CATHOLIC SECONDARY SCHOOLS' ASSOCIATION OF NEW SOUTH WALES

YEAR TWELVE FINAL TESTS 1996

MATHEMATICS

2/3 UNIT

Morning session

Wednesday 7th August 1996.

Time Allowed — Three Hours
(Plus 5 minutes reading time)

EXAMINERS

K. Breen
B. Cosgrove
M. Donaghy
C. Fernandes
A. Kollias
J. McMillan
J. Mann
R. Pantua
E. Rainert
P. Rockett
J. Wheatley

DIRECTIONS TO CANDIDATES :

* **ALL** questions may be attempted.
* **ALL** questions are of equal value.
* All necessary working should be shown in every question.
* Full marks may not be awarded for careless or badly arranged work.
* Approved slide rules or calculators may be used.
* Table of standard integrals is provided.
* The answers to the ten questions in this paper are to be returned in separate writing sheets clearly marked **QUESTION 1, QUESTION 2** etc. on the top of the sheet..
* If required, additional writing sheets may be obtained from the examinations supervisor upon request.

QUESTION 1 *(Begin a new sheet)*

Marks

(a) Factorise $5x^2 - 2x - 3$ 1

(b) Evaluate correct to one decimal place 2

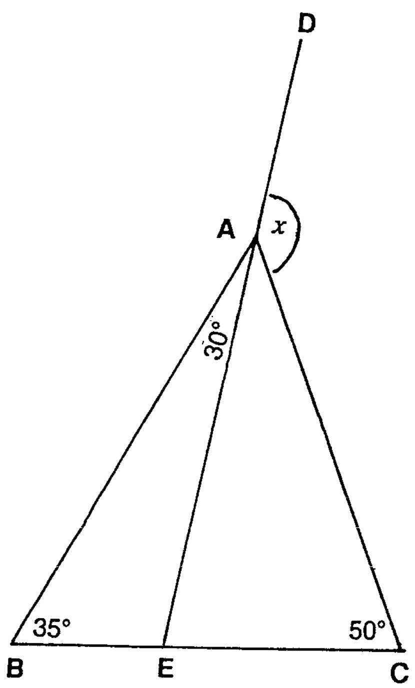
$$\frac{3 \cdot 24}{\sqrt{9 \cdot 75 - 3 \cdot 58}}$$

(c) Differentiate $4x^2 + 2$ 1

(d) Solve $|x - 5| = 6$ 2

(e) Simplify $\frac{2x - 3}{5} - \frac{3x + 4}{10}$ 2

(f) 2



NOT TO SCALE

Find the value of x

(g) Solve $2 - 3p < 7$ 2

Cont....

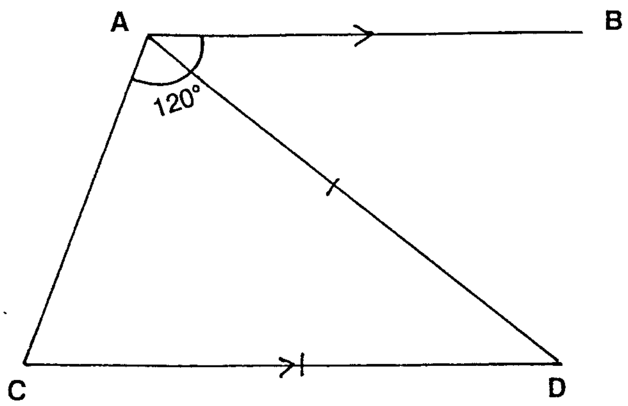
QUESTION 2 *(Begin a new sheet)* **Marks**

(a) Differentiate: 3

(i) $\sin(3x - 2)$

(ii) $\dfrac{e^x}{x}$

(b) 3



NOT TO SCALE

In the diagram, AB $\parallel$ CD, AD = CD and $\angle BAC = 120°$.

Copy the diagram onto your answer sheet.

(i) Explain why $\angle ACD = 60°$.

(ii) Show that Δ ADC is equilateral, giving reasons.

