Name:

Teacher:

DANEBANK

An Anglican School for Girls

2006

Year 12

Mathematics

Time Allowed – 2 hours
(Plus 5 minutes reading time)

(This examination paper does not necessarily reflect the content or format of the final Higher School Certificate Examination Paper for this subject)

Assessment Task #2 Weight: 25%

Outcomes examined: P2 P3,P4,P6,P7,H2,H3,H4,H5,H6,H8,H9

DIRECTIONS TO CANDIDATE:

- Attempt all questions.
- Start each new question in a new booklet.
- Show all necessary working otherwise full marks may not be awarded
- Marks may be deducted for careless or badly arranged work.
- All questions are of equal value.
- Approved calculators may be used.
- Write your name on this paper.
- A table of standard integrals is provided on the back page.

Question 1 (10 marks) **(Start a new booklet)**

a. Factorise fully:

i. x^3+27 — 1

ii. $a^2-n^2+3a+3n$ — 2

b. Solve the following:

i. $|2x+5|=9$ — 2

ii. $(x-3)^2=16$ — 2

c. Find the exact value of $t+\frac{1}{t}$ given $t=3-\sqrt{5}$ — 3

Question 2 (10 marks) **(Start a new booklet)**

$A(-1,3)$, $B(-3,-2)$, $C(2,-1)$ and D are the vertices of a parallelogram with AC as diagonal.

i. Plot the points and write down the coordinates of D — 1

ii. Find the length of BC — 1

iii. Show that the gradient of BC is $\frac{1}{5}$ — 1

iv. Show that the equation of the line BC is $x-5y-7=0$ — 1

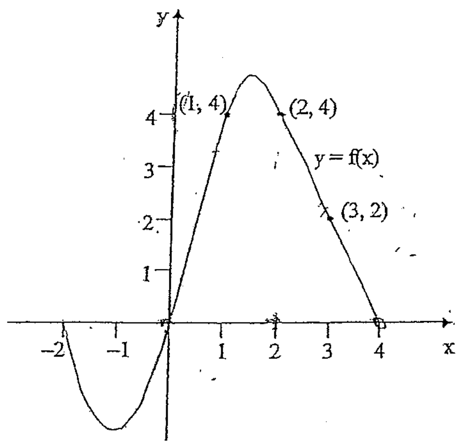
v. Find the perpendicular distance from A to BC — 1

vi. State the definition of a parallelogram and use it to determine the equation of AD. — 3

vii. Find the area of the parallelogram $ABCD$ — 2

Question 3 (10 marks) (Start a new booklet)

a. Below is the graph of $y = f(x)$ for $-2 \le x \le 4$ — 2



Use two applications of the Trapezoidal Rule to approximate the area enclosed by the curve, the x axis and the lines $x = 0$ and $x = 4$.

b. Differentiate :

i. $y = e^{x^2+3}$ — 1

ii. $f(x) = (x^3 + 2)e^{-2x}$ — 3

c. Bricks are stacked in a pile such that there are 20 bricks on the top row, 21 on the next, 22 on the next, and so on until 245 bricks in total are in the pile.

i. How many rows are there? — 3

ii. How many bricks are in the bottom row? — 1

Question 4 (10 marks) (Start a new booklet)

a. The gradient of a curve is given by $\frac{dy}{dx} = 3x^2 - 6x - 9$

i. The curve passes through the point $(1, -2)$, show that the equation of the curve is $y = x^3 - 3x^2 - 9x + 9$ — 1

ii. Find the coordinates of the stationary points and determine their nature. — 3

iii. Find the coordinates of the point of inflexion. — 2

iv. Sketch the curve, make sure you show its stationary points, point of inflexion and y-intercept. — 2

b. The limiting sum of a geometric series is 30. Give a possible example of a geometric series that fits these conditions. — 2

Question 5 (10 marks) (Start a new booklet)

