CATHOLIC SECONDARY SCHOOLS' ASSOCIATION OF NEW SOUTH WALES

YEAR TWELVE FINAL TESTS 1997

MATHEMATICS

2/3 UNIT

Morning session

Wednesday 6th August 1997.

Time Allowed — Three Hours
(Plus 5 minutes reading time)

EXAMINERS

K. Breen
B. Cosgrove
M. Donaghy
C. Fernandes
A. Kollias
J. McMillan
J. Mann
R. Pantua
E. Rainert
P. Rockett
J. Wheatley

DIRECTIONS TO CANDIDATES :

* **ALL** questions may be attempted.
* **ALL** questions are of equal value.
* All necessary working should be shown in every question.
* Full marks may not be awarded for careless or badly arranged work.
* Approved slide rules or calculators may be used.
* Table of standard integrals is provided.
* The answers to the ten questions in this paper are to be returned in separate writing sheets clearly marked **QUESTION 1, QUESTION 2** etc. on the top of the sheet..
* If required, additional writing sheets may be obtained from the examinations supervisor upon request.

QUESTION 1 **Use a *separate* writing sheet** **Marks**

(a) Find the value of $\log_3 9$ **1**

(b) Simplify $\frac{2x}{3} - \frac{x+2}{5}$ **2**

(c) Find the primitive function of $4 + \sqrt{x}$ **2**

(d) Find $\int (x-7)^5 \, dx$ **1**

(e) Express $\frac{7}{\sqrt{3} - 2}$ with a rational denominator. **2**

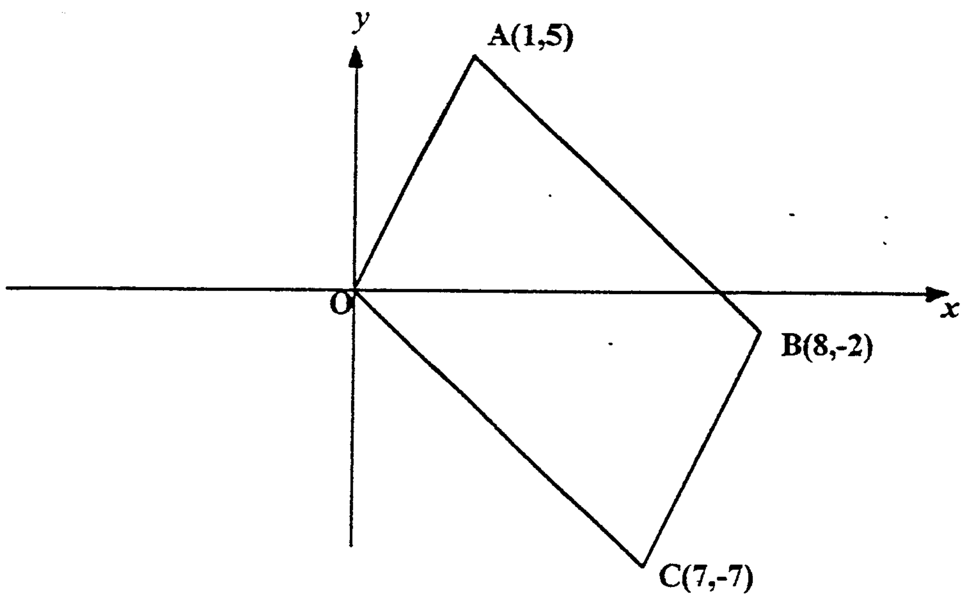
(f) Find the limiting sum of the series **2**

$$1 + \frac{3}{4} + \frac{9}{16} + \frac{27}{64} + \ldots$$

(g) How much will \$6 000 accumulate to at the end of five years if it is invested in a fund which pays an interest rate of 4% p.a. compounded quarterly? **2**

QUESTION 2 Use a *separate* **writing sheet** **Marks**

In the diagram O(0,0), A(1,5), B(8,−2) and C(7,−7) are the vertices of quadrilateral OABC.



(DIAGRAM NOT DRAWN TO SCALE)

(a) Find the midpoint of the interval joining AC. **1**

(b) Find the gradient of AB. **1**

(c) Show that the equation of AB is $x \div y = 6$ **1**

(d) Find the exact length of AB. **2**

(e) Show that AB is parallel to OC. **1**

(f) Explain why OABC is a parallelogram. **2**

(g) Find the exact perpendicular distance from O to AB. **2**

(h) Hence find the area of parallelogram OABC. **2**

QUESTION 3 **Use a *separate* writing sheet** **Marks**

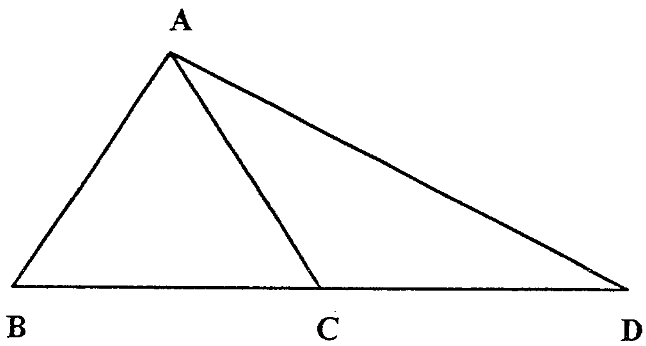
(a) Differentiate: **6**

(i) $\sqrt[3]{x}$

(ii) $(e^{x} - e^{-x})^2$

(iii) $ln\left(\dfrac{x+2}{x-1}\right)$

(b) **4**



(DIAGRAM NOT DRAWN TO SCALE)

ABC is an equilateral triangle. BC is produced to D so that BC = CD.

(i) Copy the diagram onto your answer sheet and mark on it all given information.

