

CURVE SKETCHING No 6 ASYMPTOTES

An asymptote is a line that a curve approaches but does not cross at its extremities
(That means that a non vertical asymptote can be crossed in the middle regions of the graph)

a) Vertical: These occur when the denominator is zero

i) ODD
single (or odd number of factors)

$$y = \frac{1}{x+1}$$

ii) EVEN
double (or even number of factors)

$$y = \frac{1}{(x+1)^2}$$

b) Horizontal

i) $y = 0$
degree of denominator > deg numerator

$$y = \frac{1}{x+1}$$

ii) $y = k$
degree of denominator = deg numerator

$$y = \frac{x}{(x+1)}$$

c) Inclined

i) deg num - deg den = 1

$$y = \frac{x^2}{(x+1)}$$

ii) square root

$$y = \sqrt{x^2 - 4}$$

Exercises: Sketch the following curves without the use of calculus. Note that where the numerator is equal to zero, the equation will have a root.

1. $y = \frac{2}{x-2}$ ✓ 2. $y = \frac{2}{2-x}$ 3. $y = \left(\frac{2}{x-2}\right)^2$ ✓

4. $y = \frac{x-1}{x+1}$ ✓ 5. $y = \frac{2x}{x-2}$ ✓ 6. $y = \frac{2}{2x-1}$

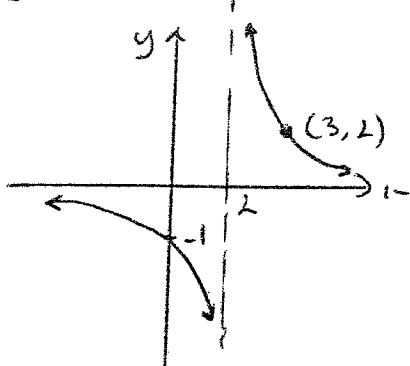
7. $y = \frac{x^2+1}{x}$ ✓ 8. $y = \frac{x^2+1}{x+1}$ ✓ 9. $y = \frac{x^2+1}{x^2}$ ✓

10. $y = \frac{1}{x^2-1}$ ✓ 11. $y = \frac{x-1}{(x+1)(x+3)}$ 12. $y = \frac{x-1}{x(x+3)}$ ✓

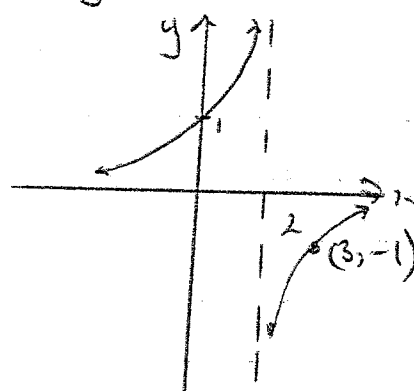
13. $y = \frac{2x}{(x-1)^2}$ 14. $y = \frac{2}{(x-1)(x+1)^2}$ 15. $y = \sqrt{x^2-1}$

Solutions Curve Sketching 6

1. $y = \frac{2}{x-2}$

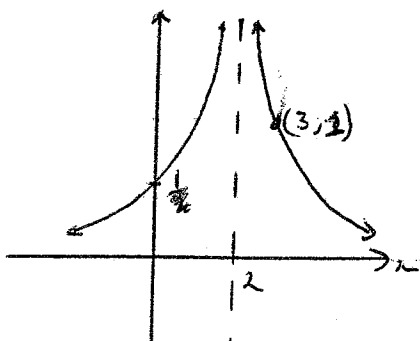


2. $y = \frac{2}{2-x}$



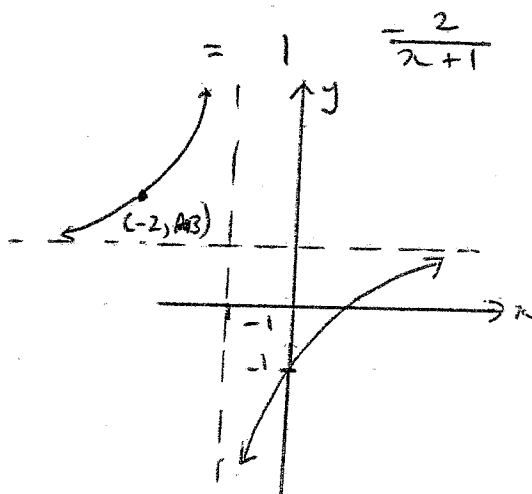
3.

