CATHOLIC SECONDARY SCHOOLS' ASSOCIATION OF NEW SOUTH WALES

YEAR TWELVE FINAL TESTS 1992

MATHEMATICS

2/3 UNIT

Morning session

Wednesday 19th August 1992

Time Allowed — Three Hours
(Including reading time)

EXAMINERS

B. Cosgrove
D. DiBona
C. Fernandes
J. Howard
C. Longhurst
R. Pantua
P. Rockett
S. Thomson
J. Wheatley

DIRECTIONS TO CANDIDATES :

* **ALL** questions may be attempted.
* **ALL** questions are of equal value.
* In every question, all necessary working should be shown.
* Marks may be deducted for careless or badly arranged work.
* Approved slide rules or calculators may be used.
* Table of standard integrals is provided.
* The answers to the ten questions in this paper are to be returned in separate writing booklets clearly marked **QUESTION 1, QUESTION 2,** etc. on the cover.
* If required, additional writing booklets may be obtained from the examinations supervisor upon request.

QUESTION 1 (Begin a new booklet)

(a) Calculate correct to 2 decimal places :

$$\frac{31\cdot 18 - \sqrt{40\cdot 7}}{8\cdot 5}$$

(b) Factorise fully :

$$a^2 - b^2 + 3a + 3b$$

(c) If $g(x) = x^2 + 1$

(i) Evaluate $g(-3)$

(ii) For what values of x is $g(x) = 2$?

(d) Graph on the number line the solution set of :

$$|3 - 2x| < 11$$

(e) After a discount of 40% is allowed, the cost of insuring a car is \$312. Find the cost of insuring this car when no discount is allowed.

Cont...

QUESTION 2 (Begin a new booklet)

In the diagram below, the lines $4y = 7x + 21$ and $4y = 31 - 3x$ intersect at the point B. Point A has coordinates $(-3,0)$ and point C has coordinates $(5,4)$.

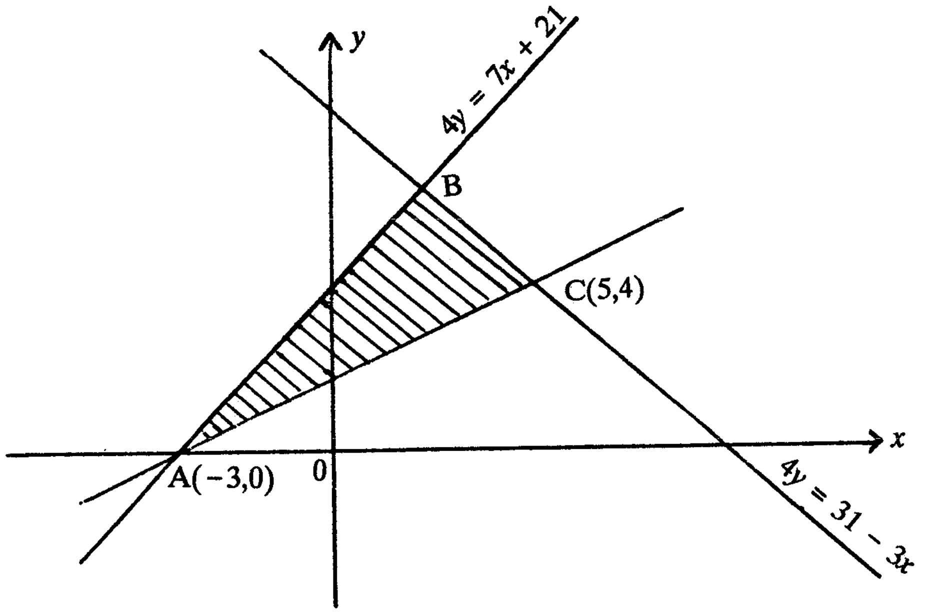


FIGURE NOT TO SCALE

(a) Calculate the gradient of AC.

(b) Show that the line AC has equation $2y = x + 3$.

(c) Show that B has coordinates $(1,7)$.

(d) Show that the perpendicular distance from B to the line AC is $2\sqrt{5}$ units.

(e) Find the exact length of the interval AC. Express your answer as a simplified surd.

(f) Find the area of Δ ABC.

Cont...

