CSSA

CATHOLIC SECONDARY SCHOOLS ASSOCIATION OF NSW

Centre Number

Student Number

2015

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

Morning Session
Thursday 30 July 2015

General Instructions

- Reading time – 5 mins
- Working time – 3 hours
- Write using blue or black pen Black pen is preferred
- Use Multiple Choice Answer Sheet provided
- Board-approved calculators may be used
- A table of standard integrals is provided on a SEPARATE sheet
- In Questions 11-16, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number at the top of this page

Total marks – 100

Section I Pages 2 - 5

10 marks

- Attempt Questions 1 - 10
- Allow about 15 minutes for this section

Section II Pages 6 - 15

90 marks

- Attempt Questions 11 - 16
- Allow about 2 hours and 45 minutes for this section

6200-1

STANDARD INTEGRALS

$\int x^n\,dx$	$=\frac{1}{n+1}x^{n+1},\ n\neq -1;\ x\neq 0, \text{if } n<0$
$\int \frac{1}{x}\,dx$	$=\ln x,\ x>0$
$\int e^{ax}\,dx$	$=\frac{1}{a}e^{ax},\ a\neq 0$
$\int \cos ax\,dx$	$=\frac{1}{a}\sin ax,\ a\neq 0$
$\int \sin ax\,dx$	$=-\frac{1}{a}\cos ax,\ a\neq 0$
$\int \sec^2 ax\,dx$	$=\frac{1}{a}\tan ax,\ a\neq 0$
$\int \sec ax\tan ax\,dx$	$=\frac{1}{a}\sec ax,\ a\neq 0$
$\int \frac{1}{a^2+x^2}\,dx$	$=\frac{1}{a}\tan^{-1}\frac{x}{a},\ a\neq 0$
$\int \frac{1}{\sqrt{a^2-x^2}}\,dx$	$=\sin^{-1}\frac{x}{a},\ a>0,\ -a<x<a$
$\int \frac{1}{\sqrt{x^2-a^2}}\,dx$	$=\ln\left(x+\sqrt{x^2-a^2}\right),\ x>a>0$
$\int \frac{1}{\sqrt{x^2+a^2}}\,dx$	$=\ln\left(x+\sqrt{x^2+a^2}\right)$

NOTE: $\ln x=\log_e x,\ x>0$

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 Evaluate $\log_2 2 + \log_2 4 + \log_2 8$:

(A) 6

(B) 64

(C) $\log_2 14$

(D) $\log_6 14$

2 Simplify $(\tan\theta - 1)^2$.

(A) $\sec^2\theta$

(B) $\text{cosec}^2\theta - 2\tan\theta$

(C) $\cot^2\theta - 2\tan\theta$

(D) $\sec^2\theta - 2\tan\theta$

3 Given that $f(x) = x^2 + x$, find the values of a if $f''(a) = f(a)$

(A) $a = 2$ and 1

(B) $a = -1$ and 2

(C) $a = -2$ and -1

(D) $a = -2$ and 1

4 Evaluate $\lim\limits_{h \to 4} \dfrac{4-h}{16-h^2}$

(A) 0

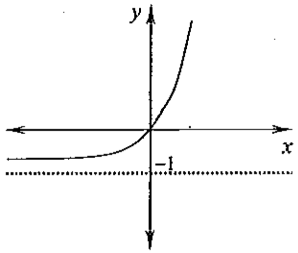
(B) $\dfrac{1}{8}$

(C) $\dfrac{1}{4}$

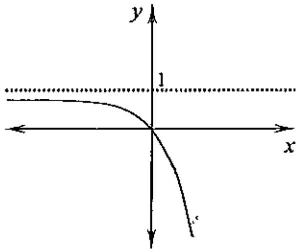
(D) 4

5 Which of the following graphs could have the equation $y = 1 - e^x$?

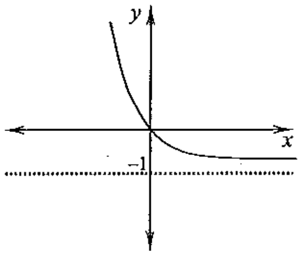
(A)



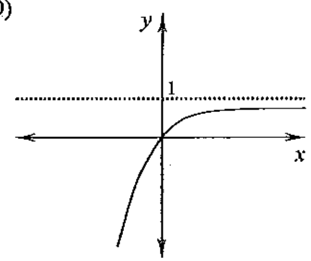
(B)



(C)



(D)



6 Differentiate $(x^2+\ln 2)^3$.

(A) $3\times(x^2+\ln 2)^2$

(B) $3\times 2x\times\frac{1}{2}(x^2+\ln 2)^2$

(C) $3\times 2x(x^2+\ln 2)^2$

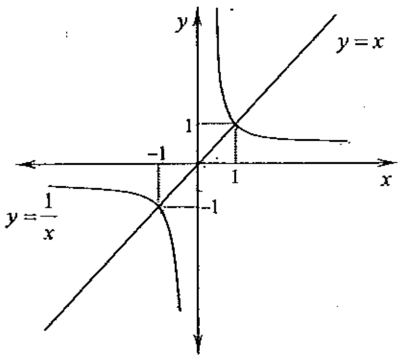
(D) $\frac{(x^2+\ln 2)^4}{4\times 2x}$

7 Fifty tickets are sold in a raffle. There are three prizes. Katherina buys 6 tickets. Which expression gives the probability that Katherina wins none of the three prizes?