$y = \frac{4}{(x-2)^2}$



$x=2$ vert asympt (even)

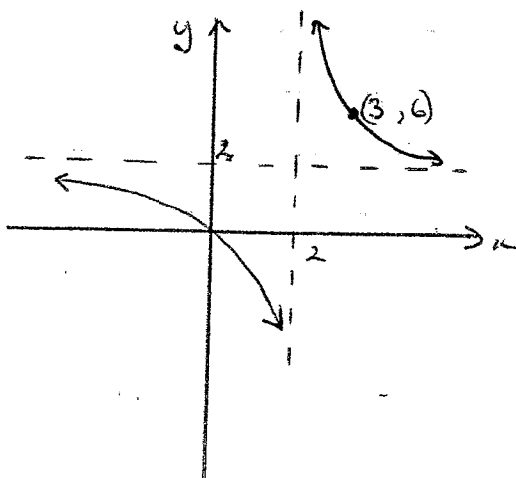
4. $y = \frac{x-1}{x+1}$
 $= \frac{x+1-2}{x+1}$



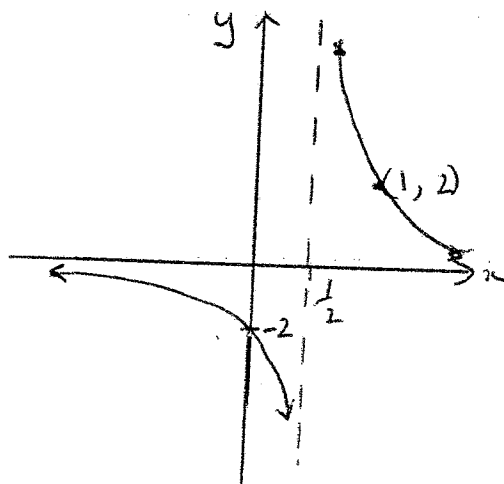
5. $y = \frac{2x}{x-2}$

Horiz asympt $y=2$

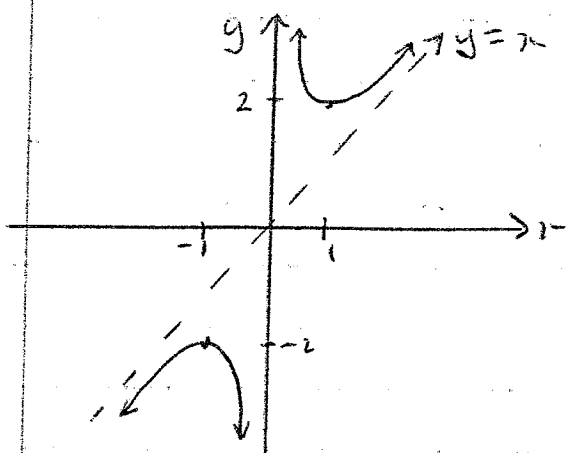
vert $x=2$



6. $y = \frac{2}{2x-1}$

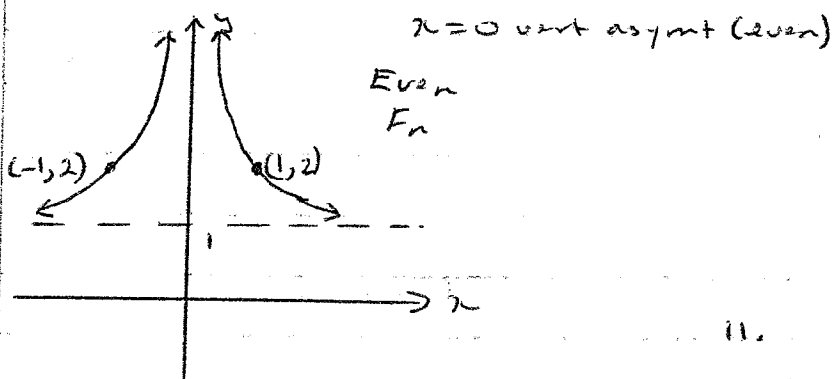