QUESTION 3 (Begin a new booklet)

(a) Solve $x + \frac{3 - x}{5} = 12$

(b) Differentiate with respect to x :

(i) $2x^3 - 5$

(ii) $(2x + 5)^3$

(iii) $\frac{3x^2}{\sin x}$

(c) Find the coordinates of the point on the curve $y = 3x^2 - 2x + 1$ where the tangent is parallel to the straight line $4x - y - 1 = 0$.

Cont...

QUESTION 4 (Begin a new booklet)

(a) Consider the series $3 + 8 + 13 + 18 + ... + 488$:

(i) how many terms are in this series?

(ii) find the sum of all the terms in this series.

(b) A parabola has equation $x^2 = 8(y + 2)$, find the :

(i) coordinates of the vertex and the focus.

(ii) equation of the directrix.

(iii) x intercepts of this parabola.

(c) Find the equation of the normal to the curve $y = \ln x^2$ at the point $x = 1$.

QUESTION 5 (Begin a new booklet)

(a)

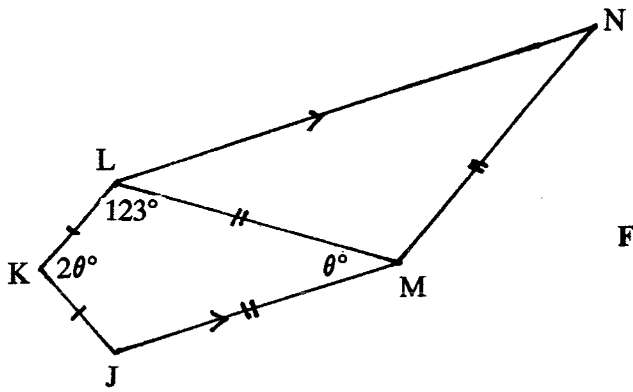


FIGURE NOT TO SCALE

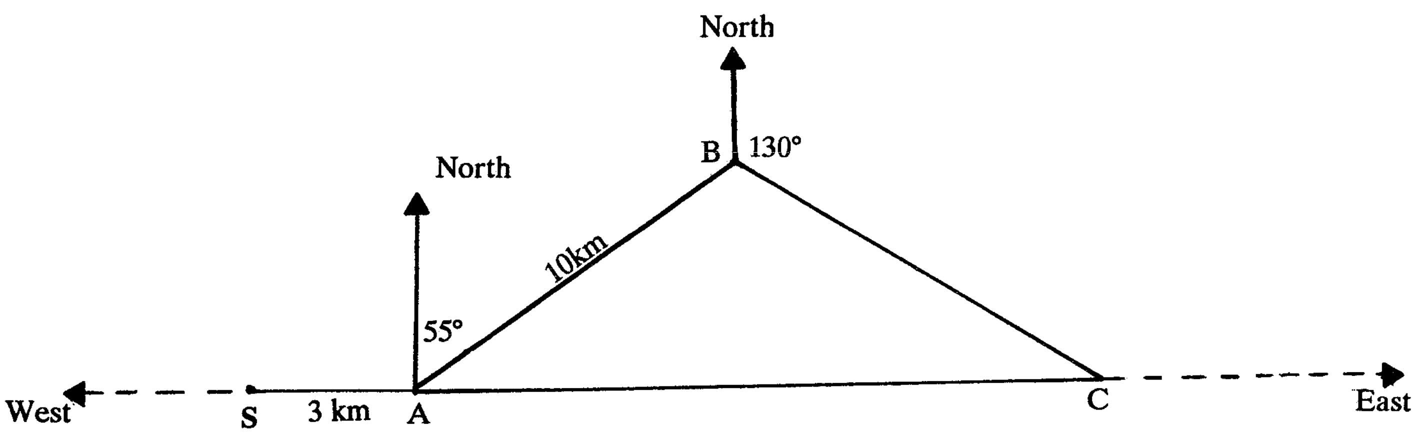
In the diagram above JKLM is a quadrilateral and LMN is a triangle.
JM ∥ LN, JK = KL, JM = ML = MN, ∠KLM = 123°, ∠JKL = 2θ° and ∠JML = θ°.

(i) Copy this diagram onto your answer sheet.

(ii) Show that ∠JML = 38° giving reasons.

(iii) Determine the size of ∠LNM giving reasons.

