Centre Number

Student Number

CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NEW SOUTH WALES

2012
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

Morning Session
Monday, 6 August 2012

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of the paper
- All necessary working should be shown in every question
- Write your Centre Number and Student Number at the top of this page

Total marks – 100

Section I Pages 2–6

10 marks
- Attempt Questions 1–10
- Allow about 20 minutes for this section

Section II Pages 7–17

90 marks
- Attempt Questions 11–16
- Allow about 2 hours and 40 minutes for this section

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a \neq 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Section I

10 marks

Attempt Questions 1 – 10
Allow about 20 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 The cost of a Blu-ray player is \$189. This includes 10% GST. What is the cost of the Blu-ray player excluding GST?

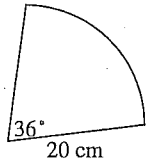
(A) \$170.10

(B) \$171.82

(C) \$207.90

(D) \$210.00

2 What is the perimeter, P, of the sector below with angle 36° and radius 20cm?



(A) $P = 0.5 \times 400 \times \left(\frac{\pi}{5} - \sin\frac{\pi}{5}\right)$ cm

(B) $P = \left(0.5 \times 400 \times \frac{\pi}{5}\right)$ cm

(C) $P = (40 + 36^\circ)$ cm

(D) $P = (40 + 4\pi)$ cm

3 Which term represents the distance that $y = a\sin bx$ extends out from the centre of its graph on the y-axis?

(A) Amplitude

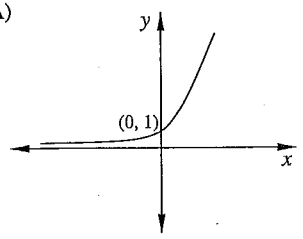
(B) Domain

(C) Period

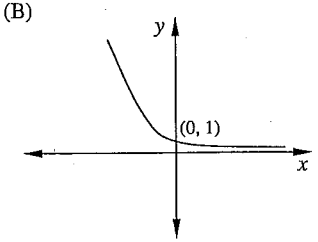
(D) Range

4 Which graph best represents the function $y = 3^{-x}$?

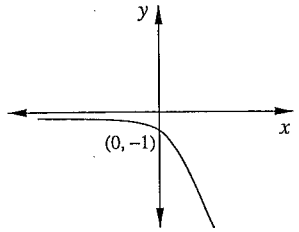
(A)



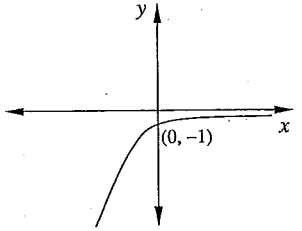
(B)



(C)



(D)



5 Helen planted a bed of gardenias in rows on her commercial property. Each row had to be fertilised before she started planting.

There were 13 gardenia plants in the first row, 19 gardenia plants in the second row, and so on. Each succeeding row had 6 more gardenia plants than the row before it.

If Helen wanted to plant 1453 gardenias, how many rows will she need to fertilise?

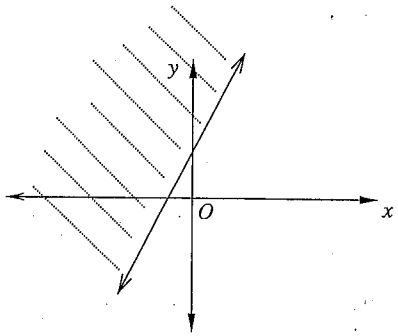
(A) 20

(B) 21

(C) 23.75

(D) 241

6 The graph of $2x-y+4=0$ is shown below.



Which of the following inequalities best describes the shaded region?

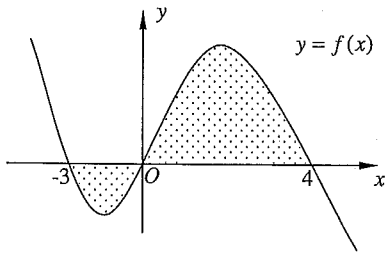
(A) $2x-y+4>0$

(B) $2x-y+4<0$

(C) $2x-y+4\geq 0$

(D) $2x-y+4\leq 0$

7 Consider the diagram below.



Which of the following represents the shaded area?

(A) $\int_{-3}^{4} f(x)\,dx$

(B) $2\int_{0}^{4} f(x)\,dx$

(C) $\int_{0}^{4} f(x)\,dx - \int_{-3}^{0} f(x)\,dx$

(D) $\int_{-3}^{0} f(x)\,dx + \int_{0}^{4} f(x)\,dx$

8 The population, N, of a colony of ants grows exponentially according to the formula $N(t)=550e^{kt}$, where k is the growth constant and t is the time in days.

If $N(\ln 4)=13\,750$, what is the exact value of k?

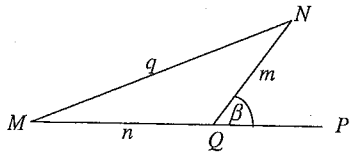
(A) $\dfrac{25}{\ln 4}$

(B) $\dfrac{\ln 4}{25}$

(C) $\dfrac{\ln 25}{\ln 4}$

(D) $\ln\left(\dfrac{25}{4}\right)$

9 If $\angle NQP = \beta$ in the diagram below, which of the following could be true?



(A) $q^2 = m^2 + n^2 + 2mn\cos\beta$

(B) $q^2 = m^2 + n^2 - 2mn\cos\beta$

(C) $m^2 = q^2 + n^2 - 2qn\cos\beta$

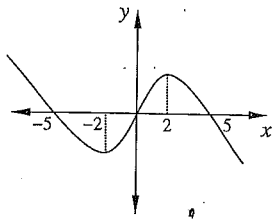
(D) $\dfrac{m}{\sin\angle NMQ} = \dfrac{q}{\cos\beta}$

10 Consider a curve with the following properties:

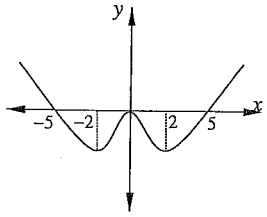
$g(x)$ is odd.
$g(5) = 0$ and $g'(2) = 0$.
$g'(x) > 0$ for $x > 2$.