(A) $\frac{6}{50}\times\frac{5}{49}\times\frac{4}{48}$ (B) $\frac{6}{50}\times\frac{5}{50}\times\frac{4}{50}$

(C) $\frac{44}{50}\times\frac{43}{49}\times\frac{42}{48}$ (D) $\frac{44}{50}\times\frac{43}{50}\times\frac{42}{50}$

8 Use the diagram below to solve the inequation $\frac{1}{x}<x$:



(A) $-1<x<0$ or $x>1$

(B) $x<-1$ or $0<x<1$

(C) $-1<x<1$

(D) $x<0$ or $x>1$

9 What values of k result in $\int_0^{\pi} \sin kx \ dx=0$, where k is an integer.

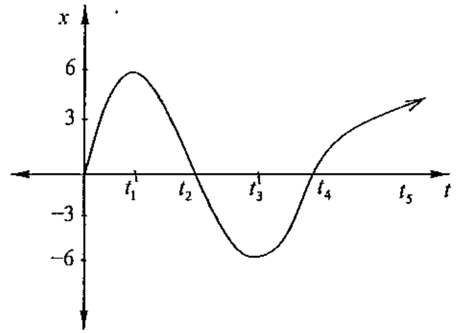
(A) Even values

(B) Negative values

(C) Odd values

(D) Positive values

10 The displacement, x metres, from the origin of a particle moving in a straight line at any time (t seconds) is shown in the graph. When is the particle at rest?



(A) t_1 and t_2

(B) t_1 and t_3

(C) t_2 and t_4

(D) t_3 and t_5

Section II

90 marks
Attempt Questions 11 - 16
Allow about 2 hours and 45 minutes for this section

Answer each question in a separate writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Evaluate $\dfrac{e^{\pi}}{\ln \pi}$ correct to 3 significant figures. **2**

(b) Solve $\dfrac{m-1}{4} - 6 = \dfrac{m}{2}$. **2**

(c) If $g'(t) = 6t^2 - 1$ and $g(-1) = 2$, find an expression for $g(t)$. **2**

(d) Evaluate $\sum_{n=1}^{5} \dfrac{1}{2^n}$. **2**

(e) Factorise completely $7x - y + 49x^2 - y^2$. **2**

(f) Differentiate $\dfrac{2x^4}{\cos x}$ with respect to x. **2**

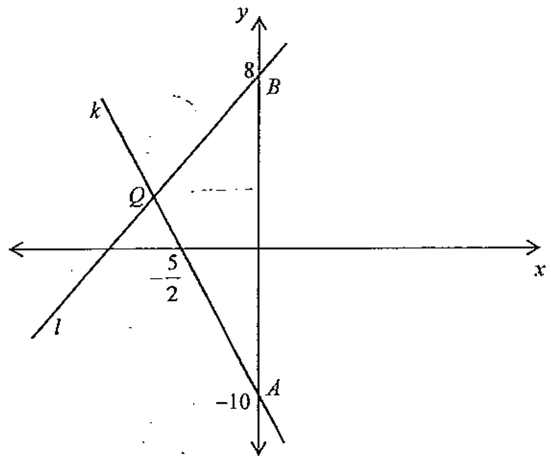
(g) For what values of p is the quadratic function $px^2 + 2x + p = 0$ positive definite. **3**

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) Find $\displaystyle\int \sec^2 \frac{x}{2}\, dx$. **1**

(b) Evaluate $\displaystyle\int_2^4 \frac{6x}{x^2 - 3}\, dx$. **2**

(c)



Line l has equation $2x - y + 8 = 0$. Line k intersects line l at Q and has an x-intercept at $-\dfrac{5}{2}$ and y-intercept at -10.

(i) Show that the equation of line k is $4x + y + 10 = 0$. **1**

(ii) Find the coordinates of Q. **2**

(iii) Write down the inequalities which define the region bounding ΔAQB. **2**

(iv) Calculate the area of ΔAQB. **1**

(d) A dodecagon has 12 sides. The angles of a dodecagon are in an arithmetic progression.

(i) Given that the size of the smallest angle is 62°, find the common difference. **2**

(ii) How many of these angles are obtuse? **1**

(e) Consider the function $y=\left(\frac{1}{2}\right)^{-x}$

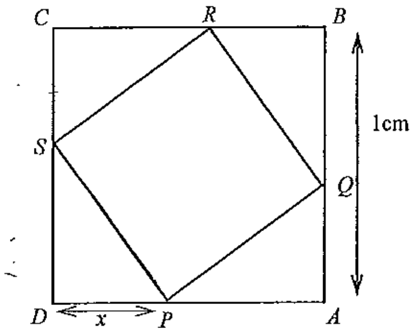
(i) Copy and complete the following table of values in your writing booklet. **1**

x	-2	-1	0	1	2
y					

(ii) Hence, use Simpson's rule with 5 function values to find an approximation to the value of $\int_{-2}^{2}\left(\frac{1}{2}\right)^{-x}dx$. **2**

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a)



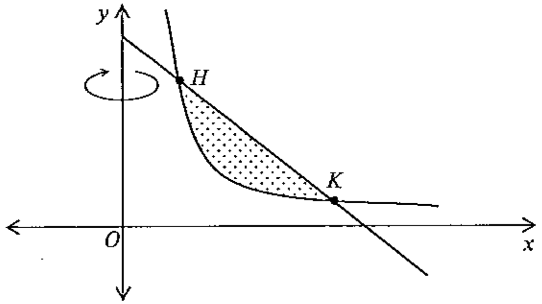
In the diagram $ABCD$ and $PQRS$ are both squares and $AB=1\text{cm}$.
Let $DP=CS=x$.