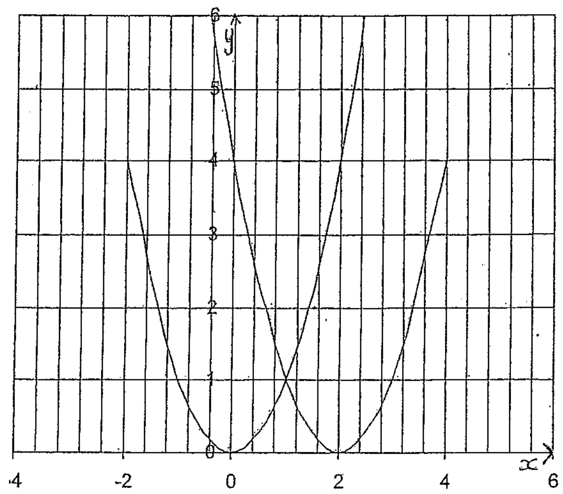
a. Consider the curve $y = 2 + e^x$.

i. Find the equation of the tangent to the curve at the point when $x = 0$ — 2

ii. **Without the use of calculus** sketch the curve $y = 2 + e^x$. On your graph also show the tangent at $x = 0$, clearly showing its x and y intercepts. — 3

iii. Find the area of the region between the curve $y = 2 + e^x$ and the tangent, from $x = a$ to $x = 0$, where a is the point of intersection of the tangent and the x-axis. — 3

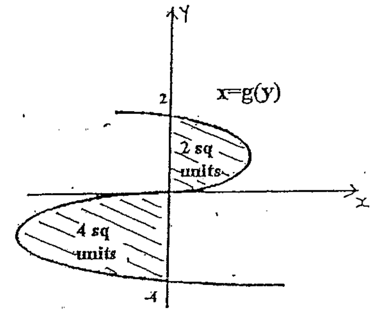
b. The diagram below shows the graph of $y=x^2$ and $y=(x-2)^2$ 2



Fiona needs to calculate the area enclosed by the two curves and the x-axis. Explain how she would go about this, include any definite integrals she would need to evaluate in your explanation. **DO NOT DO THE CALCULATION.**

Question 6. (10 marks) **(Start a new booklet)**

a. The area between each loop of the curve $x=g(y)$ and the y-axis is shown on the diagram below.



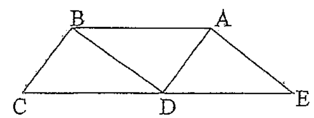
Use this information to find the value of:

i. $\int_0^2 g(y)dy$ 1

ii. $\int_{-4}^0 g(y)dy$ 1

iii. $\int_{-4}^2 g(y)dy$ 1

b. In the figure, ABCD is a parallelogram. CD is produced to E so that AE = ED and is AE∥BD. Let ∠ AED = x and ∠ EAD = y.



i. Copy the diagram and mark on all the data. 1

ii. Prove that Δ DBC is isosceles. 3

c. The curve $y=\frac{1}{x}$ is rotated about the x-axis between $x=1$ and $x=2$. Use Simpson's Rule with three function values to estimate the **volume** of the solid formed. 3

Question 7 (10 marks) (Start a new booklet)

a. Below are four integral statements. **Select the correct statement** needed to answer the following problem. 1

I. $\int_0^4 y\,dy$ II. $\pi\int_0^4 y^2\,dy$ III. $\int_0^4 x\,dx$ IV. $\pi\int_0^4 x^2\,dx$

Find the volume generated when $y = x$ is rotated about the y-axis between $y = 0$ and $y = 4$.

b. A woman borrows $20 000 at 18% p.a. reducible interest, and pays it off in equal monthly instalments.

i. Express the interest rate as a monthly rate in decimal form. 1

ii. Show that the amount she owes at the end of the second month is 1

$A_2 = 20000(1.015)^2 - 1.015M - M$

where M is the monthly repayment.

iii. Write an expression of A_n, the amount owed after n months. 1

iv. Find the amount of the monthly repayment if she repays the loan in 5 years. 3

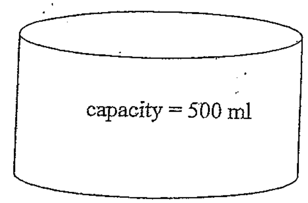
c. Find the volume of the solid formed when $y = e^{3x}$ is rotated about the x-axis, from $x = 0$ to $x = 2$. Leave your answer in exact form. 3

Question 8 (10 marks) (Start a new booklet)

a. i. Differentiate $y = (e^x + 1)^5$ 1

ii. Hence evaluate $\int_0^1 20e^x(e^x + 1)^4\,dx$ 3

b. A soft drink company is designing a drink can to hold 500 ml, in the shape of a closed cylinder.