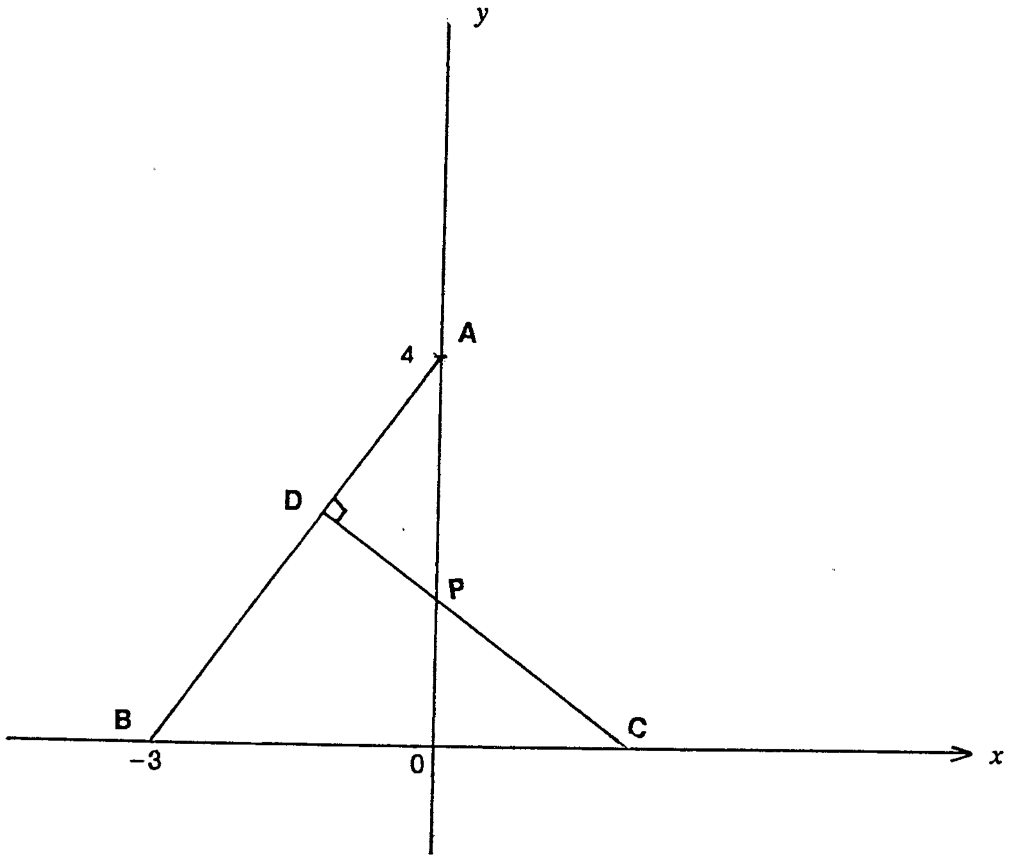
(c) Find 6

(i) $\int \sec^2 3x \, dx$

(ii) $\int (5x - 3)^5 \, dx$

(iii) $\displaystyle\int_{-1}^{0} \frac{dx}{2x + 3}$

QUESTION 3 *(Begin a new sheet)* **Marks**



NOT TO SCALE

In the diagram AB = BC and CD is perpendicular to AB.
CD intersects the y axis at P.

Copy the diagram onto your answer sheet.