Which of the following could be the graph of $y = g(x)$?

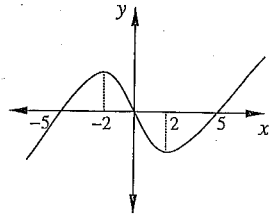
(A)



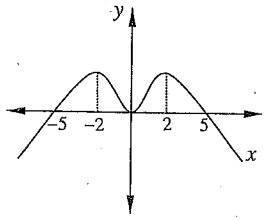
(B)



(C)



(D)



CATHOLIC SECONDARY SCHOOLS ASSOCIATION OF NSW
2012 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

Section II
90 marks

Attempt Questions 11–16
All questions are of equal value.
Allow about 2 hours and 40 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Evaluate $\dfrac{2.1^3 - 29}{\sqrt{4.01^2 - 0.8^2}}$ to 3 significant figures. **2**

(b) Simplify $\dfrac{2(x^2 - 4)}{x + 2}$. **2**

(c) Solve $|2m + 3| \le 3$. Graph your solution on a number line. **3**

(d) If $a = 4$ and $b = \sqrt{3}$, evaluate $\dfrac{a + b^2}{ab}$. **2**
Leave your answer with a rational denominator.

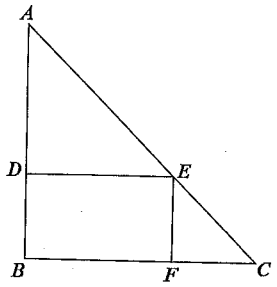
(e) The probability that Matt gets a concert booking with his band on any given weekend is 0.65. What is the probability that over two consecutive weekends he will get at least one concert booking? **2**

(f) Find the exact value of $\tan\dfrac{2\pi}{3}$. **2**

(g) Find a primitive for $\dfrac{5}{x^2} - 8x$. **2**

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) $BDEF$ is a rectangle inside ΔABC.



Prove that ΔABC is similar to ΔADE. **2**

(b) Differentiate:

(i) $(1-x)^3$ **2**

(ii) $\dfrac{\ln x^3}{x}$. **2**

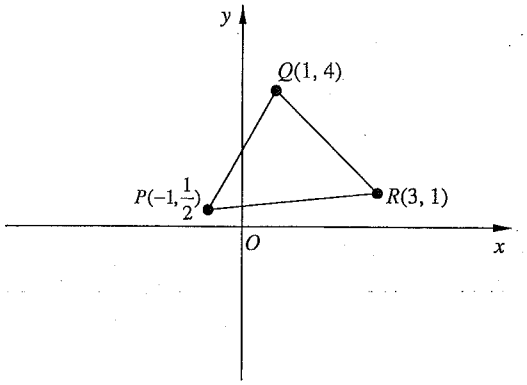
(c) (i) Find $\displaystyle\int \frac{2x}{x^2-7}\,dx$. **1**

(ii) Find $\displaystyle\int_0^{\frac{\pi}{4}} \sec^2 3x\ dx$. **2**

(d) Find the equation of the tangent to the curve $y = e^{2x}$ at the point (0, 1). **2**

Question 12 continues on page 9

(e) $P(-1,\frac{1}{2})$, $Q(1, 4)$ and $R(3, 1)$ are the vertices of a triangle, as shown below.



(i) Find the midpoint, M, of the interval joining QR. **1**

(ii) Find the gradient of PM. **1**

(iii) Show that PM is the perpendicular bisector of QR. **2**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) The height of the tide at Descartes Beach can be modelled by the equation

$$H = 4 + 2\cos\left(\frac{\pi t}{6}\right), \quad t > 0$$

where H is the height in metres of the tide after t hours.

(i) Find the height of the tide after 8 hours. **1**

(ii) Find the rate of change of the height of the tide after 8 hours. **2**

(iii) State whether the tide was rising or falling after 8 hours. Support your answer with appropriate calculations. **1**

(b) Let $f(x) = (3-x)(x-2)^2$.

(i) Show that $f(x) = -x^3 + 7x^2 - 16x + 12$. **1**

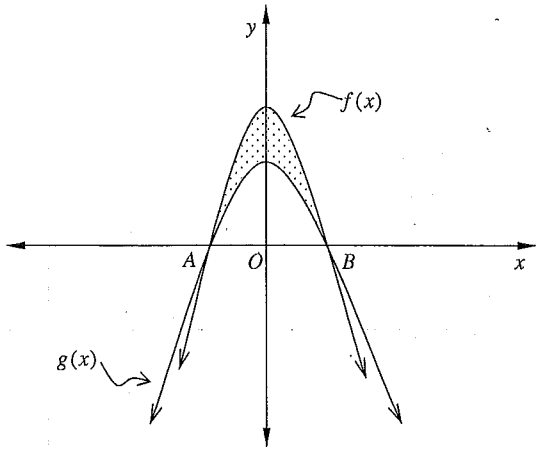
(ii) Find the coordinates of the stationary points of $y = f(x)$, and determine their nature. **3**

(iii) Hence, sketch the graph of $y = f(x)$ showing all stationary points and the y-intercept. **2**

(c) Solve $\log_3(2x-7) = 2$. **2**

Question 13 continues on page 11

(d)



The graphs of $f(x) = (5+x)(5-x)$ and $g(x) = \frac{2}{5}(5+x)(5-x)$ intersect at points A and B as shown in the diagram above.