(ii) Prove that $\angle BAD = 90°$

(c) Evaluate $\displaystyle\int_{2}^{3} e^{2x-4}\,dx$ **2**

QUESTION 4 **Use a *separate* writing sheet** **Marks**

(a) Two cards are chosen at random from the four cards shown below. **2**

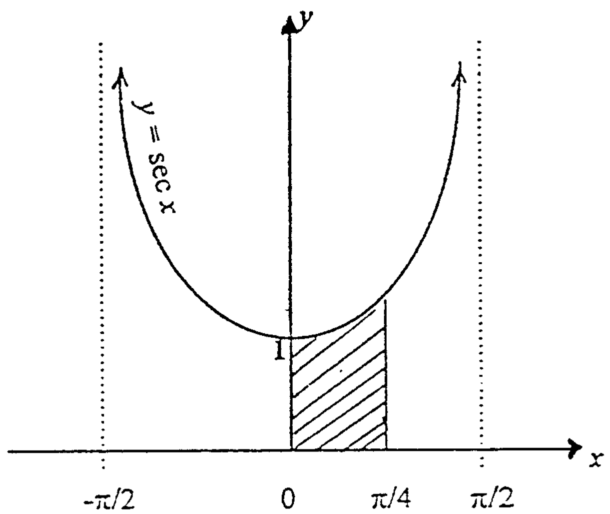
$\boxed{-1}$ $\boxed{0}$ $\boxed{1}$ $\boxed{2}$

(i) Using a tree diagram, or otherwise, list all the possible outcomes.

(ii) What is the probability that the sum of the numbers on the cards chosen is zero?

(b) Find the equation of the normal to the curve $y = x \sin x$ at the point $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$. **4**

(c) **3**

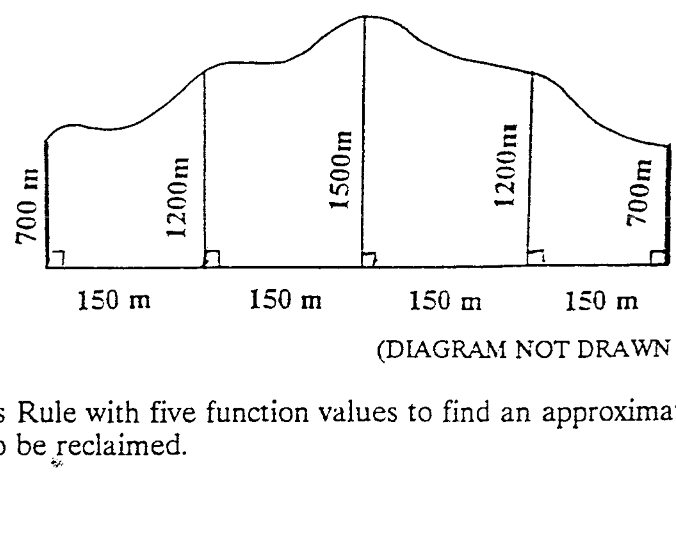


(DIAGRAM NOT DRAWN TO SCALE)

The shaded region which lies between the x axis and the curve $y = \sec x$ from $x = 0$ to $x = \frac{\pi}{4}$ is rotated about the x axis to form a solid.

Find the volume of the solid.

(d) Prime land along a foreshore is to be reclaimed and developed as part of a housing estate. A plan of the land to be reclaimed is shown below: **3**

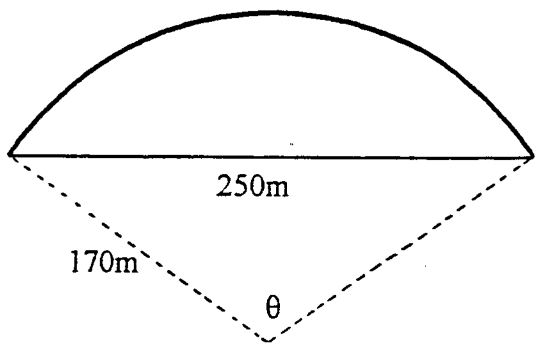


(DIAGRAM NOT DRAWN TO SCALE)

Use Simpson's Rule with five function values to find an approximation of the area of land to be reclaimed.

QUESTION 5 Use a *separate* writing sheet **Marks**

(a) A straight road was constructed to cut a dangerous bend on a country road. It was found that the bend was part of an arc of radius 170 metres and the straight road was 250 metres long. 5



(DIAGRAM NOT DRAWN TO SCALE)

(i) Use the cosine rule to find the size of θ correct to the nearest degree.

(ii) Find the distance by which the old road was shortened. Answer correct to the nearest metre.

(b) For the curve $y = 2x^3 - 6x^2 - 18x + 1$: 7

(i) find the stationary points and determine their nature.

(ii) for what values of x is the curve rising?

(iii) sketch the curve in the domain $-2 \le x \le 5$

QUESTION 6 **Use a *separate* writing sheet** **Marks**

(a) The population of a small country town is decreasing at an increasing rate. Given that N is the population of the town at a given time t, what does this statement imply about $\frac{dN}{dt}$ and $\frac{d^2N}{dt^2}$? **2**

(b) During a week, the expected amount of time A minutes that a high school student spends on Maths homework is determined by the equation: **5**

$$A = 20e^{kt} \text{ where } t \text{ is the year the student is in.}$$

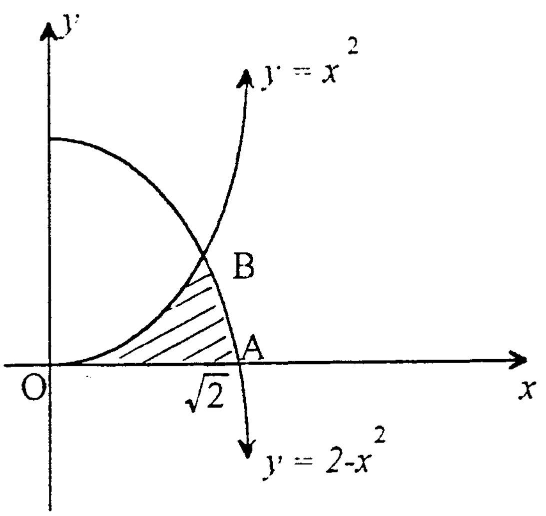
In Year 7, students are expected to spend a total of 120 minutes on Maths homework each week.

(i) Find the value of k correct to 3 decimal places.

(ii) At what rate is the expected amount of time spent on Maths homework increasing when a student is in Year 10? Answer correct to the nearest minute/year.

(iii) How many *more* minutes is a Year 12 student expected to spend on Maths homework than a Year 7 student. (Answer correct to the nearest integer)

(c)



(DIAGRAM NOT DRAWN TO SCALE)

The shaded region OAB is bounded by the parabolas $y = x^2$ and $y = 2 - x^2$ and the x axis from $x = 0$ to $x = \sqrt{2}$ **5**

(i) B is the point of intersection of the two parabolas in the first quadrant. Find the co-ordinates of B.

(ii) Calculate the area of the region OAB. Give your answer correct to 2 decimal places.