(i) Show that the area, A, of the square $PQRS$ is given by **2**

$$A=2x^2-2x+1.$$

(ii) Find the minimum area of $PQRS$. **2**

(b) The diagram shows the graphs of $y=\frac{2}{x}$ and $y=3-x$ for $x>0$, the shaded area enclosed between the two graphs and their points of intersection H and K as shown.



(i) Find the coordinates of the points H and K. **2**

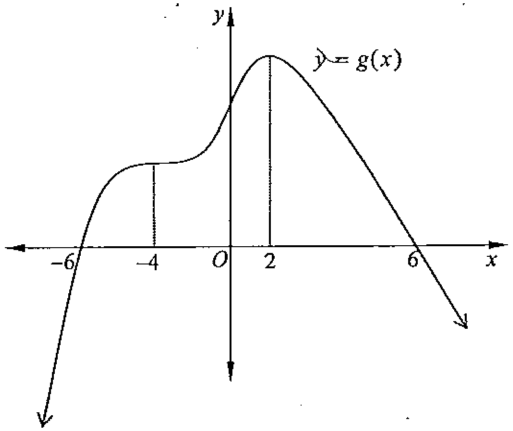
(ii) The shaded area is rotated about the y-axis. **2**
Show that the volume, V, of the solid formed is given by

$$V=\pi\int_{1}^{2}\left(9-6y+y^2-\frac{4}{y^2}\right)dy$$

(iii) Hence, or otherwise, find the volume V. **2**

(c) Solve $2e^{2x} - 7e^{x} + 3 = 0$. Leave your answer in exact form. **3**

(d) The graph shows the function $y = g(x)$.

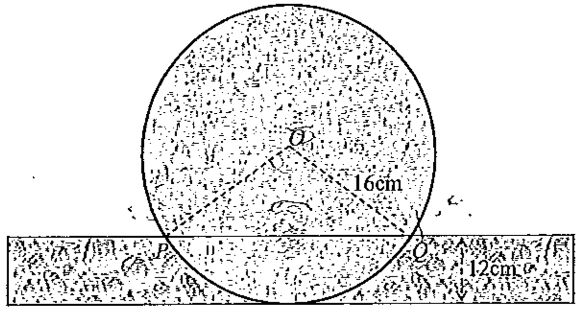


(i) For what values of x is the curve stationary? **1**

(ii) For what values of x is the curve decreasing? **1**

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a)



The diagram above represents a circular log of wood of radius 16 cm. The log floats in water to a depth of 12 cm as shown in the diagram.

(i) Show that $\angle POQ = 2.64$ radians. **2**

(ii) Hence find the area of the circular end of the log that is above the surface of the water. Give your answer to the nearest cm^2. **2**

(b) (i) Show that $x = \frac{2\pi}{3}$ is a solution of $\cos x = \cos 2x$. **1**

(ii) Sketch on the same set of axes the functions $y = \cos x$ and $y = \cos 2x$ for $0 \le x \le 2\pi$. **2**

(iii) How many solutions does $\cos x = \cos 2x$ have for the domain $0 \le x \le 2\pi$? **1**

(c) The number N of bacteria in a mouldy loaf of bread at time t hours is given by the equation $N = 21e^{kt}$.
After 7 hours the number of bacteria present is 30.

(i) Find the value of k? **2**

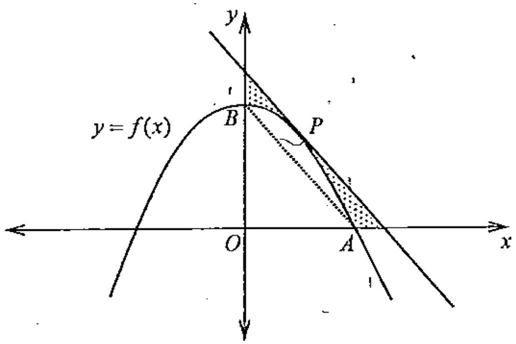
(ii) Determine the number of bacteria after 1 day. **1**

(iii) At what rate is the number of bacteria increasing after 1 day? **2**

(iv) Mouldy bread is considered unsafe to eat when the number of bacteria present reaches 3000. For how many days can the bread be still considered safe to eat? **2**

Question 15 (15 marks) Use a SEPARATE writing booklet.

(a) The graph of the function $f(x) = \dfrac{9-x^2}{3}$ is shown below.



The graph intersects the x-axis and the y-axis at the points A and B respectively.
The tangent to the graph at point P is parallel to the line AB.
The coordinates of B are $(0,3)$.

(i) Find the coordinates of the point A. **1**

(ii) Show that the coordinates of point P are $\left(1\frac{1}{2}\,,\,2\frac{1}{4}\right)$. **3**

(iii) Find the equation of the tangent at point P. **1**

(iv) The shaded region shown in the diagram above is bounded by the curve $y = f(x)$, the tangent at P, the x-axis and y-axis. **3**

Show that the area of this shaded region is $\dfrac{33}{32}$ units^2.

(b) Helen borrows \$30 000 over 4 years to purchase a 4WD from a car dealership.
The dealer offers an "interest free" period for the first 6 months of the loan.

After 6 months, the remainder of the loan is charged at 18%p.a. with interest calculated each month, just before each repayment.

The loan is to be repaid in 48 equal monthly repayments of \$$M$.

