ASSESS. TASK (LOGS/EXPON'S) 30/7/98.

(1) a) $\frac{d}{dx} \ln(5x-2) = \frac{5}{5x-2}$ using $\frac{d \ln U}{dU} \cdot \frac{dU}{dx}$
where $U = 5x-2$

b) $\frac{d e^{3-4x}}{dx} = -4e^{3-4x}$ using $\frac{d e^U}{dU} \cdot \frac{dU}{dx}$
where $U = 3-4x$.

c) $\frac{d}{dx}\left(\frac{e^{2x}}{4x}\right) = \frac{4x.(2e^{2x}) - e^{2x}.4}{(4x)^2}$ using $\frac{VU' - UV'}{V^2}$

$= \frac{4e^{2x}\{2x-1\}}{16x^2} = \frac{e^{2x}\{2x-1\}}{4x^2}$

d) $\frac{d(4+e^{7x})^3}{dx} = 3(4+e^{7x})^2 . 7e^{7x}$ using $\frac{dU^3}{dU} \cdot \frac{dU}{dx}$
where $U = (4+e^{7x})$

e) $\frac{d(x^2 \ln x)}{dx} = x^2 . \frac{1}{x} + \ln x . 2x$

$= x(1 + 2\ln x)$

(2) a) $\int \frac{6}{2x-3}\,dx = 3.\int \frac{2}{2x-3}\,dx = 3\ln(2x-3) + c$

using $\int f'(x) . \frac{1}{f(x)}\,dx = \ln f(x) + c$

b) (i) $\int_{-1}^{2} e^{2x+3}\,dx = \frac{1}{2}\left[e^{2x+3}\right]_{-1}^{2}$ using $\int f'(x) e^{f(x)}\,dx = e^{f(x)} + c$.

$= \frac{1}{2}\left[e^7 - e\right]$

(ii) $\int_1^2 x - x^{\frac{1}{2}} - \frac{3}{x}\,dx = \left[\frac{x^2}{2} - \frac{2x^{3/2}}{3} - 3\ln x\right]_1^2$

$= \left[\frac{4}{2} - \frac{2.2\sqrt{2}}{3} - 3\ln 2\right] - \left[\frac{1}{2} - \frac{2}{3} - 0\right]$

$= 2 - \frac{4\sqrt{2}}{3} - 3\ln 2 + \frac{1}{6}$

$= \frac{13 - 8\sqrt{2}}{6} - 3\ln 2 = \frac{13 - 8\sqrt{2} - 18\ln 2}{6}$.

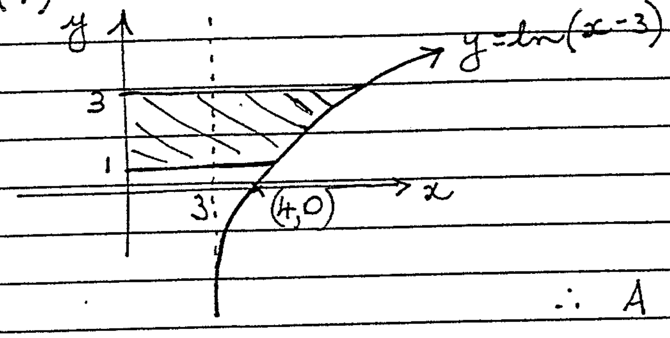
(3) a) $\log_a 12 = \log_a(4\times 3) = \log_a 4 + \log_a 3 = 4\cdot 3$

b) $\log_a 27 = \log_a(3^3) = 3\log_a 3 = 5\cdot 82.$

c) $\log_a\sqrt{3} = \log_a 3^{\frac{1}{2}} = \frac{1}{2}\log_a 3 = 0\cdot 97$

d) $\log_a 0\cdot 75 = \log_a \frac{3}{4} = \log_a 3 - \log_a 4 = \cancel{-1\cdot 58}\ -0\cdot 41$

(4)