QUESTION 7 **Use a *separate* writing sheet** **Marks**

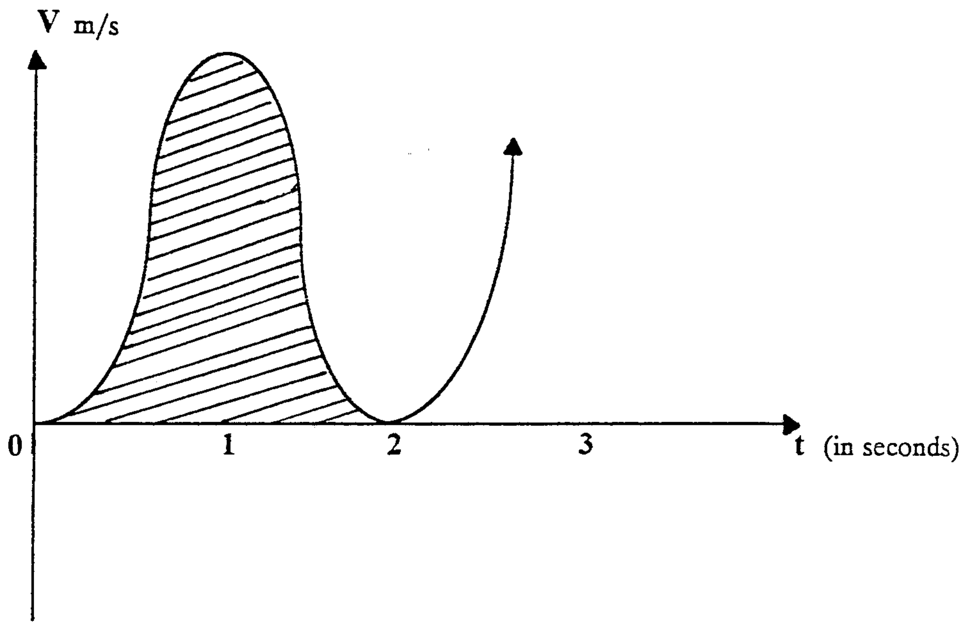
(a) The graph of $y = f(x)$ passes through the point (2,5) and $f'(x) = 6x^2 - 8$. Find the equation of the curve. **2**

(b) Jihan is planning to go on a holiday to Fiji in 12 months time. In the first month, she decided to deposit \$15 into a savings account. In the second month, she deposited \$30 into the account. She continued to increase the amount deposited by \$15 each month. **4**

(i) Show that she will deposit \$180 in the twelfth month.

(ii) How much will she have deposited into the account at the end of twelve months?

(c) **6**



The velocity-time graph of a particle starting from rest at the origin is shown in the diagram.

(i) When is the particle at rest again?

(ii) Does the particle change its direction during the motion? Give reasons.

(iii) Give the first time that the acceleration of the particle is zero.

(iv) Give a physical interpretation of the shaded area bounded by the t axis and the curve.

(v) Copy this diagram onto your answer sheet.

(vi) On this diagram, change the vertical axis to x **and** sketch the displacement-time graph of the particle.

QUESTION 8 **Use a *separate* writing sheet** **Marks**

(a) For the curve $y = ln\ (x - 2)$: 3

(i) write down its domain

(ii) sketch the curve

(b) (i) On the same coordinate axes, draw the graphs of $y = \cos 2x$ and $y = \sin x$ for $0 \le x \le 2\pi$ 3

(ii) Find the number of solutions to the equation $\cos 2x = \sin x$ in the domain $0 \le x \le 2\pi$

(c) Joe deposited \$20 000 at the beginning of January into an account which paid interest at the rate of ½% per month compounded monthly. He withdrew \$50 each month from the account immediately after the interest was paid. 6

(i) How much money did he have in the account immediately after making the first withdrawal?

(ii) Show that after making the nth withdrawal, his balance in the account is given by the expression:

$$\$(10\ 000 \times 1.005^n + 10\ 000)$$

(iii) Find the minimum number of withdrawals needed for his account balance to show at least \$50 000.

QUESTION 9 **Use a *separate* writing sheet** **Marks**

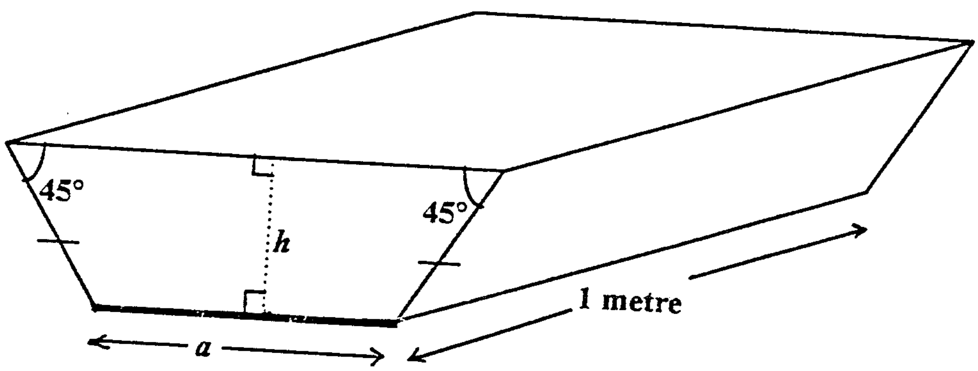
(a) Using $log_a 2 = 0.387$ and $log_a 3 = 0.613$, find the value of $log_a 12$. **2**

(b) In a touch football competition between the Red team and the Gold team, on average the Red team has won 3 games out of every 4 games. **3**

(i) Find the probability that the Red team wins the next two games.

(ii) In the next three games, what is the probability that the Red team wins more games than the Gold team?

(c) **7**



(DIAGRAM NOT DRAWN TO SCALE)

A trough of depth h metres and length 1 metre was constructed out of stainless steel sheeting. The cross-section of the trough was an isosceles trapezium with the acute angles being 45 ° each. The width of the bottom of the trough was a metres. The area of the cross-section measured 60 m^2.