(i) Show that the area of the shaded region is given by **1**

$$A = \frac{6}{5}\int_0^5 \left(25 - x^2\right)\, dx.$$

(ii) Hence, or otherwise, evaluate the area of the shaded region. **2**

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) What is the period of $y = 5 - 3\cos 2x$ for $0 \le x \le 2\pi$? **1**

(b) Find values of a, b and c such that **2**

$3x^2 - 7x - 6 \equiv a(x-2)^2 + b(x-2) + c$ for all x.

(c) A block of ice is removed from the fridge. The rate at which the ice melts is given by $\frac{dM}{dt} = -kM$, where M is measured in grams and time in minutes.

(i) Show that $M = M_0 e^{-kt}$ satisfies $\frac{dM}{dt} = -kM$. **1**

(ii) After 35 minutes only half the ice remains. Find the value of k. **2**

(iii) If, at a certain time, only 5% of the block remains, how long would it have been since the block was removed from the fridge? **2**

Leave your answer in hours and minutes.

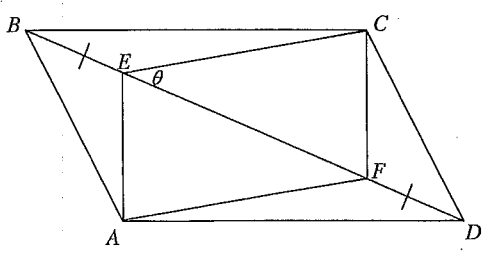
(d) Use Simpson's rule with 5 function values to find an approximation to the value of **3**

$\int_0^2 2^x \, dx$.

Give your answer correct to 3 decimal places.

Question 14 continues on page 13

(e)



In the diagram $AECF$ is a parallelogram and $\angle CEF = \theta$.
The diagonal EF is produced to D and FE to B, such that $BE=FD$.

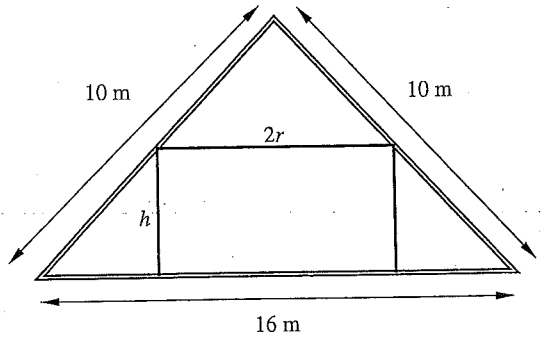
(i) Show that $\angle BEC = \angle DFA$. **2**

(ii) Prove that $\Delta ECB \equiv \Delta FAD$. **2**

End of Question 14

Question 15 (15 marks) Use a SEPARATE writing booklet.

(a) In some rural areas, hot water tanks are installed in the roofs of large homesteads. The diagram below shows the cross-section of a cylindrical tank in such a homestead's roof. The cylindrical tank fits exactly into the roof with diameter $2r$ metres and height h metres. The cross-section of the roof is an isosceles triangle with dimensions shown.



(i) Show that the height of the roof is 6 metres. **1**

(ii) Hence show that $h=\frac{3}{4}(8-r)$. **2**

(iii) Show that the volume of the cylindrical tank can be expressed by $V=\frac{3\pi}{4}(8r^2-r^3)$. **1**

(iv) Find the value of r which gives the hot water tank its greatest volume. **3**

(b) Ester sets up a savings policy for herself in order to save for a deposit to buy a car. She invests \$250 at the beginning of every month. The savings policy earns 8.4% per annum and is compounded monthly.

The policy matures at the end of the month of her final investment, 5 years after her first investment.

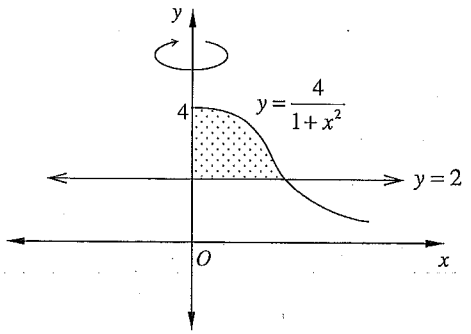
(i) Show that the value of the first investment is \$379.93. **2**

(ii) Calculate the total value of Ester's savings policy after 5 years. **3**

Question 15 continues on page 15

Question 15 (continued)

(c) The area enclosed by the curve $y=\frac{4}{1+x^2}$ and the line $y=2$ is shaded as shown in the diagram below.

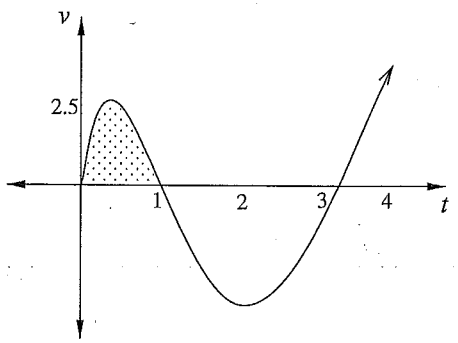


Calculate the volume of the solid formed if the shaded area is rotated about the y-axis. Leave your answer in exact form. **3**

End of Question 15

Question 16 (15 marks) Use a SEPARATE writing booklet.

(a) The graph below represents the velocity (v m/s) with respect to time (t sec) of a particle moving in a straight line.



(i) What is the velocity of the particle when $t = 3$ seconds? **1**

(ii) At what time(s) is the particle's acceleration zero? **1**

(iii) What does the shaded area on the graph represent? **1**

(iv) The particle is initially at the origin. At approximately what time does the particle next pass through the origin during its motion? **1**

(b) There are 4 red and 3 black discs in a bag. Sophie and Emma are playing a game in which they take turns drawing a disc from the bag and then replacing it.

To win the game, Sophie must draw a red disc and for Emma to win she must draw a black disc. They continue taking turns until there is a winner. Sophie goes first.