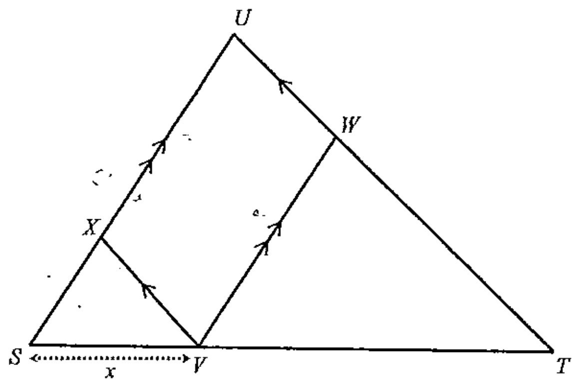
Let A_n be the amount owing after the nth repayment.

(i) Find an expression for A_6 **1**

(ii) Show that $A_8 = (30\ 000 - 6M)(1.015)^2 - M(1 + 1.015)$. **2**

(iii) Find the value of Helen's monthly repayment \$$M$. **2**

(c) **2**



In triangle STU, $SU \parallel VW$ and $WU \parallel VX$.

The length of $SU = 12\,\text{cm}$, $SX = 4\,\text{cm}$, and $TV = 24\,\text{cm}$.

Find the value of x.

Question 16 (15 marks) Use a SEPARATE writing booklet.

(a) The velocity, $\dot{x}$, in m/s of a particle moving in a straight line is given by $\dot{x} = 3 - \dfrac{9}{t-2}$ for $t > 2$, where t is the time in seconds.

(i) In which direction is the particle travelling when $t = 3$. **1**

(ii) Find the time the particle changes direction during its motion. **1**

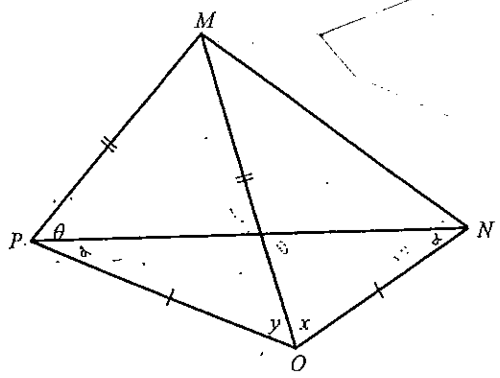
(iii) Hence, or otherwise, find the distance travelled by the particle between $t = 3$ and $t = 7$. **2**

(b) The local RSL is having a special promotion during 2015 where there will be a jackpot prize of $5000 available at the end of each day. All patrons may enter only once each day. The prize is drawn daily and there is approximately a $\dfrac{1}{75}$ chance of the jackpot prize being won.

(i) What is the probability of a patron winning the jackpot prize on two consecutive days? **1**

(ii) How many consecutive draws must be made for the RSL to be 95% certain that a jackpot prize will be won? **3**

(c)



In the diagram above, $PM = OM$ and $PO = NO$.

Also, $\angle MPN = \theta$, $\angle POM = y$ and $\angle NOM = x$.

Show that $\theta = \dfrac{x + 3y - \pi}{2}$. **3**

(d) (i) If $y = (X-1)X(X+1)$, show that $y = X^3 - X$. **1**

(ii) Now consider $y = (x+1999)(x+2000)(x+2001)$. **3**

By using part (i), or otherwise, find the y-value of the minimum turning point.

End of Paper

Catholic Trial 2015 - 2U

Question 1

$$\log_2 2 + \log_2 4 + \log_2 8 = 1 + \log_2 2^2 + \log_2 2^3$$
$$= 1 + 2 + 3$$
$$= 6$$

A

Question 2

$$(\tan\theta - 1)^2 = \tan^2\theta - 2\tan\theta + 1$$
$$= \tan^2\theta + 1 - 2\tan\theta$$
$$= \sec^2\theta - 2\tan\theta$$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$
$$\tan^2\theta + 1 = \sec^2\theta$$

D

Question 3

$$f(x) = x^2 + x$$
$$f'(x) = 2x + 1$$
$$f''(x) = 2$$

$$f''(a) = f(a)$$
$$a^2 + a = 2$$

D

$$a^2 + a - 2 = 0$$
$$(a+2)(a-1) = 0$$
$$a = -2 \text{ or } 1$$

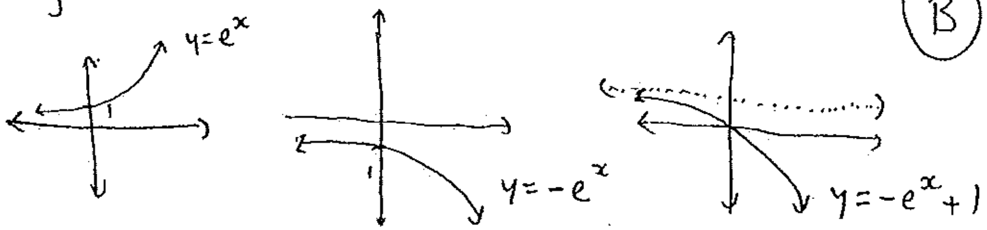
Question 4

$$\lim_{h\to 4} \frac{4-h}{16-h^2} = \lim_{h\to 4} \frac{4-h}{(4-h)(4+h)}$$
$$= \lim_{h\to 4} \frac{1}{4+h}$$
$$= \frac{1}{8}$$