7. $y = \frac{x^2+1}{x}$
 $= x + \frac{1}{x}$ odd Fn



9. $y = \frac{x^2+1}{x^2}$

$y = 1 + \frac{1}{x^2}$

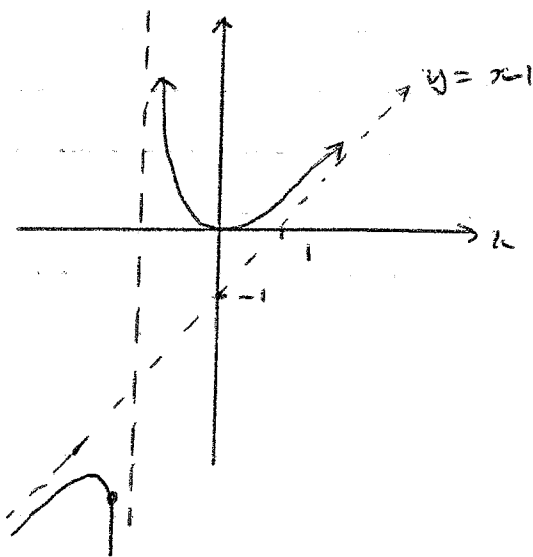


8. $y = \frac{x^2+1}{x+1}$

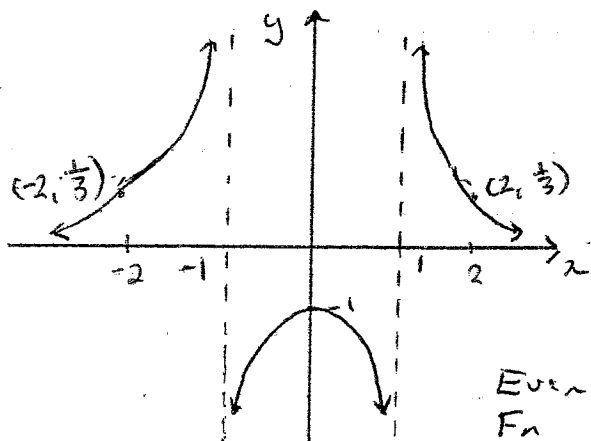
$$x+1 \overline{) x^2 - 1}$$

$$\begin{array}{r} x^2 + 0x + 1 \\ -x + 0 \\ \hline -x - 1 \\ -x - 1 \\ \hline 2 \end{array}$$