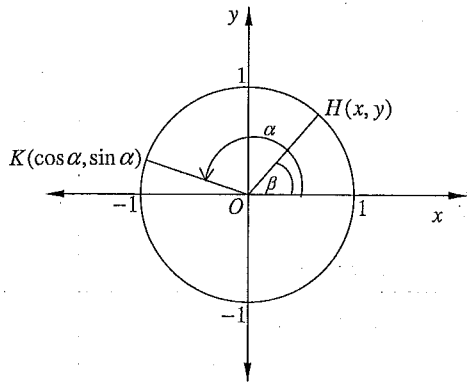
(i) Find the probability that Sophie wins on her first draw. **1**

(ii) Find the probability that Sophie wins in three or less of her turns. **2**

(iii) Find the probability that Sophie wins the game. **2**

Question 16 continues on page 17

(c) The diagram below shows the circle $x^2 + y^2 = 1$.



(i) Show that the point $H(x, y)$ can be expressed as $H(\cos\beta, \sin\beta)$. **1**

(ii) Given that $K(\cos\alpha, \sin\alpha)$, show that the distance between H and K is given by **2**

$$HK^2 = 2 - 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta).$$

(iii) By considering the triangle KOH, show that **1**

$$HK^2 = 2 - 2\cos(\alpha - \beta).$$

(iv) Hence show that $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$. **1**

(v) If $\alpha = \frac{\pi}{3}$ and $\beta = \frac{\pi}{4}$, using the result from part (iv) or otherwise, **1**

show that

$$\cos\frac{\pi}{12} = \frac{1+\sqrt{3}}{2\sqrt{2}}.$$

End of Question 16

End of Paper

CSSA

CATHOLIC SECONDARY SCHOOLS ASSOCIATION

2012 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

MATHEMATICS – SUGGESTED SOLUTIONS

These marking guidelines show the criteria to be applied to responses along with the marks to be awarded in line with the quality of responses. These guidelines are suggested and not prescriptive. This is not intended to be an exhaustive list but rather an indication of the considerations that students could include in their responses.

Section I
10 marks

Questions 1-10 (1 mark each)

Question	Answer	Outcomes Assessed	Targeted Performance Bands
1	B	P3	2-3
2	D	H5	3-4
3	A	H5	3-4
4	B	P4, P5	3-4
5	B	H5	3-4
6	D	P5	4-5
7	C	H8	4-5
8	C	H3	4-5
9	A	P4	5-6
10	C	H6	4-5

Section II
90 Marks

Question 11 (15 marks)
(a) (2 marks)
Outcomes Assessed: P3
Targeted Performance Bands: 2

Criteria	Marks
• Gives the correct answer	2
• Progresses towards answer	1

Sample answer

$$\frac{2.1^3 - 29}{\sqrt{4.01^2 - 0.8^2}} = -5.023426967 = -5.02 \text{ to 3 significant figures.}$$

(b) (2 marks)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 2-3

Criteria	Marks
• Gives the correct expression.	2
• Factorises numerator correctly	1

Sample answer

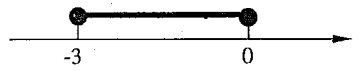
$$\frac{2(x^2-4)}{x+2} = \frac{2(x-2)(x+2)}{x+2}$$
$$= 2(x-2)$$

(c) (3 marks)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 2-3

Criteria	Marks
• Gives the correct solution and graph	3
• Progress towards answer with graph	2
• Gives one solution	1

Sample answer

$$|2m+3| \le 3$$
$$= -3 \le 2m+3 \le 3$$
$$= -6 \le 2m \le 0$$
$$= -3 \le m \le 0$$



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(d) (2 marks)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 2-3

Criteria	Marks
• Gives the correct solution with rational denominator	2
• Progress towards answer	1

Sample answer

$$\frac{a+b^2}{ab}=\frac{7}{4\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{7\sqrt{3}}{12}$$

(e) (2 marks)
Outcomes Assessed: H5
Targeted Performance Bands: 3-4

Criteria	Marks
• Calculates the probability correctly	2
• Progress towards solution	1

Sample answer

$$1-P(\text{no booking, no booking})$$
$$=1-(0.35)^2$$
$$=0.8775$$

(f) (2 marks)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 3-4

Criteria	Marks
• Correctly applies ASTC to give solution as an exact value	2
• Progress towards solution	1

Sample answer

$$\tan\frac{2\pi}{3}=\tan 120^\circ$$
$$=-\tan 60^\circ$$
$$=-\sqrt{3}$$

(g) (2 marks)
Outcomes Assessed: H5
Targeted Performance Bands: 2-3

Criteria	Marks
• Gives the correct solution	2
• Progress towards correct primitive	1

Sample answer

Primitive of

$$\frac{5}{x^2}-8x=5x^{-2}-8x=\frac{5x^{-1}}{-1}-\frac{8x^2}{2}=-\frac{5}{x}-4x^2+C$$

Question 12 (15 marks)

(a) (2 marks)

Outcomes Assessed: P2, P4

Targeted Performance Bands: 2-3

Criteria	Marks
• Shows ΔABC is similar to ΔADE, with appropriate reasoning	2
• Gives one correct statement	1

Sample answer

In ΔABC and ΔADE,

$\angle A$ is common

$\angle ADE = \angle ABC = 90°$ ($BDEF$ is a rectangle, given)

$\therefore \Delta ABC$ is similar to ΔADE (equiangular)

(b) (i) (2 marks)

Outcomes Assessed: H4, H5

Targeted Performance Bands: 3-4

Criteria	Marks
• Gives the correct derivative	2
• Progress towards solution	1

Sample answer

$$\frac{d}{dx}(1-x)^3 = 3(1-x)^2(-1) = -3(1-x)^2$$

(b) (ii) (2 marks)