B

5. $y = 1 - e^x$

B

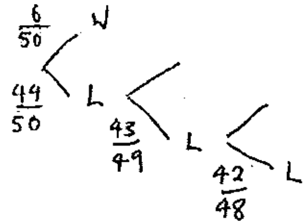


Question 6

$$y = (x^2 + \ln 2)^3$$
$$y' = 3(x^2+\ln 2)^2 \cdot 2x$$
$$= 3 \cdot 2x(x^2 + \ln 2)^2$$

C

Question 7



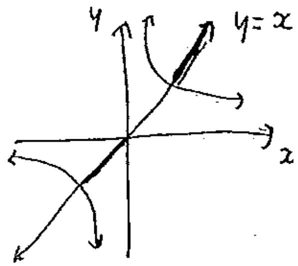
$$\frac{44}{50} \times \frac{43}{49} \times \frac{42}{48}$$

C

Question 8

$$\frac{1}{x} < x$$

$$-1 < x < 0 \quad \text{or} \quad x > 1$$



A

Question 9

$\int_0^{\pi} \sin kx \, dx = 0$

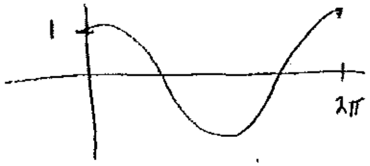
$\left[-\frac{1}{k}\cos kx\right]_0^{\pi} = 0$

$-\frac{1}{k}\cos k\pi + \frac{1}{k}\cos 0 = 0$

$-\frac{1}{k}\cos k\pi + \frac{1}{k} = 0$

$-\cos k\pi + 1 = 0$

$\cos k\pi = 1.$



$k\pi = 0, 2\pi, 4\pi$

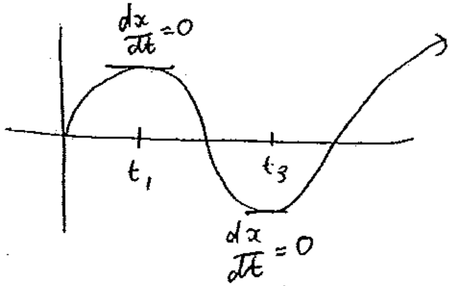
$k = 0, 2, 4, \ldots$

$\therefore$ even

A

Question 10

Particle at rest when $\frac{dx}{dt} = 0$ (ie velocity)



B

t_1 & t_3 at rest

Section II

Q.11 a) $\frac{e^{\pi}}{\ln \pi} = 20.2$

b) $\frac{m-1}{4} - 6 = \frac{m}{2}$

$m - 1 - 24 = 2m$

$m - 25 = 2m$

$-25 = m$

$m = -25$

c) $g'(t) = 6t^2 - 1$

$g(t) = \int 6t^2 - 1 \, dt$

$= 2t^3 - t + C$

$g(-1) = 2$

$2 = -2 + 1 + C$

$\therefore C = 3$

$\therefore g(t) = 2t^3 - t + 3$

d) $\sum_{n=1}^{5} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32}$

$= \frac{31}{32}$

11) e) $7x - y + 49x^2 - y^2$

$= 7x - y + (7x-y)(7x+y)$

$= (7x-y)(1+7x+y)$

Note: If there are 4 terms you should be able to factorise to have a common factor. If not, try a different set of pairs.

f) $y = \dfrac{2x^4}{\cos x}$

$u = 2x^4$

$u' = 8x^3$

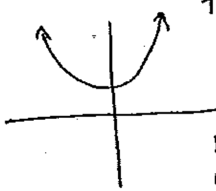
$v = \cos x$

$v' = -\sin x$

$y' = \dfrac{vu' - uv'}{v^2}$

$= \dfrac{\cos x\,(8x^3) + 2x^4 \sin x}{(\cos x)^2}$

$= \dfrac{2x^3(4\cos x + x\sin x)}{\cos^2 x}$



this parabola is positive definite
All the y-coordinates are positive

g) $px^2 + 2x + p = 0$

To be positive definite $a > 0$

$\Delta < 0$ (no roots)

$\Delta = b^2 - 4ac$

$= 2^2 - 4p^2$

$4 - 4p^2 < 0$

$(1+p)(1-p) < 0 \Longrightarrow$

$p < -1$ or $p > 1$

since $a > 0$, $p > 0$

$\therefore$ $p > 1$ is the solution

(5)

12a) $\displaystyle\int \sec^2 \frac{x}{2}\,dx = 2\tan\frac{x}{2} + C$

b) $\displaystyle\int_2^4 \frac{6x}{x^2-3}\,dx = 3\int_2^4 \frac{2x}{x^2-3}\,dx$

Note: denominator

$u = x^2 - 3$

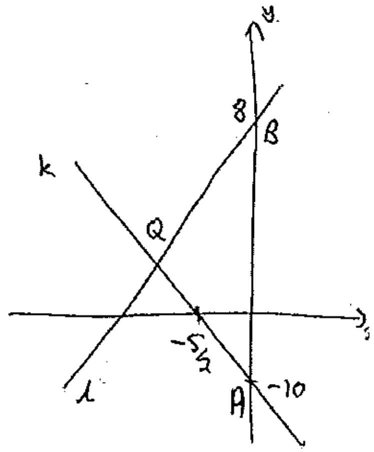
$u' = 2x \Rightarrow$ ln with $2x$ as numerator