(b) The course for a trail bike competition is shown in the diagram below :



(FIGURE NOT DRAWN TO SCALE)

From the start S, Sharon rode 3 km due east to A. At A, she proceeded on a bearing of 055° for 10 km to B. At B, she changed course to a bearing of 130° and continued in this direction until she reached the finish at C. (C is due east of the start S and A).

(i) Copy this diagram onto your answer sheet.

(ii) Show that ∠ACB = 40°.

(iii) Use the sine rule to find the distance from B to C. Give your answer to the nearest km.

(iv) It took Sharon 24 minutes to travel from the start to the finish. What was her average speed in km/h?

Cont...

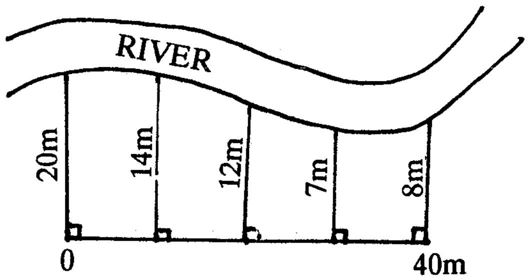
QUESTION 6 (Begin a new booklet)

(a) Find the following :

(i) $\int (1 + \frac{4}{x})\, dx$

(ii) $\int 2 \sec^2 3x\, dx$

(b) The diagram below shows a paddock with one side bounded by a river.



(FIGURE NOT TO SCALE)

Use Simpson's Rule with the five function values shown on the diagram to approximate the area of the paddock.

(c) PIN numbers are used for electronic banking. They consist of 4 digits with no restrictions on the 4 digits. (e.g. 5222, 8383, 0126, etc.) James remembered that the first two digits of his PIN number were 4 and 7 but he forgot the last two digits.

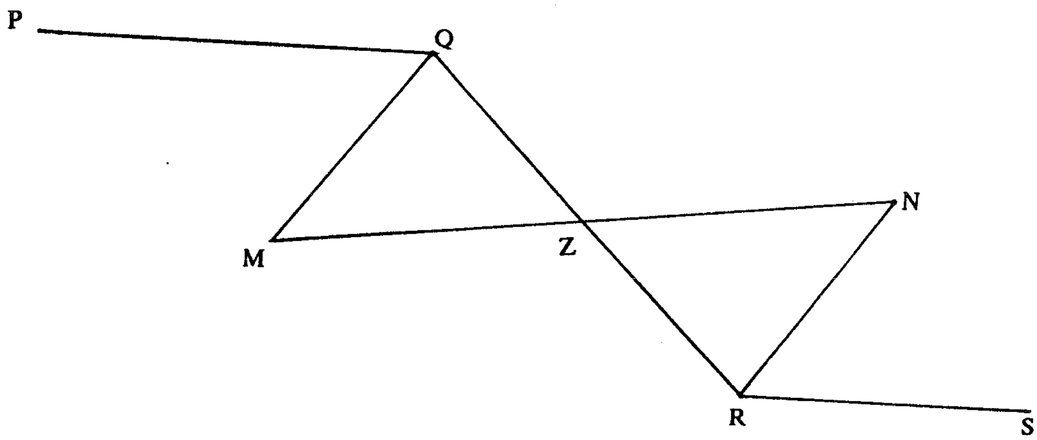
(i) What is the probability that he randomly guessed both of the next two digits correctly?

(ii) James knew that his PIN number was even. What is the probability that he can guess the correct PIN number?

Cont...

QUESTION 7 (Begin a new booklet)

(a)



In the given diagram PQ ∥ RS. MQ bisects ∠PQR, NR bisects ∠QRS and MQ = NR.

(i) Copy this diagram onto your answer sheet and mark on it all the given information.

(ii) Explain how you know that ∠MQZ = ∠NRZ

(iii) Prove that ΔQMZ ≡ ΔRNZ.

(iv) Hence prove that the intervals QR and MN bisect each other.

(b) When Susan was born her father deposited \$130 into a Trust Account earning 12% p.a. interest compounded annually. He decided to deposit \$130 into this account each time Susan had a birthday and he made his last payment on her seventeenth birthday.