(a) Find the length of AB. 1

(b) Hence show the co-ordinates of C are (2,0) 1

(c) Show the equation of CD is $3x + 4y = 6$ 3

(d) Show the co-ordinates of P are (0, 1½) 1

(e) Use Pythagoras' Theorem on Δ POC to show the length of CP is 2½ units. 1

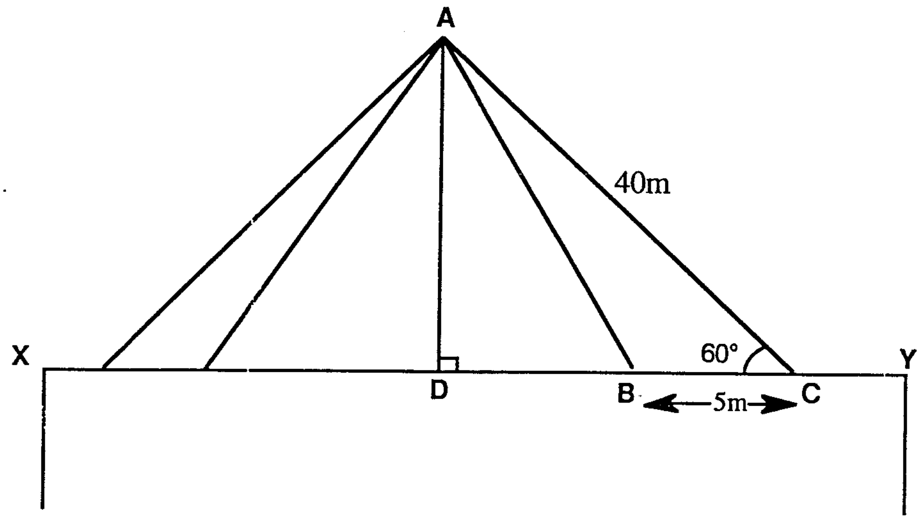
(f) Prove that Δ ADP is congruent to Δ COP. 3

(g) Hence calculate the area of the quadrilateral DPOB. 2

QUESTION 4 *(Begin a new sheet)* **Marks**

(a) Find the equation of the tangent to the curve $y = x \ln x$ at the point (1,0). 4

(b) 4



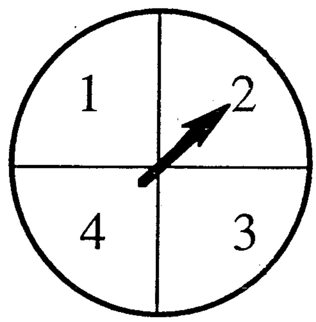
NOT TO SCALE

A horizontal bridge was built between points X and Y. Cables were used to support the bridge as shown in the diagram above. The distance between the cables AB and AC was 5 metres. Cable AC was 40 metres long and $\angle ACB = 60°$.

(i) Show that the height of A above the horizontal bridge is $20\sqrt{3}$ metres.

(ii) Use the cosine rule to show the exact length of the cable AB is $5\sqrt{57}$ metres.

(c) 4



Dino and Chris used the spinner shown above to play a game. Dino spun the spinner twice and added the results of the two spins to get his score. Chris then took his turn. The player with the highest score won the game.

(i) Use a tree diagram or a sample space to show all the possible scores Dino could have achieved when he played the game.

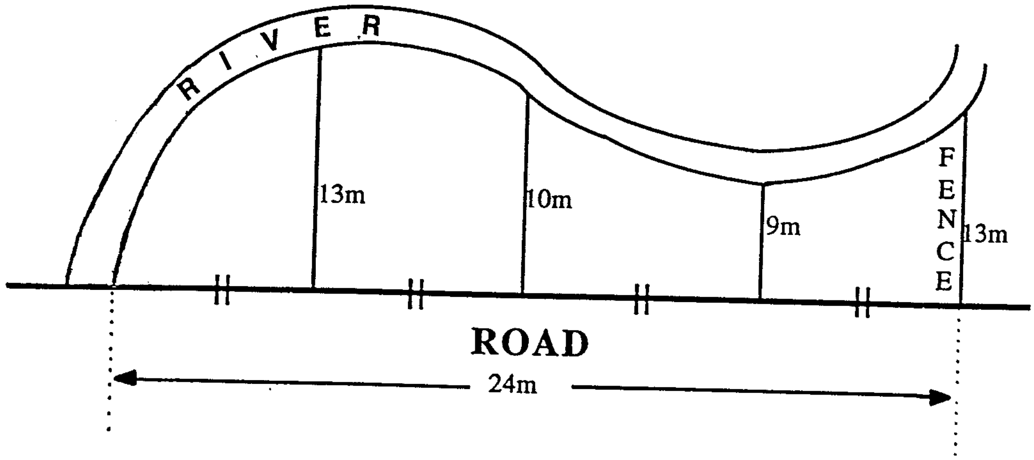
(ii) What is the probability that Dino scored 6 in the game?