The metal used on the side of the can will cost 2 cents/ 100 cm^2. The metal used on the top and bottom of the can will cost twice as much.
Let the radius of the can be r cm and the height be h cm.

i. Show that the total area of metal needed to make the can is given by 1

$$A = \frac{1000}{r} + 2\pi r^2$$

ii. If C is the cost (in cents) of making the can, show that:

$$C = 0.02(4\pi r^2 + \frac{1000}{r})$$ 1

iii. Find the dimensions of the can so as to minimise the cost. 4

END OF PAPER

2u. 2006 ½ Yrly Ass2 sol^{n}s.

Q1 a) i) $x^3+27=(x+3)(x^2-3x+9)$ (½ or nothing) 1

ii) $a^2-n^2+3a+3n=(a+n)(a-n)+3(a+n)$

$=(a+n)(a-n+3)$ 2

b) i) $|2x+5|=9$

$2x+5=9$ or $2x+5=-9$ 2

$2x=4$ $\quad 2x=-14$

$x=2$ $\quad x=-7$

ii) $(x-3)^2=16$

$x-3=\pm 4$

$x=4+3,\ -4+3$ 2

$=7,\ -1$

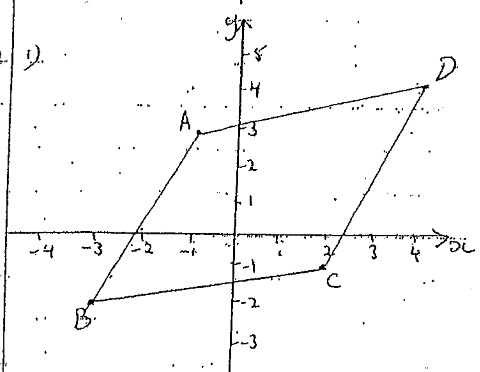
c) $t+\frac{1}{t}=3-\sqrt5+\frac{1}{3-\sqrt5}$ 3

$=3-\sqrt5+\frac{3+\sqrt5}{4}$

$=\frac{12-4\sqrt5+3+\sqrt5}{4}$

$=\frac{15-3\sqrt5}{4}$

Q2 i)



D (4, 4)

ii) $BC=\sqrt{(2--3)^2+(-1--2)^2}$

$=\sqrt{25+1}$

$=\sqrt{26}$

iii) $m_{BC}=\frac{-1--2}{2--3}$

$=\frac{1}{5}$

iv) Eq^n BC

$y--1=\frac{1}{5}(x-2)$

$5y+5=x-2$

$x-5y-7=0$

v) $d=\frac{|1\times 1-5\times 3-7|}{\sqrt{1^2+(-5)^2}}$

$=\frac{|-1-15-7|}{\sqrt{26}}$

$=\frac{23}{\sqrt{26}}$

vi) two pair of opp sides parall

AD ∥ BC

$\therefore m_{AD}=\frac{1}{5}$

eq^n AD $\quad y-3=\frac{1}{5}(x+1)$

$5y-15=x+1$

$x-5y+16=0$

vii) Area = base × height

$=\sqrt{26}\times\frac{23}{\sqrt{26}}$

$=23$ units^2

Q3 a)

x	$f(x)$		
0	0		0
2	4	×2	8
4	0		0
		Total =	8

Area $\doteqdot \frac{h}{2}\times$ total

$=\frac{2}{2}\times 8$

$=8$ units^2

b) i) $y=e^{x^2+3}$

$\frac{dy}{dx}=(2x)e^{x^2+3}$

ii) $f(x)=(x^3+2)e^{-2x}$

$f'(x)=(x^3+2)\times -2e^{-2x}+3x^2 e^{-2x}$

$=e^{-2x}(-2x^3-4+3x^2)$

c) 20, 21, 22 ...