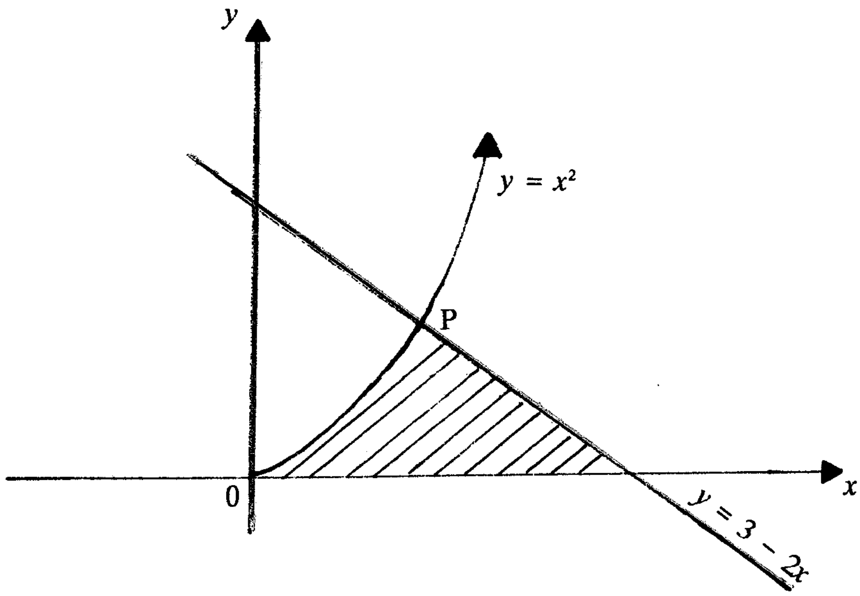
(i) Show that the initial deposit of \$130 amounted to \$999.70 on her eighteenth birthday.

(ii) Calculate the total amount that was in the account on Susan's eighteenth birthday.

Cont...

QUESTION 8 (Begin a new booklet)

(a)



(FIGURE NOT TO SCALE)

The diagram shows the parabola $y = x^2$ and the line $y = 3 - 2x$ intersecting at the point P, in the first quadrant.

(i) Show that the coordinates of the point P are (1,1).

(ii) The shaded region is rotated about the x axis. Find the volume of the solid formed.

(b) A particle moves so that its velocity, v metres per second, at any time t is given by :

$$v = e^{-2t}$$

Initially the particle is at $x = 2$.

(i) Find the acceleration, a, of the particle as a function of time.

(ii) What is the acceleration of the particle after 1 second?

(iii) Find an expression for the displacement, x metres, of the particle in terms of t.

(iv) Find the distance the particle travelled during the first two seconds.

(v) Describe what happens to the velocity of the particle for large values of t.

Cont...

QUESTION 9 (Begin a new booklet)

(a) For the equation $x^2 + (k+6)x - 2k = 0$ find the :

(i) discriminant in terms of k.

(ii) values of k for which this equation has real roots.

(b) Find the domain and range of $f(x) = \sqrt{1 - x^2}$.

(c) The gradient function of a curve $y = f(x)$ is given by :

$$f'(x) = x^2 (3 - x)$$

(i) Show that the curve has two stationary points and determine their nature.

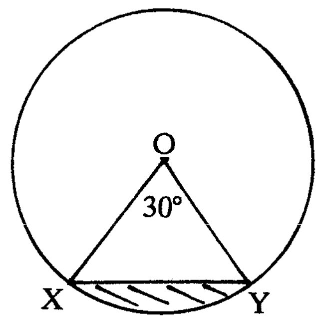
(ii) If $f(0) = 2$ and $f(x)$ has a maximum value of $8\frac{3}{4}$,

sketch $y = f(x)$

Cont...

QUESTION 10 (Begin a new booklet)

(a) The diagram shows an arc, XY, of length 10πcm subtending an angle of 30° at O, the centre of the circle.

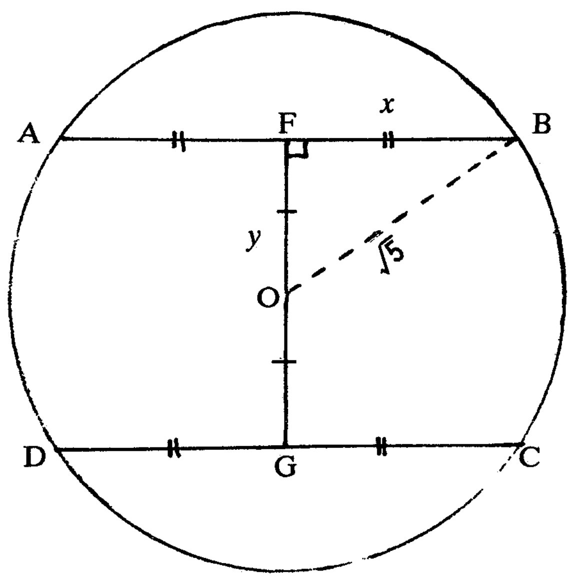


(i) Show that the radius of the circle is 60cm.