(iii) Dino's score was 6. What is the probability that Chris won the game?

QUESTION 5 *(Begin a new sheet)*

Marks

(a) 4



NOT TO SCALE

Wasteland bordering a river bank and a straight road was fenced off and used as a recreational park. Perpendicular distances from the road to the river bank are shown on the diagram.

Use Simpson's Rule, with 5 function values, to approximate the area of the recreational park.

(b) A pool is being drained and the number of litres of water, L, in the pool at time t minutes is given by the equation: 4

$$L = 120(40 - t)^2$$

(i) At what rate is the water draining out of the pool when $t = 6$ minutes?

(ii) How long will it take for the pool to be completely empty?

(c) Alex accepted a job that pays an initial salary of \$28 000 per annum. After each year of service she will receive an increment of \$950 until she reaches the maximum salary of \$40 350. 4

(i) What will her salary be after 5 years of service?

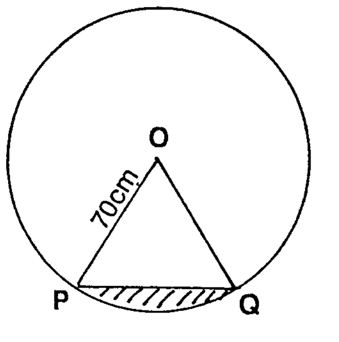
(ii) How long will she have to work before she reaches her maximum salary?

(iii) Calculate her total earnings for the first 10 years of service.

Cont....

QUESTION 6 *(Begin a new sheet)* **Marks**

(a) 4



NOT TO SCALE

In the diagram the length of the arc PQ is 30 cm.
The radius of the circle is 70 cm.

(i) Show that $\angle POQ = 25°$ to the nearest degree.

(ii) Find the shaded area correct to the nearest cm^2.

(b) The region bounded by the curve $y = x^3$, the y–axis and the line $y = 8$ is rotated about the y axis.

Find the volume of the solid formed. 3

(c) An island was found to be contaminated by fallout from a nuclear test explosion. One of the most dangerous components of nuclear fallout is strontium – 90. 5

The rate at which the amount, A, of strontium – 90 decays t years after an explosion is given by:

$$\frac{dA}{dt} = -kA$$

where k is a constant.

(i) Show that $A = A_0 e^{-kt}$, where A_0 is the amount of strontium – 90 present immediately after an explosion, satisfies the given equation.

(ii) It was found that 28 years after the explosion there was half the original amount of strontium – 90 present on the island. Find the value of k correct to 4 decimal places.

(iii) Another explosion has left 99% of the original amount of strontium – 90 present on an island. How many months ago did this explosion occur?

QUESTION 7 *(Begin a new sheet)* **Marks**

(a) The price of one tonne of copper, $P, was studied over a period of t years. 2

(i) Throughout the period of study $\frac{dP}{dt} > 0$. What does this say about the price of copper?

(ii) It was also observed that the rate of change of the price of copper decreased over the period of study.

What does this statement imply about $\frac{d^2P}{dt^2}$?

(b) The position, x cm, of a particle moving along an x-axis is given by: 6

$$x = 3t + e^{-2t}$$

where t is the time in seconds.

(i) What is the position of the particle when $t = \frac{1}{2}$ second?

(ii) What is the initial velocity of the particle?

(iii) Show the initial acceleration of the particle is 4 cm/s^2.

(iv) Explain why the particle will never come to rest.