i) $S_n=245$

$S_n=\frac{n}{2}(2\times 20+(n-1)\times 1)$

$245=\frac{n}{2}(40+n-1)$

$490=39n+n^2$

$n^2+39n-490=0$

$(n+49)(n-10)=0$

$\therefore n=-49$ not valid $\quad n=10$

there are 10 rows

ii) $T_{10}=20+9\times 1$

$=29$

Q4. a)

i) $\frac{dy}{dx}=3x^2-6x-9$

$y=x^3-3x^2-9x+C$

when $x=1$ $\quad y=-2$

$-2=1-3-9+C$

$C=+9$

$\therefore y=x^3-3x^2-9x+9$

ii) stationary pts when $\frac{dy}{dx}$ = 0

$3x^2-6x-9=0$

$x^2-2x-3=0$

$(x-3)(x+1)=0$

$x=-1,\ 3$

when $x=-1$

$y=-1-3+9+9$

$=14$

$x=3$

$y=27-27-27+9$

$=-18$

$\frac{d^2y}{dx^2}=6x-6$

when $x=-1$

$\frac{d^2y}{dx^2}<0$ $\therefore$ local max^m $(-1, 14)$

when $x=3$

$\frac{d^2y}{dx^2}>0$ $\therefore$ local min $(3, -18)$

Q4 cont.

iii) $\frac{d^2y}{dx^2} = 6x - 6$

pts of inflexion when $\frac{d^2y}{dx^2} = 0$

& change in concavity

$\therefore 6x - 6 = 0$

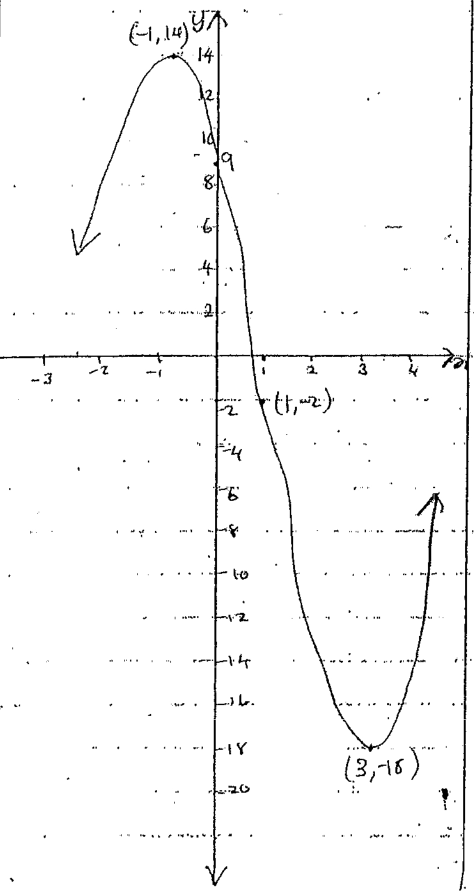
$x = 1$

test $\frac{d^2y}{dx^2}$

x	1^-	1	1^+
$\frac{d^2y}{dx^2}$	$-$	0	$+$

$\therefore$ pt of inflexion at $(1, -2)$

iv)



b) $S_\infty = \frac{a}{1-r}$

$|r| < 1$

let $r = \frac{1}{2}$

$\therefore 30 = \frac{a}{1-\frac{1}{2}}$

$a = 15$

$\therefore$ Series $15, 7\frac{1}{2}, 3\frac{3}{4}$ etc.

they need to choose an appropriate value of r and solve for a.

Q5.