(i) Show that $a = \frac{60}{h} - h$

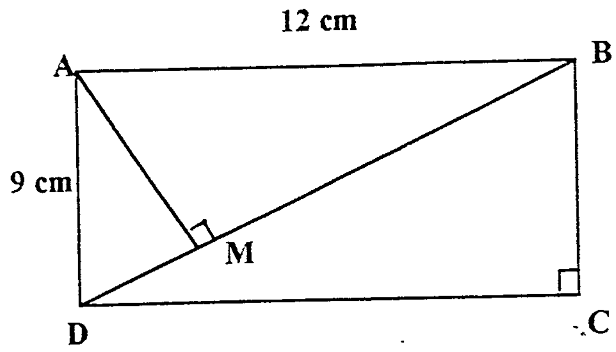
(ii) Show that the amount of stainless steel, A, in m^2, required to construct the trough was given by

$$A = \frac{60}{h} - h + 2h\sqrt{2} + 120$$

(iii) Find the depth of the trough, to the nearest mm, if the amount of stainless steel used is kept to a minimum.

QUESTION 10 **Use a *separate* writing Sheet** **Marks**

(a) **5**



(DIAGRAM NOT DRAWN TO SCALE)

ABCD is a rectangle with AB = 12cm, AD = 9cm and AM is perpendicular to BD.

(i) Copy the diagram onto your answer sheet.

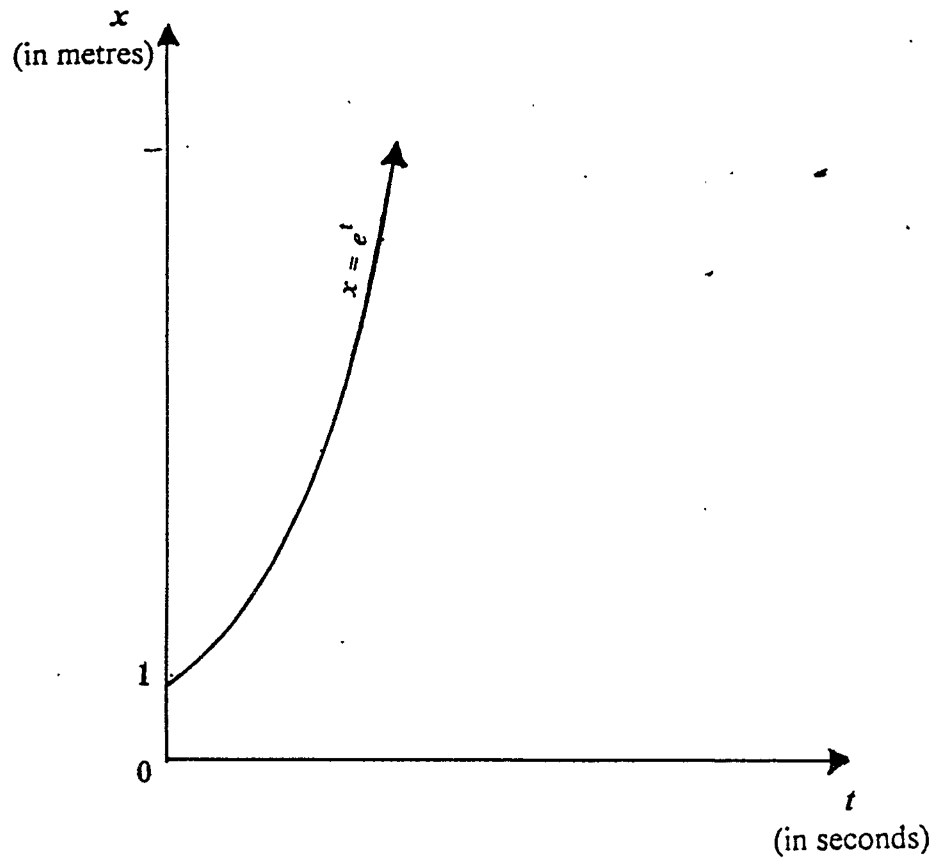
(ii) Find the length of BD.

(iii) Prove that $\Delta ABM \,|||\, \Delta DBA$.

(iv) Hence find the length of BM.

Marks

(b) Two particles P and Q are moving along a horizontal line. **7**
At any time t seconds, the position of particle P is given by $x = e^{t}$ and the position of particle Q is given by $x = 1 + 6e^{-t}$. The diagram below shows the position of particle P, x metres, at any time t seconds.



(DIAGRAM NOT DRAWN TO SCALE)

(i) As time increases indefinitely, what position does the particle Q approach?

(ii) On a pair of coordinate axes, sketch the path of particle Q.

(iii) Calculate the position where the two particles meet.

(iv) Explain why P and Q will never travel at the same velocity.

END OF EXAMINATION

CSSA 1997 – 2/3 UNIT MATHS – Suggested Solutions

QUESTION 1

a) $\log_3 9 = x$

$3^x = 9$

$3^x = 3^2$

$\therefore x = 2$ $\boxed{1}$

b) $\frac{2x}{3} - \frac{x+2}{5}$

$= \frac{10x - 3x - 6}{15}$

$= \frac{7x-6}{15}$ $\boxed{2}$

c) $\int 4 + \sqrt{x}\, dx$

$= \int 4 + x^{\frac{1}{2}}\, dx$

$= 4x + \frac{2x^{3/2}}{3} + C$ $\boxed{2}$

d) $\int (x-7)^5\, dx$

$= \frac{(x-7)^6}{6} + C$

e) $\frac{7}{\sqrt{3}-2} \times \frac{\sqrt{3}+2}{\sqrt{3}+2} = \frac{7(\sqrt{3}+2)}{(\sqrt{3})^2 - 2^2}$

$= \frac{7(\sqrt{3}+2)}{3-4}$

$= \frac{7(\sqrt{3}+2)}{-1}$

$= -7(\sqrt{3}+2)$

$= -7\sqrt{3} - 14$ $\boxed{2}$

f) $a = 1$, $r = \frac{3}{4}$

$S = \frac{a}{1-r}$

$= \frac{1}{1-3/4}$

$= \frac{1}{1/4}$

$= 4$ $\boxed{2}$

g) $P = 6000$

$r = 4\%$ pa

$= \frac{4}{4}\%$ per quarter

$= 1\%$

$n = 5$ yr

$= 5 \times 4$ quarters

$= 20$

$\therefore A = P\left(1 + \frac{r}{100}\right)^n$

$= 6000(1.01)^{20}$

$= \$7321.14$

QUESTION 2

a) $A(1,5)$ $C(7,-7)$

Midpoint $x = \frac{1+7}{2}$ $y = \frac{5-7}{2}$

$= (4, -1)$ [1]