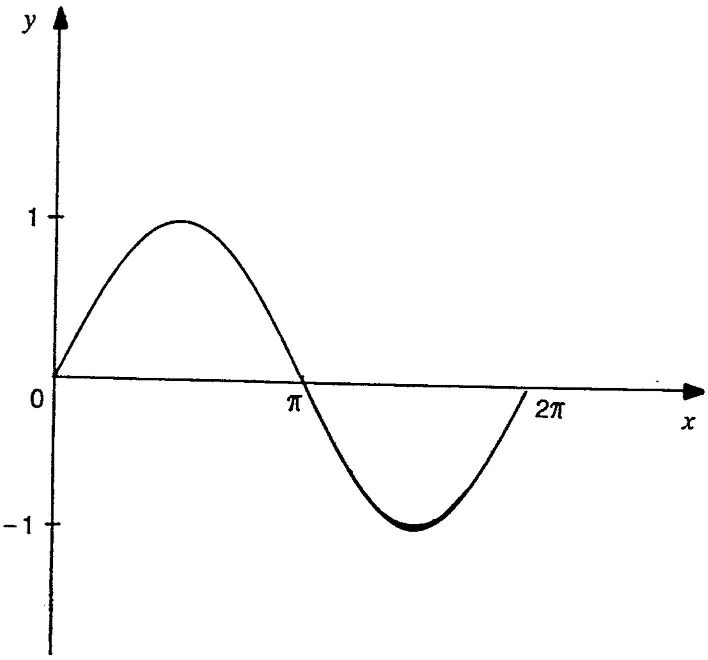
(c) Remy found a bank book belonging to her great-great-grandmother in an old cedar chest. 116 years ago, the balance in the account was $26. Remy calculated the savings account should be worth $3 738 today if the interest was compounded annually. What rate of interest did Remy use in her calculation? Give your answer correct to one decimal place. 4

QUESTION 8 *(Begin a new sheet)* **Marks**

(a) (i) Express $0 \cdot 1\dot{5}$ as an infinite series. 2

(ii) Hence express $0 \cdot 1\dot{5}$ as a fraction with no common factors.

(b) 2



NOT TO SCALE

The diagram shows the graph of $y = \sin x$ for $0 \leq x \leq 2\pi$
Copy the diagram onto your answer sheet.

(i) On the same co-ordinate axes, sketch the graph of $y = \cos 2x$ for $0 \leq x \leq 2\pi$.

(ii) Hence state the number of solutions in $0 \leq x \leq 2\pi$ to the equation

$$\sin x = \cos 2x$$

(c) Consider the curve given by the equation 8

$$y = 9x(x - 2)^2$$

(i) Find the co-ordinates of the stationary points and determine their nature.

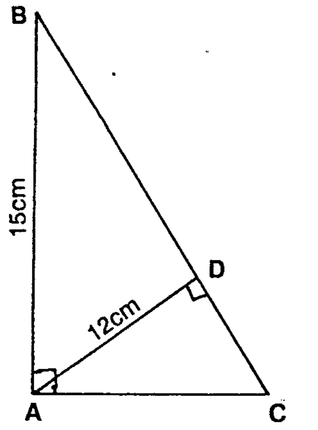
(ii) Find the co-ordinates of any points of inflexion.

(iii) Sketch the curve in the domain $-1 \leq x \leq 3$.

(iv) What is the maximum value of $9x(x - 2)^2$ in the domain $-1 \leq x \leq 3$?

QUESTION 9 *(Begin a new sheet)* **Marks**

(a) 5



Δ ABC is right-angled at A and AD is drawn perpendicular to BC.
AB = 15 cm and AD = 12 cm.

Copy the given diagram onto your answer sheet.

(i) Show that BD = 9 cm.

(ii) Prove that ΔABC is similar to ΔDBA.