Outcomes Assessed: P7, H5

Targeted Performance Bands: 3-4

Criteria	Marks
• Gives the correct derivative	2
• Progress towards solution	1

Sample answer

Let $y = \dfrac{\ln x^3}{x}$ using quotient rule $y' = \dfrac{vu' - uv'}{v^2}$

$u = 3\ln x, \quad v = x$

$u' = \dfrac{3}{x}, \quad v' = 1$

$$y' = \frac{x \times \frac{3}{x} - 3\ln x \times 1}{x^2}$$

$$= \frac{3 - 3\ln x}{x^2}$$

(c) (i) (1 mark)

Outcomes Assessed: H3, H8

Targeted Performance Bands: 2-3

Criteria	Mark
• Gives the correct answer	1

Sample answer

$$\int \frac{2x}{x^2 - 7}\,dx = \ln\left(x^2 - 7\right) + C$$

(c) (ii) (2 marks)

Outcomes Assessed: H5, H8

Targeted Performance Bands: 3-4

Criteria	Marks
• Gives the correct solution	2
• Progress towards correct primitive	1

Sample answer

$$\int_0^{\frac{\pi}{4}} \sec^2 3x\,dx = \left[\frac{1}{3}\tan 3x\right]_0^{\frac{\pi}{4}} = \left(\frac{1}{3}\tan\frac{3\pi}{4}\right) - \left(\frac{1}{3}\tan 0\right) = -\frac{1}{3} - 0 = -\frac{1}{3}$$

(d) (2 marks)

Outcomes Assessed: P6, P7

Targeted Performance Bands: 3-4

Criteria	Marks
• Gives the correct equation	2
• Differentiates and finds the gradient of the tangent	1

Sample answer

$y = e^{2x}$

$\dfrac{dy}{dx} = 2e^{2x}$ at $x = 0, \dfrac{dy}{dx} = 2$

$\therefore$ gradient of tangent $= 2$

$\therefore$ equation of tangent at (0, 1) is: $y - 1 = 2(x - 0)$

$y = 2x + 1$ or $2x - y + 1 = 0$

(e) (i) (1 mark)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 2-3

Criteria	Mark
• Gives the correct midpoint	1

Sample answer

$$M = \left(\frac{1+3}{2}, \frac{4+1}{2}\right)$$

$$= \left(2, \frac{5}{2}\right)$$

(e) (ii) (1 mark)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 2-3

Criteria	Mark
• Gives the correct gradient	1

Sample answer

Gradient: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{\frac{5}{2} - \frac{1}{2}}{2 - -1} = \frac{2}{3}$$

(e) (iii) (2 marks)
Outcomes Assessed: P3, P4, H2
Targeted Performance Bands: 3-4

Criteria	Mark
• Correctly deduces that PM is the perpendicular bisector of QR	2
• Finds gradient of QR	1

Sample answer

Gradient of $QR = m_2 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1-4}{3-1}$

$$= -\frac{3}{2}$$

For PM to be perpendicular to QR, $m_1 m_2 = -1$

$\therefore m_1 m_2 = \dfrac{2}{3} \times -\dfrac{3}{2} = -1$

$\therefore$ Since M is the midpoint of QR and PM is perpendicular to QR, then PM is the perpendicular bisector of QR.

Question 13 (15 marks)
(a) (i) (1 mark)
Outcomes Assessed: H4, H5
Targeted Performance Bands: 4

Criteria	Mark
• Gives the correct solution	1

Sample answer

at $t = 8$, $H = 4 + 2\cos\left(\dfrac{4\pi}{3}\right) = 4 + 2\left(-\dfrac{1}{2}\right) = 3\text{m}$

(a) (ii) (2 marks)
Outcomes Assessed: H4, H5
Targeted Performance Bands: 4-5

Criteria	Marks
• Correctly substitutes $t = 8$ into $\dfrac{dH}{dt}$	2
• Correctly differentiates $H = 4 + 2\cos\dfrac{\pi t}{6}$	1

Sample answer

$$\frac{dH}{dt} = -\frac{2\pi}{6}\sin\left(\frac{\pi t}{6}\right)$$

$$= -\frac{\pi}{3}\sin\left(\frac{\pi t}{6}\right)$$

At $t = 8$, $\dfrac{dH}{dt} = -\dfrac{\pi}{3}\sin\left(\dfrac{4\pi}{3}\right) = -\dfrac{\pi}{3} \times -\dfrac{\sqrt{3}}{2} = \dfrac{\pi\sqrt{3}}{6}$

$\cong 0.90689... \cong 0.9$ m/h (1 significant figure)

(a) (iii) (1 mark)
Outcomes Assessed: H6
Targeted Performance Bands: 4

Criteria	Mark
• Gives the correct solution	1

Sample answer

When $t = 8$, $H = 3$ metres and when $t = 9$, $H = 4$ metres
$\therefore$ The tide is rising after 8 hours.

Alternatively,

At $t = 8$, the tide is rising because $\dfrac{dH}{dt} > 0$, which indicates that the height of the tide is rising.

(b) (i) (1 mark)

Outcomes Assessed: P3

Targeted Performance Bands: 3

Criteria	Mark
• Expands $f(x)=(3-x)(x-2)^2$ correctly	1

Sample answer

$$\begin{aligned} f(x) &= (3-x)(x-2)^2 \\ &= (3-x)(x^2-4x+4) \\ &= 3x^2-12x+12-x^3+4x^2-4x \\ &= -x^3+7x^2-16x+12 \end{aligned}$$

(b)(ii) (3marks)

Outcomes Assessed: P7, P8, H6

Targeted Performance Bands: 3-4

Criteria	Marks
• Determines the nature of the stationary points correctly	3
• Finds the stationary points and their corresponding y-coordinates	2
• Differentiates $f(x)=(3-x)(x-2)^2$ correctly and finds the stationary points	1

Sample answer

$f(x)=-x^3+7x^2-16x+12$

$f'(x)=-3x^2+14x-16$

Stationary points at $f'(x)=0$

So, $-3x^2+14x-16=0$

$(8-3x)(x-2)=0$

$x=\frac{8}{3}$ or $x=2$ and $f\left(\frac{8}{3}\right)=\frac{4}{27}$ and $f(2)=0$

So the stationary points are

$\left(\frac{8}{3},\frac{4}{27}\right)$ and $(2,0)$.