$= 3\left[\ln(x^2-3)\right]_2^4$

$= 3\left[\ln 13 - \ln 1\right]$

$= 3\ln 13$

c) line k $(-\frac{5}{2}, 0)$ & $(0, -10)$ lie on the line



i) $m = \dfrac{-10-0}{0+\frac{5}{2}}$

$= -4$

$y = mx + b$

$y = -4x - 10$

$4x + y + 10 = 0$

ii) $y = 2x + 8$ — ①

$y = -4x - 10$ — ②

Point of intersection ① + ②

$2x + 8 = -4x - 10$

$6x = -18$

$x = -3$

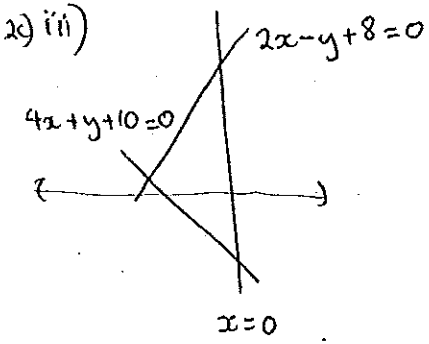
$y = 2(-3) + 8$

$= 2$

Point of intersection $(-3, 2)$

(6)

2c) iii)



All lines are full ∴ equal to !!

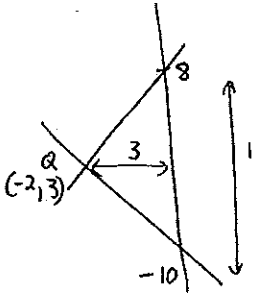
$x \leq 0$

$2x - y + 8 \geq 0$

$4x + y + 10 \geq 0$

iv) $A = \frac{1}{2} \times 3 \times 18$

$= 27 \text{ units}^2$



2 d) i) Angle sum of dodecagon $= (12-2) \times 180 = 1800°$

$\text{Sum} = \frac{n}{2}(2a + (n-1)d)$

$S_{12} = 6(124 + 11d)$

$1800 = 6(124 + 11d)$

$11d = 176$

$d = 16$

ii) $\underbrace{64, 78}_{\text{acute}}, 94, 110, 126, 142, 158, 174, \underbrace{190, \ldots}_{\text{reflex}}$

6 angles are obtuse.

e) $y = \left(\frac{1}{2}\right)^{-x}$

i)

	y_0	y_1	y_2	y_3	y_4
x	-2	-1	0	1	2
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4

ii) $\int_{-2}^{2} \left(\frac{1}{2}\right)^{-x} dx = \frac{h}{3}\left(y_0 + y_4 + 2(y_2) + 4(y_1 + y_3)\right)$

$= \frac{1}{3}\left(\frac{1}{4} + 4 + 2(1) + 4\left(\frac{1}{2} + 2\right)\right)$

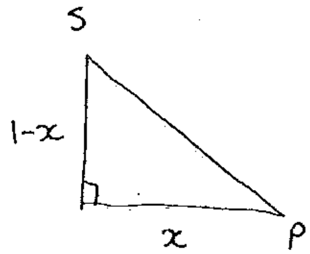
$= \frac{1}{3} \times \frac{65}{4}$

$= \frac{65}{12} \text{ units}^2$

13 a i) $SP^2 = x^2 + (1-x)^2$

$SP^2 = x^2 + 1 - 2x + x^2$

$SP^2 = 2x^2 - 2x + 1$



$SP^2 =$ Area of square

$= 2x^2 - 2x + 1$

ii) $A = 2x^2 - 2x + 1$

$\frac{dA}{dx} = 4x - 2$

Stationary point when $\frac{dA}{dx} = 0$

$4x - 2 = 0$

$x = \frac{1}{2}$

$\frac{d^2A}{dx^2} = 4 > 0$ ∴ minimum

Minimum when $x = \frac{1}{2}$

$A = 2x^2 - 2x + 1$

$= \frac{1}{2} - 1 + 1$

$= \frac{1}{2} \text{ units}^2$

b) i) $\frac{2}{x} = 3 - x$

$2 = 3x - x^2$

$x^2 - 3x + 2 = 0$

$(x-2)(x-1) = 0$

$x = 2, \quad x = 1$

$y = 1, \quad y = 2$

$H(2,1), \quad K(1,2)$

$y = \frac{2}{x}$

$x = \frac{2}{y}$

$x^2 = \frac{4}{y^2}$

$y = 3 - x$

$x = 3 - y$

$x^2 = (3-y)^2$

$= 9 - 6y + y^2$

ii) $V = \pi \int_a^b x^2 \, dy$

$= \pi \int_1^2 \left(9 - 6y + y^2 - \frac{4}{y^2}\right) dy$

iii) $V = \pi \left[9y - 3y^2 + \frac{y^3}{3} + \frac{4}{y} \right]_1^2$

$= \pi \left[\left(18 - 12 + \frac{8}{3} + 2\right) - \left(9 - 3 + \frac{1}{3} + 4\right) \right]$

$= \pi \left(\frac{32}{3} - \frac{31}{3} \right)$

$= \frac{\pi}{3} \text{ units}^3$

13 c) $2e^{2x} - 7e^x + 3 = 0$

let $u = e^x$

$2u^2 - 7u + 3 = 0$

$(u-3)(2u-1) = 0$

$\therefore u = 3$ or $u = \frac{1}{2}$

$e^x = 3$ or $e^x = \frac{1}{2}$

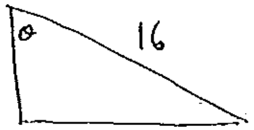
take logs of both sides

$x = \ln 3$ or $x = \ln \frac{1}{2}$

d) i) $x = -4, \quad x = 2$

ii) $x > 2$

14a) i)