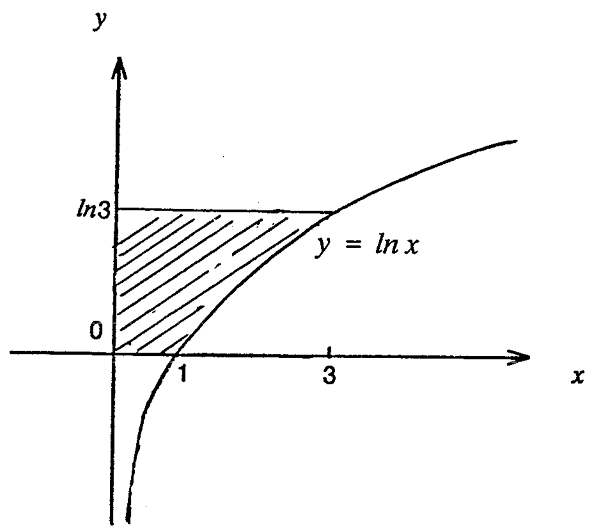
(iii) Hence find the length of AC.

(b) A horticulturist found the probability that a planted tomato seedling will eventually bear fruit was 0·85. He planted n seedlings. 3

(i) What is the probability that no seedlings will bear fruit?

(ii) How many seedlings must be planted to be at least 99% certain that at least one seedling will bear fruit?

(c) 4



NOT TO SCALE

The diagram shows the area bounded by the graph $y = \ln x$, the co-ordinate axes and the line $y = \ln 3$.

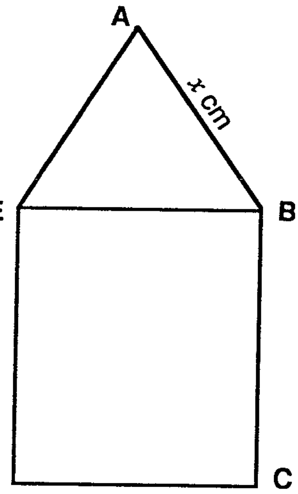
(i) Find the shaded area.

(ii) Hence find the exact value of $\int_1^3 \ln x \, dx$

Cont....

QUESTION 10 *(Begin a new sheet)* **Marks**

(a) 6



NOT TO SCALE

ABCDE is a pentagon of fixed perimeter P cm. In the figure triangle ABE is equilateral and BCDE is a rectangle. The length of AB is x cm.

(i) Show that the length of BC is $\dfrac{P - 3x}{2}$ cm.

(ii) Show that the area of the pentagon is given by:

$$A = \tfrac{1}{4}\left[2Px - (6 - \sqrt{3})x^2\right] \text{ cm}^2$$

(iii) Find the value of $\dfrac{P}{x}$ for which the area of the pentagon is a maximum.

(b) Two particles A and B are moving along a straight line so that their velocities in m/s at any time t seconds are given by: 6

$$V_A = t^2 + 1 \text{ and } V_B = 7 - 2t \text{ respectively.}$$

(i) Sketch, on the same co-ordinate axes, the velocity-time graph for both particles.

(ii) What is the significance of the point of intersection of the two graphs?

(iii) When have the two particles travelled exactly the same distance?

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

(1)(a) $(5x+3)(x-1)$

(b) 1.3

(c) $8x$

(d) $x = 11$ or -1

(e) $\frac{x-10}{10}$

(f) $x = 115^0$

(g) $p > -\frac{5}{3}$

(2) (a) (i) $3\cos(3x-2)$

(ii) $\frac{e^x(x-1)}{x^2}$

(b) (i) Cointerior angles are supplementary.

(ii) Proof

(c) (i) $\frac{1}{3}\tan 3x + c$

(ii) $\frac{(5x-3)^6}{30} + c$

(iii) $\frac{1}{2}\ln 3$

(3)(a) 5 units

(b) to (e) Proofs

(f) A.A.S Test

(g) $4\frac{1}{2}$ units^2

(4) (a) $x - y - 1 = 0$

(b) (i), (ii) Proofs

(c) (i) Tree diagram

(ii) $\frac{3}{16}$ (iii) $\frac{3}{16}$

(5) (a) 242 m^2

(b) (i) –8160 L/min

(ii) $t = 40$ mins

(c) (i) $32 750

(ii) 13 years

(iii) $322 750

(6) (a) (i) Proof

(ii) 34 cm^2

(b) $\frac{96\pi}{5}$ u^3

(c) (i) Proof

(ii) $k = 0.0248$ (to 3 d.p)

(iii) 4.9 months (to 1 d.p)

(7) (a) (i) The price is increasing.

