\therefore R = 40\sin 2\theta$

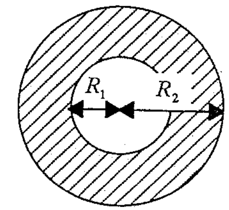
(a) (iii) If the horizontal range R varies so that $R_1 \le R \le R_2$, then the area watered is shaded in the diagram.

$15° \le \theta \le 45°$

$30° \le 2\theta \le 90°$

$0\cdot 5 \le \sin 2\theta \le 1$

$20 \le R \le 40$



Shaded area is $\pi(40^2 - 20^2)$

Area watered is $1200\pi\ \text{m}^2$.

(b) (i)

$v = \frac{1}{2}(1 - x^2)$

$\frac{dv}{dx} = -x \qquad \therefore\ a = v\frac{dv}{dx} = \frac{x^3 - x}{2}$

(b) (ii)

$$\frac{1}{1+x} + \frac{1}{1-x} = \frac{(1-x)+(1+x)}{(1+x)(1-x)} = \frac{2}{1-x^2}$$

$\frac{dx}{dt} = \frac{1-x^2}{2} \Rightarrow \frac{dt}{dx} = \frac{2}{1-x^2}$

$\frac{dt}{dx} = \frac{1}{1+x} + \frac{1}{1-x}$

$t = \ln(1+x) - \ln(1-x) + c$

when $t = 0$, $x = 0$ $\therefore\ c = 0$

$\therefore\ t = \ln\frac{1+x}{1-x}$

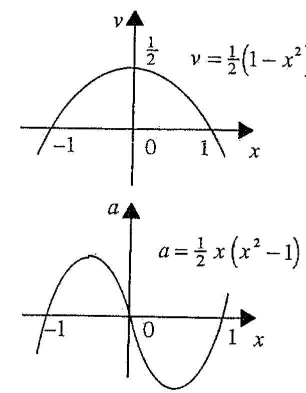
$\frac{1+x}{1-x} = e^t$

$1 + x = e^t - xe^t$

$x(e^t + 1) = e^t - 1$

$\therefore \quad x = \frac{e^t - 1}{e^t + 1} = \frac{1 - e^{-t}}{1 + e^{-t}}$

(b) (iii)



Initially the particle is at O, moving right at speed of $0\cdot 5\text{ms}^{-1}$ and slowing down (since v and a have opposite signs for $0 < x < 1$).

The particle continues to move right while slowing down for $x < 1$. As $t \to \infty$, $x \to \frac{1-0}{1+0} = 1$.

Its limiting position is 1 m to the right of O.