$y = x - 1 + \frac{2}{x+1}$

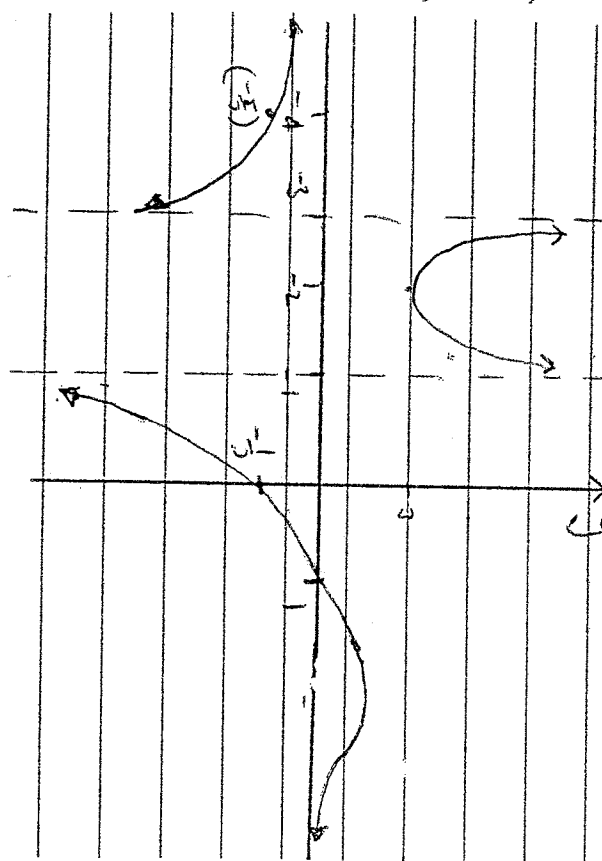


10. $y = \frac{1}{x^2-1}$
 $= \frac{1}{(x+1)(x-1)}$

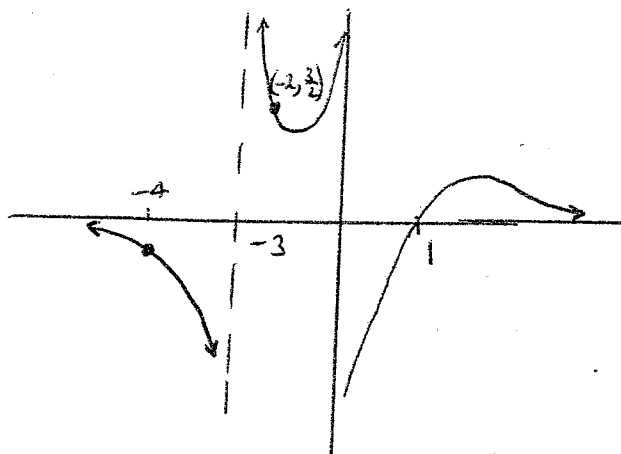


When $x = 0, y = -1$
 $x = 2, y = 3$
 $x = -2, y = 3$

11. $y = \frac{x-1}{(x+1)(x+3)}$



12. $y = \frac{x-1}{x(x+3)}$



Vert Asympt $x=0, x=-3$

Horiz x axis

x intercept 1 No y intercept

$x = -2, y = \frac{-3}{-2(-1)} = \frac{3}{2}$

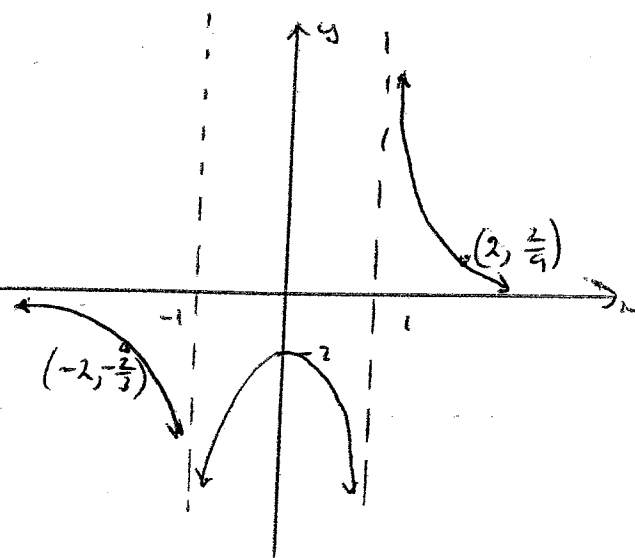
$x = -4, y = \frac{-5}{(-4)(-1)} = \frac{-5}{4}$

14. $y = \frac{2}{(x-1)(x+1)^2}$

Vert Asympt $x=1$ (odd)

" " $x=-1$ (even)

x axis horizontal

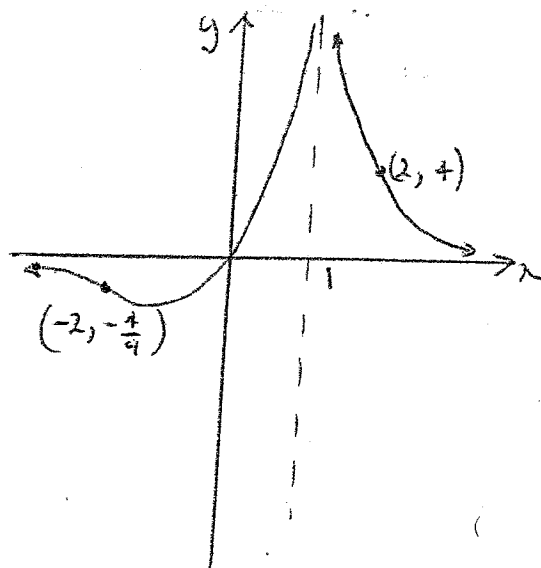


When $x=0, y = -2$

$x=2, y = \frac{2}{9}$

$x=-2, y = \frac{-2}{3}$

13. $y = \frac{2x}{(x-1)^2}$



Vert Asympt $x=1$ (even)

horiz x axis

when $x=0, y=0$

$x=-2, y = \frac{-4}{9}$

15. $y = \sqrt{x^2 - 1}$

defined for $x^2 - 1 \geq 0$

$x \leq -1, x \geq 1$

as $x \rightarrow \pm\infty, y \rightarrow \pm x$