$\cos\theta = \frac{4}{16}$

$\theta = 1.318$ radians

$\therefore \angle POQ = 2.64$ radians

ii) Area of segment $= \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$

$= \frac{1}{2}\times 16^2(2.64 - \sin 2.64)$

$\doteq 276.37\text{ cm}^2$

Area of circle $= \frac{1}{2}r^2 \times 2\pi$

$= 16^2\pi$

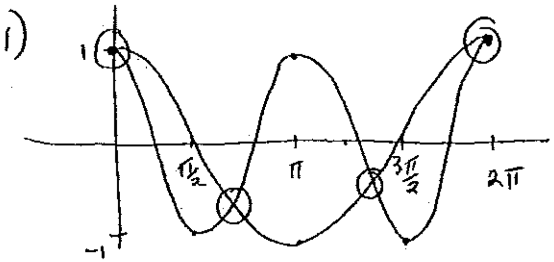
$= 804.25\text{ cm}^2$

$\therefore$ Area above the surface $= 804.25 - 276.37$

$= 528\text{ cm}^2$.

b) i) $\cos\frac{2\pi}{3} = -\frac{1}{2}$, $\cos\frac{4\pi}{3} = -\frac{1}{2}$

ii)



iii) 4 solutions

c) i) $N = 21e^{kt}$

when $t=7$, $N=30$

$30 = 21e^{7k}$

$e^{7k} = \frac{10}{7}$

$\ln e^{7k} = \ln\frac{10}{7}$

$7k = \ln\frac{10}{7}$

$k = \frac{\ln\frac{10}{7}}{7}$

$= 0.05095356342$

Note: Don't round k.

$N = 21e^{0.05095\ldots\times t}$

ii) When $t = 24$

$N = 21e^{0.05095\ldots\times 24}$

≈ 71 bacteria

iii) $N = 21e^{kt}$

$\frac{dN}{dt} = 21\times k\times e^{kt}$

when $t=24$

$\frac{dN}{dt} = 21\times 0.05095\ldots\times e^{0.05095\ldots\times 24}$

$= 3.63$

≈ 4 bacteria / hour.

14 c iv) $3000 > 21e^{0.05095...\times t}$

$e^{0.05095...\times t} < \frac{1000}{7}$

$t < \left(\ln \frac{1000}{7}\right) \div 0.05095...$

$t < 97.4$ hours

$t < 4.057...$ days

Safe upto 4 days.

15 a) i) $f(x) = \frac{9-x^2}{3}$

to find x-intercept, when $x=0$

$\frac{9-x^2}{3} = 0$

$9 - x^2 = 0$

$x^2 = 9$

$x = \pm 3$

$\therefore A(3,0)$

ii) P parallel to AB

Gradient of AB $= \frac{0-3}{3-0} = -1$. The gradient of the curve is given by $f'(x)$

$f(x) = \frac{9-x^2}{3}$

$f(x) = 3 - \frac{x^2}{3}$

$f'(x) = -\frac{2x}{3}$

$\therefore -\frac{2x}{3} = -1$

$-2x = -3$

$x = 3/2$

$\therefore y = 2\frac{1}{4}$

$P(1\frac{1}{2}, 2\frac{1}{4})$

15 a iii) Equation of tangent $(1\frac{1}{2}, 2\frac{1}{4})$ $m=-1$

$y - y_1 = m(x - x_1)$

$y - 2\frac{1}{4} = -1(x - 1\frac{1}{2})$

$4y - 9 = -4x + 6$

$4x + 4y - 15 = 0$

iv) Area = Area of Δ – Area under curve

Area of triangle $= \frac{1}{2} \times 3\frac{3}{4} \times 3\frac{3}{4}$

$= \frac{225}{32}$ units^2

x-int, $y=0$

$4x = 15$

$x = 3\frac{3}{4}$

$(3\frac{3}{4}, 0)$

similarly y-intercept

$(0, 3\frac{3}{4})$

Area under curve $= \int_0^3 3 - \frac{x^2}{3}\,dx$

$= \left[3x - \frac{x^3}{9}\right]_0^3$

$= 9 - \frac{27}{9}$

$= 6$ units^2

Area $= \frac{225}{32} - 6$

$= \frac{33}{32}$ units^2

5b) i) 18% pa = 0.015

After 1 month $A_1 = 30000 - m$

After 2 months $A_2 = 30000 - 2m$

After 6 months $A_6 = 30000 - 6m$

ii) After 7 months

$A_7 = (30000 - 6m)1.015 - m$

$A_8 = [(30000 - 6m)1.015 - m]1.015 - m$

$= (30000 - 6m)1.015^2 - 1.015m - m$

$= (30000 - 6m)1.015^2 - m(1 + 1.015^1)$

iii) $A_{48} = 0$

$A_{48} = (30000 - 6m)1.015^{42} - m(1 + 1.015 + \ldots + 1.015^{41})$

$0 = (30000 - 6m)1.015^{42} - m(66\tfrac{2}{3}(1.015^{42} - 1))$

$0 = 30000 \times 1.015^{42} - 6 \times 1.015^{42} m - m \times 66\tfrac{2}{3}(1.015^{42} - 1)$

$m[6 \times 1.015^{42} + 66\tfrac{2}{3}(1.015^{42} - 1)] = 30000 \times 1.015^{42}$

$$m = \frac{30000 \times 1.015^{42}}{(6 \times 1.015^{42} + 66\tfrac{2}{3}(1.015^{42} - 1))}$$