a) i) $y = 2 + e^x$

$\frac{dy}{dx} = e^x$

gradient of tangent $= e^0$
$= 1$

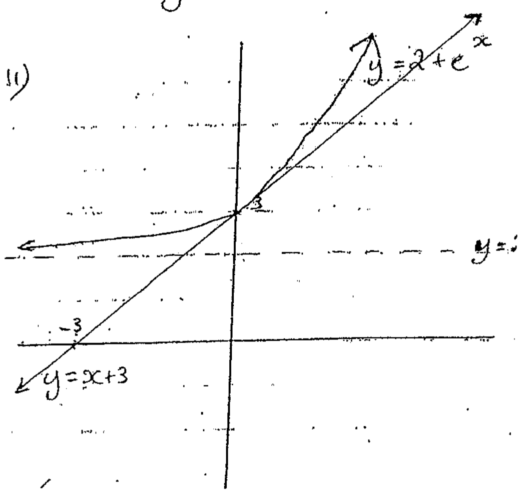
when $x = 0$ $y = 2 + e^0$
$= 3$

eq^n of tangent

$y - 3 = 1(x - 0)$

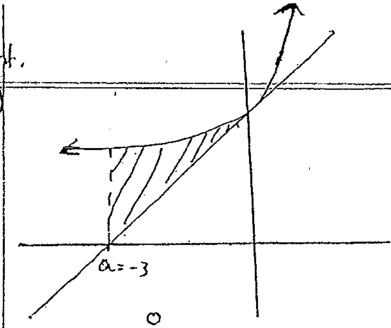
$y = x + 3$

ii)



Q5 cont.

iii)



area $= \int_{-3}^{0} 2 + e^x - (x+3)\, dx$

$= \int_{-3}^{0} (-1 + e^x - x)\, dx$

$= \left[-x + e^x - \frac{x^2}{2}\right]_{-3}^{0}$

$= \left(-0 + e^0 - \frac{0}{2}\right) - \left(3 + e^{-3} - \frac{(-3)^2}{2}\right)$

$= 1 - 3 - \frac{1}{e^3} + \frac{9}{2}$

$= \left(\frac{5}{2} - \frac{1}{e^3}\right)$ units^2

b) She needs to find pt. of intersection and find area under $y = x^2$ to this pt. and add area from this pt under $y = (x-2)^2$.

ie $\therefore$ Area $= \int_0^1 x^2\, dx + \int_1^2 (x-2)^2\, dx$

or

Area $= 2\int_0^1 x^2\, dx$ – but explanation must be given as to why.

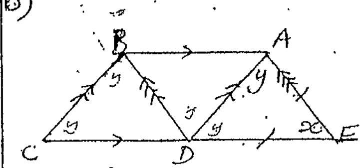
Q6.

a) i) $\int_0^2 g(y)\, dy = 2$

ii) $\int_{-4}^{0} g(y)\, dy = -4$

iii) $\int_{-4}^{2} g(y)\, dy = -4 + 2$
$= -2$

b)



ii) In $\triangle ADE$

$\angle ADE = y$ (base ∠'s of isos. $\triangle ADE$ are equal)

then $\angle ADB = \angle EAD$ (alternate ∠'s $AE \parallel DB$)
$= y$

then $\angle CBD = \angle BDA$ (alternate ∠'s $CB \parallel DA$)
$= y$

& $\angle BCD = \angle ADE$ (corresponding ∠'s =, $CB \parallel DA$)
$= y$

$\therefore \triangle BCD$ is isosceles because $\angle CBD = \angle BCD$.

Q6 cont. c) $h = \frac{b-a}{2}$

$= \frac{2-1}{2}$

$= \frac{1}{2}$

x	$f(x)$		
1	1		1
$1\frac{1}{2} = \frac{3}{2}$	$\frac{2}{3}$	$\times 4$	$\frac{8}{3}$
2	$\frac{1}{2}$		$\frac{1}{2}$
			Tot $= 4\frac{1}{6}$

Area $\doteqdot \frac{h}{3} \times$ tot

$= \frac{1/2}{3} \times 4\frac{1}{6}$

$= \frac{25}{36}$ units^2

Q7. a) (1.1)

b) i) Int rate $= \frac{18/12}{100}$

$= 0.015$

ii) $A_1 = 20000 \times 1.015 - M$

$A_2 = A_1 \times 1.015 - M$

$= (20000 \times 1.015 - M) \times 1.015 - M$

$= 20000 \times 1.015^2 - 1.015M - M$

iii) $A_n = 20000 \times 1.015^n - M(1 + 1.015 + 1.015^2 + \dots 1.015^{n-1})$

iv) $n = 5 \times 12$

$= 60$

After 60 months $A_n = 0$

$\therefore 20000 \times 1.015^{60} - M(1 + 1.015 + \dots + 1.015^{59}) = 0$

but $1 + 1.015 + \dots + 1.015^{59}$ is a G.P with $a = 1$, $r = 1.015$, $n = 60$