(ii) Calculate the shaded area shown in the diagram.

(b) A circular stained glass window of radius $\sqrt{5}$ metres requires metal strips for support along AB, DC and FG.

O is the centre of the window.



(i) Copy the diagram and information onto your answer sheet.

(ii) If OF = OG = y metres and FB = x metres, find an expression for y in terms of x.

(iii) The total length of the strips of metal used for support (i.e. AB+ DC + FG) is L metres. Show that :

$$L = 4x + 2\sqrt{5 - x^2}$$

(iv) The window will have maximum strength when the length of the supports is a maximum. Show that when FB = 2 metres, the window will have maximum strength.

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

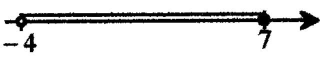
$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

(1)(a) 2.92

(b) $(a+b)(a-b+3)$

(c) (i) 10 (ii) $x=\pm 1$

(d) 

(e) \$520

(2) (a) $\frac{1}{2}$

(b) Proof

(c) Proof

(d) Proof

(e) $4\sqrt{5}$

(f) 20 units^2

(3)(a) $14\frac{1}{4}$

(b) (i) $6x^2$

(ii) $6(2x+5)^2$

(iii) $\dfrac{3x(2\sin x - x\cos x)}{\sin^2 x}$

(c) $(1,2)$

(4) (a) (i) $n=98$

(ii) 24059

(b) (i) $V(0,-2), S(0,0)$

(ii) $y=-4$

(iii) $x=\pm 4$

(c) $x+2y-1=0$

(5) (i) Diagram

(ii) Proof

(iii) $\angle LNM = 38^0$

(b) (i) Diagram

(ii) Proof

(iii) $BC = 9$ km

(iv) 55 km/hr

(6) (a) (i) $x+4\ln x+c$

(ii) $\frac{2}{3}\tan 3x + c$

(b) $453\frac{1}{3}$ units^2

(c) (i) $\frac{1}{100}$ (ii) $\frac{1}{20}$

(7)(a)(i) Diagram

(ii) $\angle PQR = \angle SRQ$, $PQ \| RS$
But $\angle PQM = \angle QMZ$ and
$\angle ZRN = \angle NRS$ (given).
$\angle MQZ = \angle NRZ$

(iii) Angle, Angle, Side Test

(iv) Proof

(b) (i) $130(1.12)^{18}$

(ii) \$7247.46

(8) (a) (i) Proof

(ii) $\frac{\pi}{3}+\frac{3}{4} = 1.38$ u^3 (to 2 d.p)

(b) (i) $-2e^{-2t}$

(ii) $-\frac{2}{e^2} = 0.27$ ms^{-2}

(iii) $x = \dfrac{5-e^{-2t}}{2}$

(iv) $\frac{1}{2}\left(1-\frac{1}{e^4}\right)$

(v) As $t\to\infty, v\to 0$

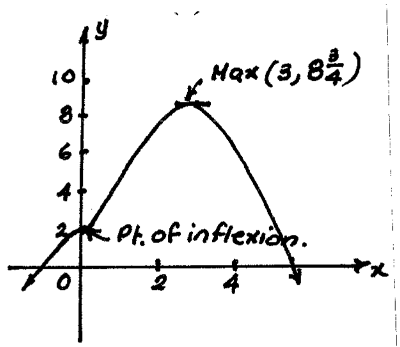
(9) (a)(i) $\Delta = k^2+20k+36$

(ii) $k\le -18$ and $k \ge -2$

(b) $D: -1\le x\le 1$
$R: 0\le y\le 1$

(c) (i) $x=0$, Pt. of inflexion

$x=3$, rel. max.

(ii) 

(10) (a)(i) Proof

(ii) $300\pi - 900 = 42.48$ cm^2

(b)(i) Diagram

(ii) $y=\sqrt{5-x^2}$

(iii) Proof

(iv) Proof