b) $A(1,5)$ $B(8,-2)$

gradient $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-2-5}{8-1}$

$= \frac{-7}{7}$

$= -1$ [1]

c) $y - y_1 = m(x - x_1)$

$y - 5 = -1(x-1)$

$y - 5 = -x + 1$

$\therefore x + y = 6$ [1]

d) $AB = \sqrt{(1-8)^2 + (5+2)^2}$

$= \sqrt{49+49}$

$= \sqrt{98}$

$= 7\sqrt{2}$ units [2]

e) gradient OC $= \frac{0+7}{0-7}$

$= -1$

$M_{AB} = M_{OC}$

$\therefore$ lines are parallel [1]

f) $d_{OC} = \sqrt{(0-7)^2 + (0+7)^2}$

$= \sqrt{49+49}$

$= \sqrt{98}$

$= 7\sqrt{2}$ units

$= d_{AB}$

OABC is a parallelogram because one pair of opposite sides are parallel + equal [2]

g) $(0,0)$ $x + y - 6 = 0$

$d = \frac{|0+0-6|}{\sqrt{1+1}}$

$= \frac{6}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{6\sqrt{2}}{2}$

$= 3\sqrt{2}$ units

h) $A = b \times h$

$= 7\sqrt{2} \times 3\sqrt{2}$

$= 21 \times 2$

$= 42u^2$ [2]

QUESTION 3

a) (i) $\frac{d}{dx}\sqrt[3]{x}$

$= \frac{d}{dx} x^{\frac{1}{3}}$

$= \frac{1}{3}x^{-\frac{2}{3}}$

$= \frac{1}{3x^{2/3}}$ [2]

(ii) $\frac{d}{dx}(e^x - e^{-x})^2$

$= 2(e^x - e^{-x}).(e^x + e^{-x})$

$= 2(e^{2x} - e^{-2x})$

$= 2e^{2x} - 2e^{-2x}$ [2]

(iii) $\frac{d}{dx}\ln\left[\frac{x+2}{x-1}\right]$

$= \frac{d}{dx} \ln(x+2) - \ln(x-1)$

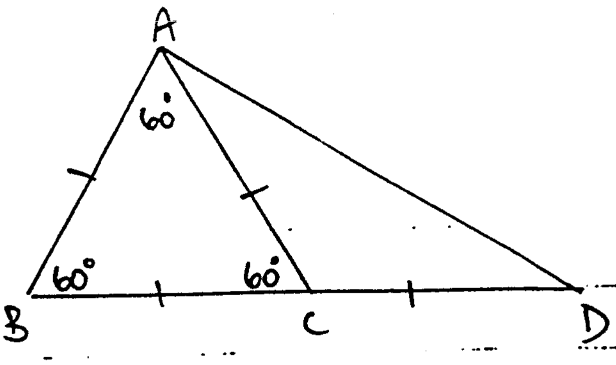
$= \frac{1}{x+2} - \frac{1}{x-1}$

$= \frac{x-1-x-2}{(x+2)(x-1)}$

$= \frac{-3}{(x+2)(x-1)}$ [2]

(b)

(i)



(ii) $\angle ACD = 180-60$

$= 120°$ (straight line)

$\triangle ACD$ is isosceles (AC = D

$\therefore \angle CAD = \angle CDA$ (base angl.

$= (180-120) \div 2$

$= 30°$

$\therefore \angle BAD = 60+30$

$= 90°$

c) $\int_2^3 e^{2x-4}\,dx$

$= \left[\frac{1}{2}e^{2x-4}\right]_2^3$

$= \frac{1}{2}\left[e^{6-4} - e^{4-4}\right]$

$= \frac{1}{2}\left[e^2 - e^0\right]$

$= \frac{e^2-1}{2}$

$[\doteqdot 3.2$ (to 2dp)$]$

QUESTION 4

a)

(i)

-1 0	1 -1
-1 1	1 0
-1 2	1 2
0 -1	2 -1
0 1	2 0
0 2	2 1

12 possible outcomes

(ii) $P = \frac{2}{12} = \frac{1}{6}$ [2]

b) $y = \underset{u}{x} \underset{v}{\sin x}$ $\left[\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\frac{dy}{dx} = \sin x . 1 + x . \cos x$

$= \sin x + x\cos x$

when $x = \frac{\pi}{2}$

$\frac{dy}{dx} = \sin\frac{\pi}{2} + \frac{\pi}{2} \times \cos\frac{\pi}{2}$

$= 1 + 0$

$= 1$

$\therefore$ gradient $= 1$

$\therefore$ gradient of normal $= -1$ $(m_1 m_2 = -1)$

eqn of normal: $y - \frac{\pi}{2} = -1(x - \frac{\pi}{2})$

$y - \frac{\pi}{2} = -x + \frac{\pi}{2}$

$x + y - \pi = 0$ [4]

c) $V = \pi \int y^2 \, dx$

$= \pi \int_0^{\frac{\pi}{4}} \sec^2 x \, dx$

$= \pi \left[\tan x\right]_0^{\frac{\pi}{4}}$

$= \pi \left[\tan\frac{\pi}{4} - \tan 0\right]$

$= \pi$ units^3 [3]

d) $A \doteqdot \frac{150}{3}\left[700 + 700 + 4(1200 + 1200) + 2(1500)\right]$

$\doteqdot 700\,000$ units^2

\

Question 5

a) (i) $\cos\theta = \dfrac{170^2 + 170^2 - 250^2}{2 \times 170 \times 170}$

$\theta \doteqdot -0.08$

$\doteqdot 95^\circ$ [2]

(ii) $\theta = \dfrac{95 \times \pi}{180}$ radians

$= \dfrac{19\pi}{36}$

$l = r\theta$

$= 170 \times \dfrac{19\pi}{36}$

$= 282$

$\therefore 282 - 250 = 32m$

old road was shortened by 32m [3]

b) $y = 2x^3 - 6x^2 - 18x + 1$

(i) $\dfrac{dy}{dx} = 6x^2 - 12x - 18$

$\dfrac{dy}{dx} = 0$ for st

$6(x^2 - 2x - 3) = 0$

$(x-3)(x+1) = 0$

$x = 3, -1$

$y = -53, 11$

$\therefore (3, -53)\ (-1, 11)$

$\dfrac{d^2y}{dx^2} = 12x - 12$

when $x = 3$, $\dfrac{d^2y}{dx^2} > 0$

$\therefore (3, -53)$ is a min

when $x = -1$

$\dfrac{d^2y}{dx^2} < 0 \therefore$

$(5, 11)$ is a max. [3]