$= \$810.94$

$= \$811$

$a = 1$, $r = 1.015$, $n = 42$

$S_{42} = \dfrac{a(r^n - 1)}{r - 1}$

$= \dfrac{1(1.015^{42} - 1)}{0.015}$

$= 66\tfrac{2}{3}(1.015^{42} - 1)$

5 c) $\dfrac{XU}{XS} = \dfrac{TV}{VS}$ (ratio of parallel intercepts are equal)

$\dfrac{8}{4} = \dfrac{24}{x}$

$8x = 96$

$x = 12$ cm

QS

$\Delta SXV ||| \Delta SUT$ (equiangular)

$\dfrac{SV}{ST} = \dfrac{SX}{SU}$

$\dfrac{x}{x+24} = \dfrac{4}{12}$

$12x = 4(x + 24)$

$8x = 96$

$x = 12$

or similar using other triangles but your ratios must match your reasons

16 a i) $\dot{x} = 3 - \dfrac{9}{t-2}$

when $t = 3$

$V = \dot{x} = 3 - \dfrac{9}{3-2}$

$= -6$ moving towards the left.

ii) changes direction when $\dot{x} = 0$

$3 - \dfrac{9}{t-2} = 0$

$\dfrac{9}{t-2} = 3$

$9 = 3t - 6$

$t = 5$

Changes direction after 5 mins

iii) $x = \int 3 - \dfrac{9}{t-2} \, dt$

$= 3t - 9\ln(t-2) + C$

Between $t = 3$ & $t = 5$, moving to left

$t = 5$ & $t = 7$ moving to right

$x = \left| \left[3t - 9\ln(t-2) \right]_3^5 \right| + \left[3t - 9\ln(t-2) \right]_5^7$

$= |(15 - 9\ln 3) - (9 - 0)| - ((21 - 9\ln 5) - (15 - 9\ln 3))$

$= |-3.8875| + 1.4026$

$= 5.29$

or

$t = 3$	$t = 5$	$t = 7$
$x = 9$	$x = 5.11$	$x = 6.52$

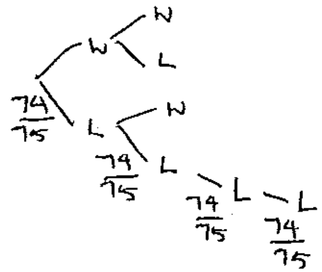
$x = (9 - 5.11) + (6.52 - 5.11)$

$= 5.2$

16 b i) $P(WW) = \dfrac{1}{75} \times \dfrac{1}{75}$

$= \dfrac{1}{5625}$

ii) $P(\text{at least 1 win}) = 1 - P(\text{no win})$



$\leftarrow P(\text{no win})$

$= 1 - P(\text{no win})$

$= 1 - \left(\dfrac{74}{75}\right)^n$

$1 - \left(\dfrac{74}{75}\right)^n = 0.95$

$\left(\dfrac{74}{75}\right)^n = 0.05$

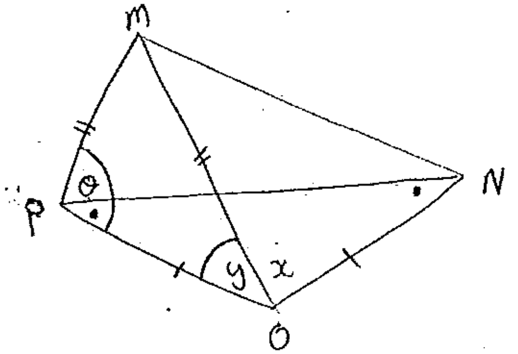
$\ln\left(\dfrac{74}{75}\right)^n = \ln 0.05$

$n = \dfrac{\ln 0.05}{\ln \frac{74}{75}}$

$= 223.1$

$\therefore$ 224 draws are needed.

16 c)



$\angle NPO = \dfrac{180 - (x+y)}{2}$ (Base ∠'s of isos △PNO are equal & △ sum of △PNO is 180°)

$\therefore \angle MPO = \theta + \dfrac{180 - (x+y)}{2}$ (adjacent ∠'s)

$\angle MPO = \angle POM = y$ (base angles in △MOP)

$\therefore \quad y = \theta + \dfrac{\pi - (x+y)}{2}$

$2y = 2\theta + \pi - x - y$

$2\theta = 3y + x - \pi$

$\theta = \dfrac{3y + x - \pi}{2}$

16 d) i) $y = (x-1)x(x+1)$

$= x(x^2-1)$

$= x^3 - x$

ii) $y = (x+1999)(x+2000)(x+2001)$

let $X = x + 2000$

$\therefore \quad y = (X-1)X(X+1)$

$\therefore \quad y = X^3 - X$ from part (i)

$\dfrac{dy}{dx} = 3X^2 - 1$

Turning point $\dfrac{dy}{dx} = 0$

$3X^2 - 1 = 0$

$X^2 = \dfrac{1}{3}$

$X = \pm\dfrac{1}{\sqrt{3}}$

$\dfrac{d^2y}{dx^2} = 6X$

when $X = \dfrac{1}{\sqrt{3}}$

$6 \times \dfrac{1}{\sqrt{3}} > 0$

$\therefore$ min

when $X = -\dfrac{1}{\sqrt{3}}$

$6 \times -\dfrac{1}{\sqrt{3}} < 0 \quad \therefore$ max

Sub to find y

$y = X^3 - X$

$= \left(\dfrac{1}{\sqrt{3}}\right)^3 - \dfrac{1}{\sqrt{3}} = \dfrac{1}{3\sqrt{3}} - \dfrac{1}{\sqrt{3}} = -\dfrac{2\sqrt{3}}{9}$