(ii) $\frac{d^2P}{dt^2} < 0$

(b) (i) 1.868 (3 d.p.)

(ii) 1 cm/s

(iii) Proof

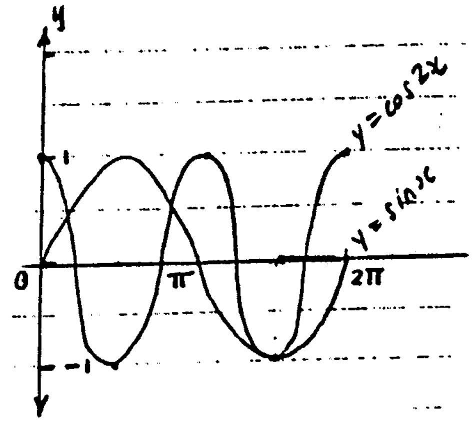
(iv) As $t \to \infty$, $e^{-2t} \to 0$, $v \to 3$

(c) 4.4%

(8) (i) $0.1 + 0.05 + 0.005 + \ldots$

(ii) $x = \frac{1}{10} + \frac{\frac{5}{100}}{1 - \frac{1}{10}} = \frac{7}{45}$

(b) (i)



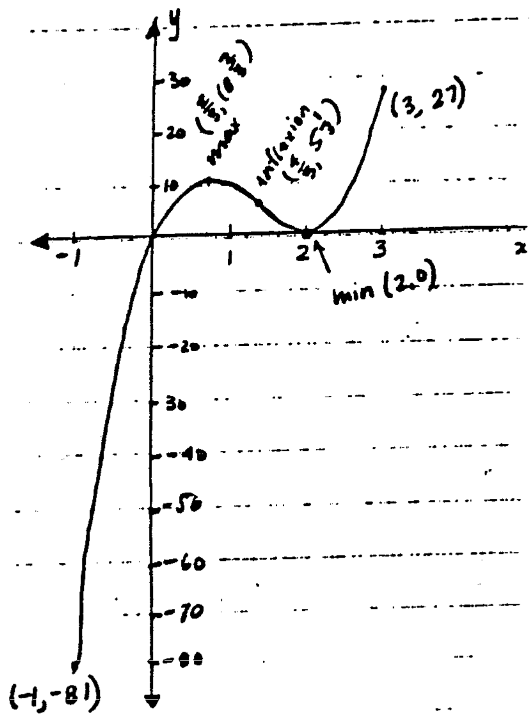
(ii) 3

(c)

(i) $(2,0)$ min $\left(\frac{2}{3}, 10\frac{2}{3}\right)$ max.

(ii) $\left(\frac{4}{3}, \frac{16}{3}\right)$ pt. of inflexion

(iii)



(iv) Max. value is $y = 27$

(9) (a) (i), (ii) Proofs

(iii) $AC = 20$ cm

(b) (i) $(0.15)^n$

(ii) $n = 3$

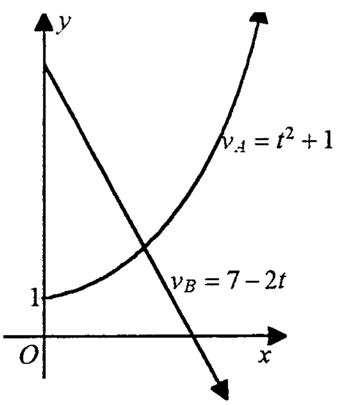
(c) (i) 2 units^2

(ii) $3\ln 3 - 2$

(10) (i), (ii) Proofs

(iii) $\left(6 - \sqrt{3}\right)$

(b) (i)



(ii) The point of intersection is the time when the two particles are travelling at the same velocity.

(iii) $t = 3$ sec