(ii) the curve is rising when $\dfrac{dy}{dx} > 0$

need to solve

$(x-3)(x+1) > 0$

$\therefore x < -1$ or $x > 3$ [2]

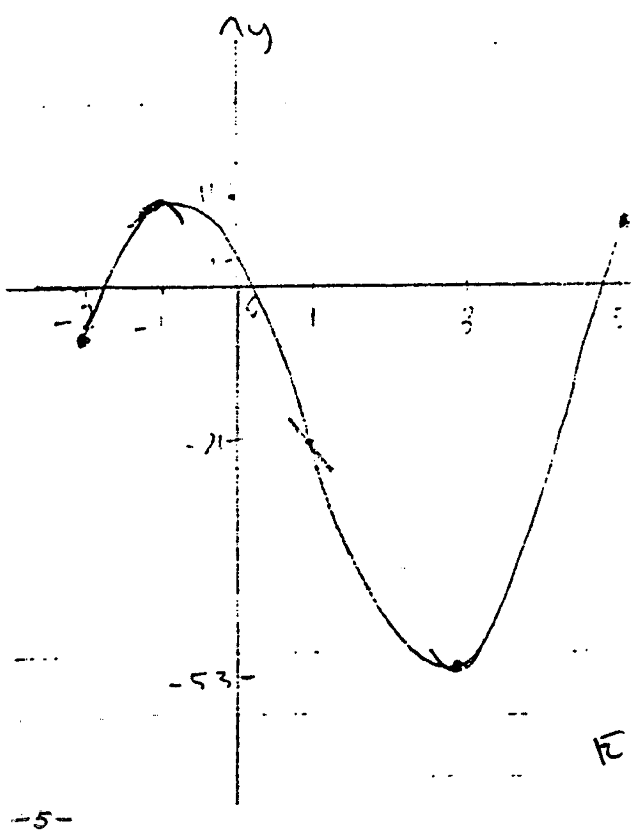
(iii) $-2 \le x \le 5$

when $x = -2$

$y = -3$

when $x = 5$

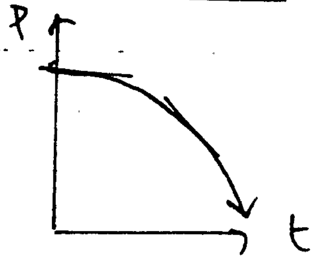
$y = 11$



[3]

QUESTION 6

(a)



$\frac{dN}{dt} < 0$ i.e population is decreasing

$\frac{d^2N}{dt^2} < 0$

[2]

b) $A = 20e^{kt}$

(i) $t = 7$, $A = 120$

$\therefore 120 = 20e^{7k}$

$e^{7k} = 6$

$7k = \ln 6$

$k = \frac{\ln 6}{7}$

$\doteqdot 0.255965638$

$= 0.256$ (3 dp)

[1]

(ii) $\frac{dA}{dt} = 20ke^{kt}$

when $t = 10$

$\frac{dA}{dt} = 20 \times \frac{\ln 6}{7} \times e^{\frac{\ln 6}{7} \times 10}$

$= 66.1997436\ldots$

$= 66$ min/year

[2]

(iii) when $t = 12$

$A = 20e^{12 \times \frac{\ln 6}{7}}$

$= 431.5526218\ldots$

$\therefore$ a Yr 12 student will spend $(431.55 - 120)$

312 min. more

(to the nearest min

c)(i) $y = x^2$, $y = 2 - x^2$

Solve

$x^2 = 2 - x^2$

$2x^2 = 2$

$x^2 = 1$

$x = \pm 1$

(first quadrant)

$\therefore x = 1$

$y = 1$

So B is $(1, 1)$

[2]

(ii) Area $= \int_0^1 x^2\,dx + \int_1^{\sqrt{2}} (2 - x^2)\,dx$

$= \left[\frac{x^3}{3}\right]_0^1 + \left[2x - \frac{x^3}{3}\right]_1^{\sqrt{2}}$

$= \frac{1}{3} + 2\sqrt{2} - \frac{2\sqrt{2}}{3} - 2 + \frac{1}{3}$

$= \frac{4\sqrt{2}}{3} - \frac{4}{3}$

$= 0.55228\ldots$

$= 0.55\,u^2$ (2dp)

QUESTION 7

a) $f'(x) = 6x^2 - 8$

$f(x) = 2x^3 - 8x + C$

when $x = 2$, $y = 5$ $\;(= f(x))$

$$2\times 2^3 - 8\times 2 + C = 5$$

$$16 - 16 + C = 5$$

$$\therefore C = 5$$

$\therefore f(x) = 2x^3 - 8x + 5$ [2]

b)(i) $T_1 = 15$

$T_2 = 15 + 15$

$T_3 = 15 + 15 + 15$

$\vdots$

$T_n = 15n$

$\therefore T_{12} = 15 \times 12$

$= \$180$

i.e. She will deposit \$180 in the twelfth month

(ii) $S_n = \frac{n}{2}(a + l)$

$= \frac{12}{2}(15 + 180)$

$= \$1170$

i.e she will have deposited \$1170 [4]

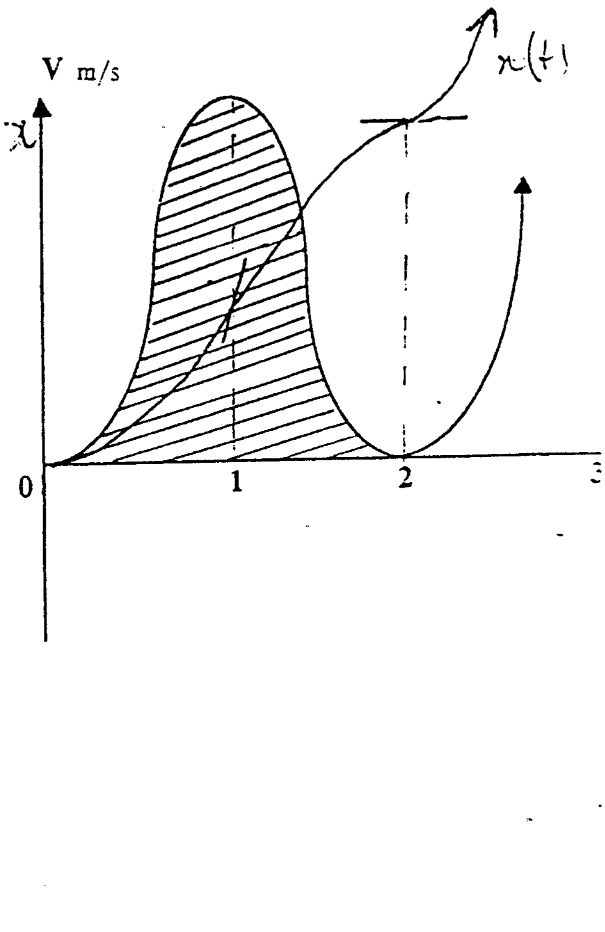
c) (i) after 2 seconds

(ii) No, the velocity does not change sign at $v=0$

(iii) $a = 0$ when $\frac{dv}{dt} = 0$ i.e where v–t graph has stationary points i.e after 1 second