$f''(x)=-6x+14$

$f''\left(\frac{8}{3}\right)=-\frac{48}{3}+14$ $\qquad$ $f''(2)=-12+14$

$=-2,\ <0$ $\qquad$ $=2,\ >0$

so $\left(\frac{8}{3},\frac{4}{27}\right)$ is a maximum turning point. $\qquad$ So $(2,0)$ is a minimum turning point.

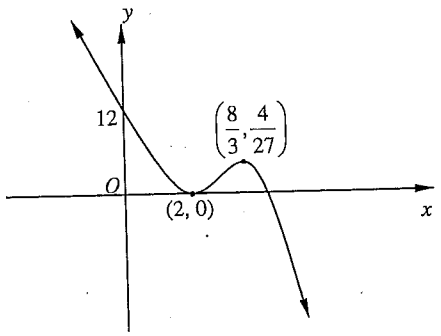
(b) (iii) (2 marks)

Outcomes Assessed: H6

Targeted Performance Bands: 3-4

Criteria	Marks
• Correct solution showing stationary points and the y-intercept	2
• Draws the correct graph with ONE error	1

Sample answer



(c) (2 marks)

Outcomes Assessed: P7, H5

Targeted Performance Bands: 4-5

Criteria	Mark
• Gives the correct solution	2
• Correctly writes in exponential form	1

Sample answer

$\log_3(2x-7)=2 \Leftrightarrow 2x-7=3^2$

$2x=16$

$x=8$

(d) (i) (1 mark)
Outcomes Assessed: P3, H5
Targeted Performance Bands: 3-4

Criteria	Mark
• Correctly shows that the area is given by $A=\frac{6}{5}\int_0^5(25-x^2)dx$	1

Sample answer

$$A=2\int_0^5\left[(5+x)(5-x)-\frac{2}{5}(5+x)(5-x)\right]dx$$

$$=2\int_0^5\frac{3}{5}(5+x)(5-x)\,dx$$

$$=\frac{6}{5}\int_0^5(5+x)(5-x)dx$$

(d) (ii) (2 marks)
Outcomes Assessed: H8
Targeted Performance Bands: 4-5

Criteria	Marks
• Gives the correct solution	2
• Finds the correct primitive	1

Sample answer

$$A=\frac{6}{5}\int_0^5(25-x^2)dx$$

$$=\frac{6}{5}\left[25x-\frac{x^3}{3}\right]_0^5$$

$$=\frac{6}{5}\left[\left(125-\frac{125}{3}\right)-(0-0)\right]$$

$$=\frac{250}{3}\times\frac{6}{5}$$

$$=100\text{ u}^2$$

Question 14 (15 marks)
(a) (1 mark)
Outcomes Assessed: H5
Targeted Performance Bands: 3

Criteria	Mark
• Gives the correct solution	1

Sample answer

$$\text{Period}=\frac{2\pi}{n}=\frac{2\pi}{2}=\pi\text{ radians}$$

(b) (2 marks)
Outcomes Assessed: P4
Targeted Performance Bands: 3-4

Criteria	Marks
• Correct solution	2
• Determine the correct value of a	1

Sample answer

$$\text{RHS}=a(x-2)^2+b(x-2)+c$$
$$=a\left(x^2-4x+4\right)+bx-2b+c$$
$$=ax^2-4ax+4a+bx-2b+c$$
$$=ax^2+(b-4a)x+(4a-2b+c)$$

Equating coeffs :

$$\left\{\begin{matrix}a=3\\b-4a=-7\\4a-2b+c=-6\end{matrix}\right\}\text{ Solving}$$

$b=-7+12=5$

$c=-6+10-12=-8$

So, $a=3, b=5$ and $c=-8$

$\therefore 3x^2-7x-6=3(x-2)^2+5(x-2)-8.$

(c) (i) (1 mark)
Outcomes Assessed: H3, H6
Targeted Performance Bands: 2-3

Criteria	Mark
• Gives the correct solution	1

Sample answer

$M = M_0 e^{-kt}$ $\quad\therefore \dfrac{dM}{dt} = -kM_0 e^{-kt} = -kM$

(c) (ii) (2 marks)
Outcomes Assessed: H3, H5
Targeted Performance Bands: 3-4

Criteria	Marks
• Gives the correct value for k	2
• Substitutes $t = 35$ and makes progress towards finding k	1

Sample answer

When $t = 35$,

$\dfrac{M_0}{2} = M_0 e^{-35k}$

$\dfrac{1}{2} = e^{-35k}$ $\quad\therefore \ln\dfrac{1}{2} = -35k$

$\therefore k = 0.0198042\ldots = 0.01980$ (4 significant figures)

(c) (iii) (2 marks)
Outcomes Assessed: H3
Targeted Performance Bands: 2-4

Criteria	Marks
• Gives the correct value for t	2
• Makes progress towards finding t	1

Sample answer

$\dfrac{5M_0}{100} = M_0 e^{-0.01980\,t}$ $\quad\therefore \ln\dfrac{1}{20} = -0.01980t$

$\therefore t = \dfrac{\ln\frac{1}{20}}{-0.01980} = 151.299$ minutes $\quad$ i.e. $t =$ 2 h 31 min.