$S_{60} = \frac{1(1.015^{60} - 1)}{1.015 - 1}$

$= \frac{1.015^{60} - 1}{0.015}$

$\therefore 20000 \times 1.015^{60} - M\left(\frac{1.015^{60} - 1}{0.015}\right) = 0$

$M\frac{(1.015^{60} - 1)}{0.015} = 20000 \times 1.015^{60}$

$\therefore M = 20000 \times 1.015^{60} \times \frac{0.015}{(1.015^{60} - 1)}$

$= 507.8685\dots$

$= \$507.87$

c. Vol $= \pi \int_0^2 y^2\, dx$

$= \pi \int_0^2 e^{6x}\, dx$

Q7c cont.

Vol $= \pi \left[\frac{e^{6x}}{6}\right]_0^2$

$= \pi \left\{\frac{e^{12}}{6} - \frac{e^0}{6}\right\}$

$= \pi \left(\frac{e^{12}}{6} - \frac{1}{6}\right)$ units^3

Q8. a) i) $y = (e^x + 1)^5$

$\frac{dy}{dx} = 5 \times e^x (e^x + 1)^4$

$= 5e^x (e^x + 1)^4$ (1)

ii) $\int_0^1 20e^x (e^x + 1)^4\, dx$

$= 4 \int_0^1 5e^x (e^x + 1)^4\, dx$ ½

$= 4\left[(e^x + 1)^5\right]_0^1$

$= 4\{(e + 1)^5 - (e^0 + 1)^5\}$ ½ sub.

$= 4\{(e+1)^5 - 2^5\}$ ½ eval.

$= 4(e+1)^5 - 2^2 \times 2^5$

$= 4(e+1)^5 - 2^7$ optional ½ 128

$4(e+1)^5 - 32)$ (3)

b) i) Area $= 2\pi r^2 + 2\pi r h$

but Vol $= \pi r^2 h$

cap $= 500$ ml

Vol $= 500$ cm^3

$500 = \pi r^2 h$

$h = \frac{500}{\pi r^2}$ ½

$\therefore A = 2\pi r^2 + 2\pi \times r \times \left(\frac{500}{\pi r^2}\right)$ ½

$= 2\pi r^2 + \frac{1000}{r}$ (1)

ii) Cost $= 2\pi r^2 \times \frac{4}{100} + \frac{1000}{r} \times \frac{2}{10}$

$= \frac{8}{100}\pi r^2 + \frac{2}{100} \times \frac{1000}{r}$

$= \frac{2}{100}\left(4\pi r^2 + \frac{1000}{r}\right)$

$= 0.02\left(4\pi r^2 + \frac{1000}{r}\right)$ (1)

iii) min^m cost $\frac{dc}{dr} = 0$ & $\frac{d^2c}{dr^2} > 0$

$\frac{dc}{dr} = 0.02\left(8\pi r - \frac{1000}{r^2}\right)$ 1

when $\frac{dc}{dr} = 0$

$0.02\left(8\pi r - \frac{1000}{r^2}\right) = 0$ ½

$8\pi r = \frac{1000}{r^2}$

$r^3 = \frac{1000}{8\pi}$ ½

$r = 10 = 5 \dots$ (4)

Q8 ii) cont.

$$\frac{d^2C}{dr^2} = 0.02\left(8\pi + \frac{2000}{r^3}\right) \quad \tfrac{1}{2}$$

when $r = \frac{5}{\sqrt[3]{\pi}}$

$\frac{d^2C}{dr^2} > 0 \quad \therefore \min^{m}$ when $r = \frac{5}{\sqrt[3]{\pi}}$

$\therefore r = \frac{5}{\sqrt[3]{\pi}}$

$$h = \frac{500}{\pi \times \frac{25}{\pi^{2/3}}}$$

$$= \frac{500}{\sqrt[3]{\pi} \times 25}$$

$$= \frac{20}{\sqrt[3]{\pi}} \doteqdot 13.66 \quad \tfrac{1}{2}$$