(iv) The distance travelled during the first 2 seconds or the displacem of the particle after 2 seconds.

v, vi



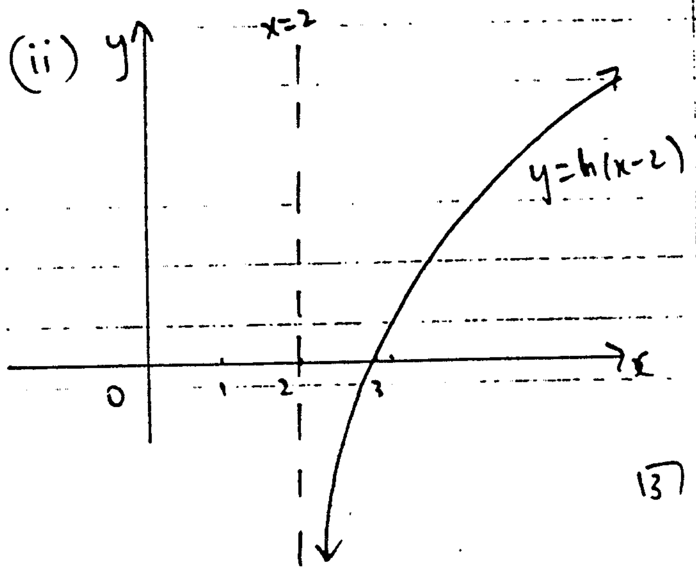
QUESTION 8

a) $y = \ln(x-2)$

Domain: $x - 2 > 0$

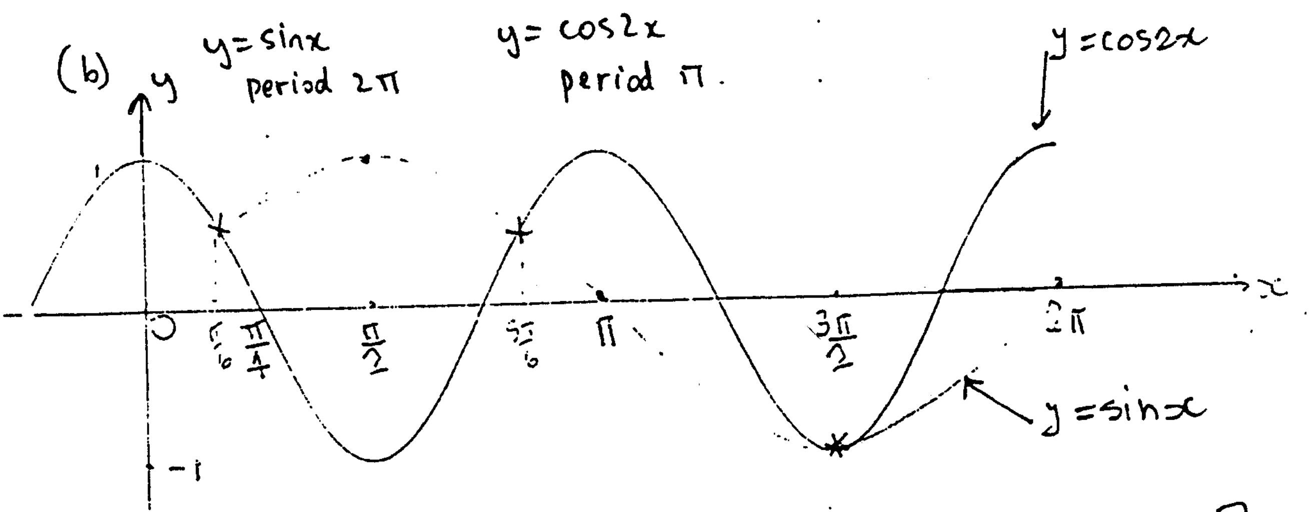
$x > 2$

(ii)



[3]

(b)



3 solutions

[3]

c)(i) Let A_n = amount in the account after nth withdrawl

$A_1 = 20000(1.005) - 50$

$= \$20050$

[1]

8 continued

(ii) $A_2 = [20000(1.005) - 50](1.005) - \$50$

$= 20000(1.005)^2 - 50(1.005 + 1)$

$\vdots$

$A_n = 20000(1.005)^n - 50(1.005^{n-1} + 1.005^{n-2} + \cdots + 1.005 +$

$= 20000(1.005)^n - 50\left[\dfrac{1.005^n - 1}{1.005 - 1}\right]$

$= 20000(1.005)^n - 10000(1.005^n - 1)$

$= 20000(1.005^n - 10000(1.005)^n + 10000$

$= 10000(1.005)^n + 10000$

(2)

(iii) Solve $A_n \geq 50000$

let $10000(1.005)^n + 10000 = 50000$

$10000(1.005)^n = 40000$

$1.005^n = 4$

$\therefore n\ln(1.005) = \ln 4$

$n = \dfrac{\ln 4}{\ln(1.005)}$

$= 277.951\ldots$

$\therefore n = 278$

Question 9

a) $\log_a 12 = \log_a(2^2 \times 3)$

$= \log_a 2^2 + \log_a 3$

$= 2\log_a 2 + \log_a 3$

$= 2 \times .387 + 0.613$

$= 1.387$

[2]

b) $P(\text{Red win}) = \frac{3}{4}$

$P(\text{Red loss}) = \frac{1}{4}$

(i) $P(WW) = \frac{3}{4} \times \frac{3}{4}$

$= \frac{9}{16}$

[1]