(d) (3 marks)
Outcomes Assessed: H3, H8
Targeted Performance Bands: 3-5

Criteria	Marks
• Correct solution correct to 3 decimal places	3
• Substitutes correctly into Simpson's rule	2
• Substitutes correctly into the five function values of 2^x	1

Sample answer

For two applications of Simpson's rule, 5 function values are required.

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
2^x	1	$\sqrt{2}$	2	$2\sqrt{2}$	4

$$\int_0^2 2^x\,dx \cong \frac{1-0}{6}\left[f(0)+4f\left(\frac{1}{2}\right)+f(1)\right]+\frac{2-1}{6}\left[f(1)+4f\left(\frac{3}{2}\right)+f(2)\right]$$

$$\cong \frac{1}{6}\left[1+4\sqrt{2}+2\right]+\frac{1}{6}\left[2+8\sqrt{2}+4\right]$$

$$\cong \frac{1}{6}\left[9+12\sqrt{2}\right]$$

$\cong 4.328427\ldots \cong 4.328$ (3 decimal places)

(e) (i) (2 marks)
Outcomes Assessed: H2, H5
Targeted Performance Bands: 3-5

Criteria	Marks
• Gives the correct proof	2
• Shows that $\angle AFE = \angle CEF$, with reasoning.	1

Sample answer

$\angle AFE = \angle CEF$ (Alternate angles, $AE \parallel FC$)

Let $\angle AFE = \angle CEF = \theta$

So $\angle DFA = 180 - \theta$ (Straight angle)

and $\angle BEC = 180 - \theta$ (Straight angle)

$\therefore \angle BEC = \angle DFA$

(e) (ii) (2 marks)
Outcomes Assessed: H2, H5
Targeted Performance Bands: 3-5

Criteria	Marks
• Gives the correct proof	2
• Progress towards the answer	1

Sample answer

In Δs ECB and FAD,

$\angle BEC = \angle DFA$ (proven in part (i))

$BE = FD$ (given)

$EC = AF$ (opposite sides of a parallelogram are equal)

$\therefore \Delta ECB \equiv \Delta FAD$ (SAS)

Question 15 (15 marks)
(a) (i) (1 mark)
Outcomes Assessed: H2, H4
Targeted Performance Bands: 2-3

Criteria	Mark
• Gives the correct solution	1

Sample answer

From the triangle's height, $x^2 = 10^2 - 8^2 = 36$ $\therefore x = 6$ metres.

(a) (ii) (2 marks)
Outcomes Assessed: H4, H5
Targeted Performance Bands: 3-4

Criteria	Marks
• Gives the correct solution	2
• Progress towards the answer	1

Sample answer

By similar triangles,

$\frac{6}{h} = \frac{8}{8-r}$ $\therefore h = \frac{3}{4}(8-r)$

(a) (iii) (1 mark)
Outcomes Assessed: H4, H5
Targeted Performance Bands: 3-4

Criteria	Mark
• Gives the correct solution	1

Sample answer

Volume of the cylinder $= \pi r^2 h = \pi \times r^2 \times \frac{3}{4}(8-r) = \frac{3\pi}{4}(8r^2 - r^3)$

(a) (iv) (3 marks)

Outcomes Assessed: H4, H5

Targeted Performance Bands: 4-5

Criteria	Marks
• Justifies the correct value of r to attain a maximum	3
• Finds the correct value of r	2
• Finds the first derivative	1

Sample answer

$V=\frac{3\pi}{4}\left(8r^2-r^3\right)$ $\quad\therefore\frac{dV}{dr}=\frac{3\pi}{4}\left(16r-3r^2\right)$ and for a greatest volume, $\frac{dV}{dr}=0$

$\therefore\left(16r-3r^2\right)=0$ $\quad\therefore r(16-3r)=0$ $\quad\therefore r=\frac{16}{3}$

Determining nature: $\frac{d^2V}{dr^2}=\frac{3\pi}{4}(16-6r)=-12\pi<0$ $\therefore$Maximum

$\therefore$ The greatest volume occurs when $r=\frac{16}{3}$ metres.

(b) (i) (2 marks)

Outcomes Assessed: H3, H5

Targeted Performance Bands: 2-4

Criteria	Marks
• Correct solution	2
• Finds the correct r or n	1

Sample answer

$r=\frac{8.4}{12}=0.7\%$ and $n=5\times12=60$

$A_1=250\left(1+\frac{0.7}{100}\right)^{60}=250(1.007)^{60}=\$379.93$

(b) (ii) (3 marks)

Outcomes Assessed: H5

Targeted Performance Bands: 3-5

Criteria	Marks
• Correct solution	3
• Progress towards the answer using S_n of a GP.	2
• Writes a geometric series for the sum	1

Sample answer

$A_1=250(1.007)^{60}$, $A_2=250(1.007)^{59}$, $A_3=250(1.007)^{58}$,…, $A_{60}=250(1.007)^1$

$\therefore\ A=250(1.007)+250(1.007)^2+250(1.007)^3+\ldots+250(1.007)^{60}$

$=\frac{250(1.007)\left[1.007^{60}-1\right]}{1.007-1}=\$18691.94$

(c) (3 marks)

Outcomes Assessed: H5, H7

Targeted Performance Bands: 3-5

Criteria	Marks
• Correct solution after substitution into primitive	3
• Correct primitive	2
• Makes x^2 the subject and finds the correct expression for V	1

Sample answer

$y=\frac{4}{1+x^2}$ $\quad\therefore1+x^2=\frac{4}{y}$ $\quad\therefore x^2=\frac{4}{y}-1$

$V=\pi\int_2^4\left(\frac{4}{y}-1\right)dy$ $\quad\therefore V=\pi\left[4\ln y-y\right]_2^4=\pi\left[(4\ln4-4)-(4\ln2-2)\right]$