(ii) WWW ✓

WWL ✓

WLW ✓

WLL

LWW ✓

LWL

LLW

LLL

$P(WWW) + P(WWL) + P(WLW) + P(LWW)$

$= \left(\frac{3}{4}\right)^3 + 3\left(\frac{3}{4} \cdot \frac{3}{4} \cdot \frac{1}{4}\right)$

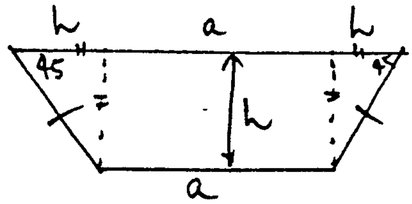
$= \frac{54}{64}$

$= \frac{27}{32}$

[2]

c) Area of cross section

$= \frac{1}{2}h(a + a + 2h)$



$A = h(a+h)$

now $A = 60$

$\therefore h(a+h) = 60$

$a + h = \frac{60}{h}$

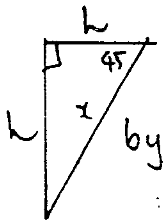
$a = \frac{60}{h} - h$

(ii) A = bottom

+ 2x sides

+ front + back.

now



by Pythagoras

$x = \sqrt{2}h$

$\therefore A = a + 2 \times \sqrt{2}h \times 1 + 60 \cdot$

$= \frac{60}{h} - h + 2\sqrt{2}h + 120$

(iii) $\frac{dA}{dh} = -\frac{60}{h^2} - 1 + 2\sqrt{2}$

Solve $\frac{dA}{dh} = 0$

$\frac{60}{h^2} = 2\sqrt{2} - 1$

$\frac{h^2}{60} = \frac{1}{2\sqrt{2} - 1}$

$$h^2 = \frac{60}{2\sqrt{2}-1}$$

$\therefore h = 5.7284\ldots$ m

$= 5728.44$ mm

$= 5728$ (nearest mm)

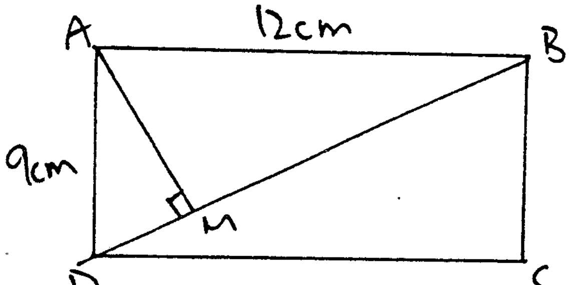
also, $\dfrac{d^2A}{dh^2} = \dfrac{120}{h^3} > 0$ for $h > 0$

i.e min.

[3]

QUESTION 10

1.



(ii) $BD^2 = AB^2 + AD^2$ (Pythagoras).

$\therefore BD^2 = 12^2 + 9^2$

$BD = 15$

(iii) In ΔABM, DBA

$\angle ABM = \angle DBA$ (common)

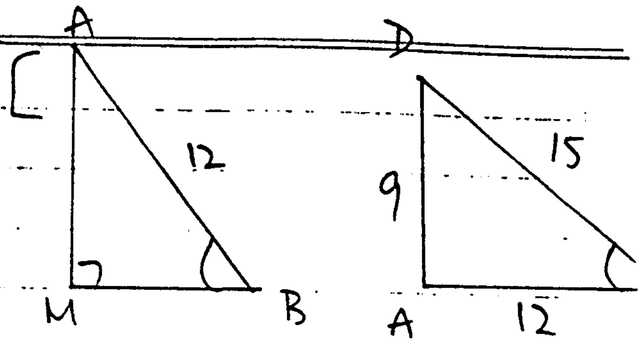
$\angle AMB = 90°$ (given)

$\angle DAB = 90°$ (angle of rectangle)

$= \angle AMB$

$\therefore \angle BAM = \angle BDA$ (remaining angles)

$\therefore \Delta ABM \,|||\, \Delta DBA$ (equiangular)



(iv) $\dfrac{BM}{BA} = \dfrac{AB}{DB}$ (corresp. side of similar Δ's)

$\therefore \dfrac{BM}{12} = \dfrac{12}{15}$

$BM = \dfrac{144}{15}$

$= 9\frac{3}{5}$ cm

(b)(i) As $t \to \infty$

$e^{-t} \to 0$

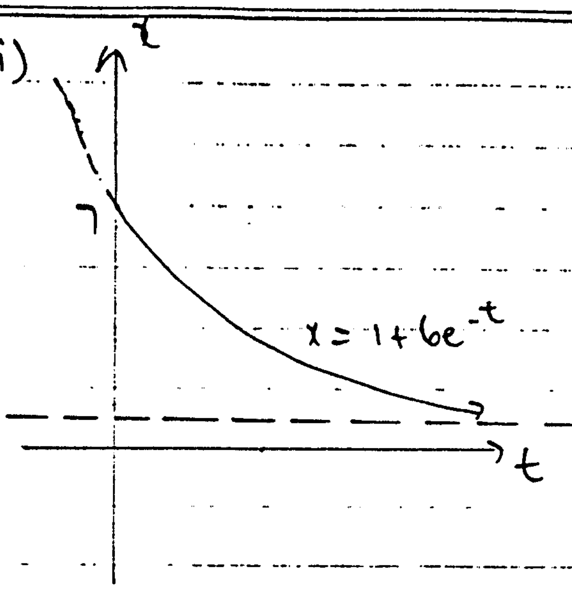
$\therefore$ as $t \to \infty$

$x = 1 + 6e^{-t}$

$\to 1$

i.e as time increases indefinitely (the) particle Q approaches the position 1 metre to the right of O.

(ii)



Q : $v = \frac{dx}{dt} = -6e^{-t} < 0$

Hence velocities cannot be equal.

<u>OR</u> they cannot have the same velocity because they are travelling in opposite directions. (1)

(iii) Particle meet when they have the same position

i.e solve $e^t = 1 + 6e^{-t}$

$$e^t = 1 + \frac{6}{e^t}$$

$$e^{2t} - e^t - 6 = 0$$

$$(e^t - 3)(e^t + 2) = 0$$

$$e^t = 3, \quad \underbrace{e^t = -2}_{\text{no solution}}$$

$e^t = 3$

$t = \ln 3$

or since $x = e^t$

$x = 3$

i.e particles meets 3m to the right of O.

(iv) P : velocity $= \frac{dx}{dt}$

$= e^t > 0$