$=\pi\left[4\ln4-4-4\ln2+2\right]=\pi(4\ln2-2)u^3$

Question 16 (15 marks)

(a) (i) (1 mark)

Outcomes Assessed: H7

Targeted Performance Bands: 3-5

Criteria	Mark
• Gives the correct solution	1

Sample answer

At $t = 3$, $v = 0$ m/s

(a) (ii) (1 mark)

Outcomes Assessed: H7

Targeted Performance Bands: 3-5

Criteria	Mark
• Gives the correct solution	1

Sample answer

At $t = \frac{1}{2}$, $t = 2$

(a) (iii) (1 mark)

Outcomes Assessed: H7

Targeted Performance Bands: 3-5

Criteria	Mark
• Gives the correct solution	1

Sample answer

The distance travelled in the first second

(a) (iv) (1 mark)

Outcomes Assessed: H7

Targeted Performance Bands: 3-5

Criteria	Mark
• Gives the correct solution	1

Sample answer

At approximately $t = 2$

(b) (i) (1 mark)

Outcomes Assessed: H5

Targeted Performance Bands: 4

Criteria	Mark
• Gives the correct answer	1

Sample answer

$$P(\text{Sophie wins}) = \frac{4}{7}$$

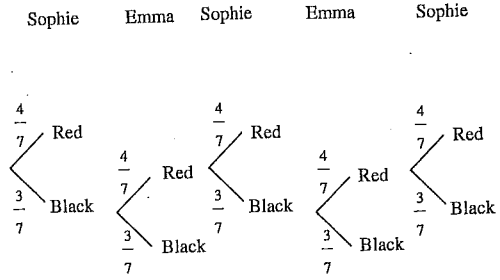
(b) (ii) (2 marks)

Outcomes Assessed: H4, H5, H9

Targeted Performance Bands: 5

Criteria	Marks
• Calculates the probability correctly	2
• Correctly identifies that for Sophie to win in 3 or less turns means that she can win in 1 turn, 2 turns or 3 turns	1

Sample answer



$$P(\text{Sophie wins in 3 or less turns})$$
$$= P(\text{Sophie wins in 1 turn}) + P(\text{Sophie wins in 2 turns}) + P(\text{Sophie wins in 3 turns})$$
$$= \frac{4}{7} + \left(\frac{3}{7} \times \frac{4}{7} \times \frac{4}{7}\right) + \left(\frac{3}{7} \times \frac{4}{7} \times \frac{3}{7} \times \frac{4}{7} \times \frac{4}{7}\right)$$
$$= \frac{4}{7} + \frac{48}{343} + \frac{576}{16807}$$
$$= 0.746$$

(b) (iii) (2 marks)
Outcomes Assessed: H5, H9
Targeted Performance Bands: 5-6

Criteria	Marks
• Correctly substitutes into the limiting sum formula to determine answer	2
• Recognises that this is a limiting sum	1

Sample answer

This is a limiting sum with $|r|<1$, where $a=\frac{4}{7}$ and $r=\frac{12}{49}$.

So, $S_\infty = \dfrac{\frac{4}{7}}{1-\frac{12}{49}}$

$= \dfrac{28}{37}$

So, the probability that Sophie wins the game is $\frac{28}{37}$.

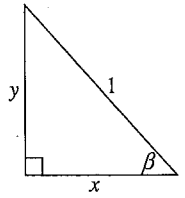
(c) (i) (1 mark)
Outcomes Assessed: P3, H5
Targeted Performance Bands: 4-5

Criteria	Mark
• Gives the correct solution	1

Sample answer

$\sin\beta = y$ and $\cos\beta = x$

$\therefore H(x,y) \equiv H(\cos\beta, \sin\beta)$



(c) (ii) (2 marks)
Outcomes Assessed: P3, H5
Targeted Performance Bands: 4-6

Criteria	Marks
• Gives the correct solution	2
• Progress towards the solution	1

Sample answer

Using the distance formula $H(\cos\beta, \sin\beta)$, $K(\cos\alpha, \sin\alpha)$

$$\begin{aligned} HK^2 &= (\cos\beta - \cos\alpha)^2 + (\sin\beta - \sin\alpha)^2 \\ &= \cos^2\beta - 2\cos\beta\cos\alpha + \cos^2\alpha + \sin^2\beta - 2\sin\beta\sin\alpha + \sin^2\alpha \\ &= 2 - 2\cos\beta\cos\alpha - 2\sin\beta\sin\alpha \\ &= 2 - 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta) \end{aligned}$$

(c) (iii) (1 mark)
Outcomes Assessed: P3
Targeted Performance Bands: 5-6

Criteria	Mark
• Gives the correct solution	1

Sample answer

By the cosine rule:

$$\begin{aligned} HK^2 &= 1^2 + 1^2 - 2(1)(1)\cos(\alpha - \beta) \\ &= 2 - 2\cos(\alpha - \beta) \end{aligned}$$

(c) (iv) (1 mark)
Outcomes Assessed: H2
Targeted Performance Bands: 4-5

Criteria	Mark
• Gives the correct solution	1

Sample answer

Equating (ii) and (iii)

$$2 - 2\cos(\alpha - \beta) = 2 - 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$$
$$\therefore \cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

(c) (iv) (1 mark)

Outcomes Assessed: H2, H5

Targeted Performance Bands: 3-5

Criteria	Mark
• Gives the correct solution	1

Sample answer

$$\cos\frac{\pi}{12}=\cos\left(\frac{\pi}{3}-\frac{\pi}{4}\right)=\cos\frac{\pi}{3}\cos\frac{\pi}{4}+\sin\frac{\pi}{3}\sin\frac{\pi}{4}$$

$$=\frac{1}{2}\times\frac{1}{\sqrt{2}}+\frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}}=\frac{1+\sqrt{3}}{2\sqrt{2}}$$