Area $= \int_1^3 x\,dy$ where $y = \log_e(x-3)$

i.e. $e^y = x-3$

So, $x = 3+e^y$

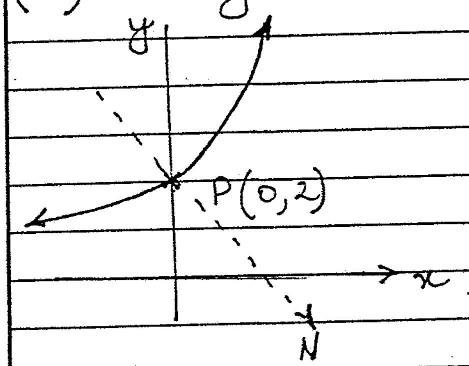
$$\therefore A = \int_1^3 (3+e^y)\,dy$$

$$= \left[3y + e^y\right]_1^3$$

$$= (9+e^3) - (3+e)$$

$$= (6+e^3-e)\ u^2$$

(5) $y = 2e^{3x}.$



Curve cuts y-axis $(x=0)$ at $(0,2)$

Now $\frac{dy}{dx} = 2(3e^{3x})$

i.e. $\frac{dy}{dx} = 6e^{3x}$

When $\underline{x=0}$, grad. of tang^t, $\frac{dy}{dx} = 6.e^0 = 6$

$\therefore$ Using $m_N \times m_T = -1$,

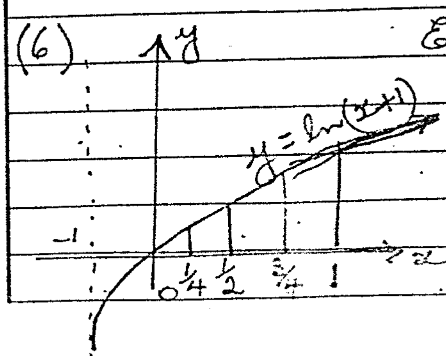
$m_N = -\frac{1}{6}$ is grad. of normal

$\therefore$ Eq^n of normal is $y-2 = -\frac{1}{6}(x-0)$

" " " $6y-12 = -x$

" " " $x+6y-12 = 0$ (OR $y = -\frac{1}{6}x+2$)

(6)



Each strip is $\frac{1}{4}$ unit wide $(b-a) = \frac{1}{4}$ for each strip.

$$A \approx \frac{\frac{1}{4}}{6}\left\{f(0)+f(\tfrac{1}{4})+4f(\tfrac{1}{8})\right\} + \frac{\frac{1}{4}}{6}\left\{f(\tfrac{1}{4})+f(\tfrac{1}{2})+4f(\tfrac{3}{8})\right\}$$

$$+ \frac{\frac{1}{4}}{6}\left\{f(\tfrac{1}{2})+f(\tfrac{3}{4})+4f(\tfrac{5}{8})\right\} + \frac{\frac{1}{4}}{6}\left\{f(\tfrac{3}{4})+f(1)+4f(\tfrac{7}{8})\right\}$$

$$\approx \frac{1}{24}\left[f(0)+f(1)+2\left\{f(\tfrac{1}{4})+f(\tfrac{3}{4})+f(\tfrac{1}{2})\right\}+4\left\{f(\tfrac{1}{8})+f(\tfrac{3}{8})+f(\tfrac{5}{8}) + f(\tfrac{7}{8})\right\}\right]$$

(6) (Cont'd)

i.e. $A \approx \frac{1}{24}\left[\ln 1 + \ln 2 + 2\left\{\ln \frac{5}{4} + \ln \frac{7}{4} + \ln \frac{6}{4}\right\} + 4\left\{\ln \frac{9}{8} + \ln \frac{11}{8} + \ln \frac{13}{8} + \ln \frac{15}{8}\right\}\right]$

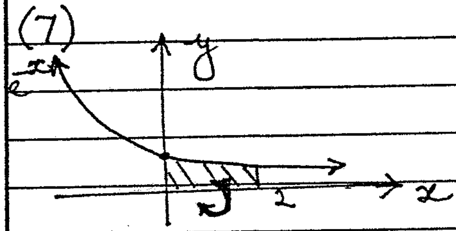
$\approx \frac{1}{24}\left[\ln 2 + 2\ln \frac{210}{64} + 4\ln \frac{19305}{4096}\right]$

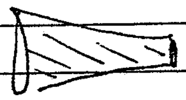
using $\ln a + \ln b = \ln ab$, etc

$\approx 0.33 \quad u^2$

(7)

$y = e^{-x}$





$V = \pi \int_0^2 y^2\,dx$

$= \pi \int_0^2 (e^{-x})^2\,dx$

$= \pi \int_0^2 e^{-2x}\,dx$

$= \frac{-\pi}{2}\left[e^{-2x}\right]_0^2 \quad u^3$.

$= \frac{-\pi}{2}\left[e^{-4} - e^0\right] \quad u^3$

$= \frac{-\pi}{2}\left[e^{-4} - 1\right] \quad u^3$.

(8) $y = x\ln x$ — (1)

$\frac{dy}{dx} = x.\frac{1}{x} + \ln x.1$

$= 1 + \ln x$ — (2)

$\frac{d^2y}{dx^2} = \frac{1}{x}$ — (3)

From (2), T.P. occur for $\boxed{\frac{dy}{dx} = 0}$

i.e. $1 + \ln x = 0$

$\ln x = -1$

ie $x = e^{-1} = \frac{1}{e} \Rightarrow y = \frac{1}{e}\ln\left(\frac{1}{e}\right) = \frac{1}{e}\ln(e^{-1}) = \frac{-1}{e}\ln e$

$= -\frac{1}{e}$

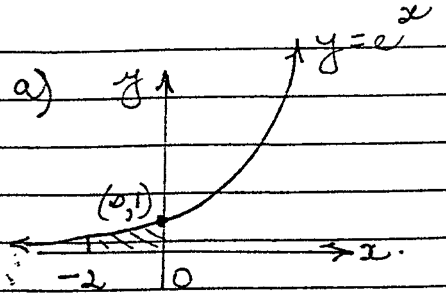
since $\ln e = 1$

If $x = \frac{1}{e}$, (3) $\Rightarrow \frac{d^2y}{dx^2} = \frac{1}{\left(\frac{1}{e}\right)} = e > 0$

i.e. At $x = \frac{1}{e}$, there is a MIN. PT.

$\left(\frac{1}{e}, -\frac{1}{e}\right)$ is a MIN. PT.

(9) a)

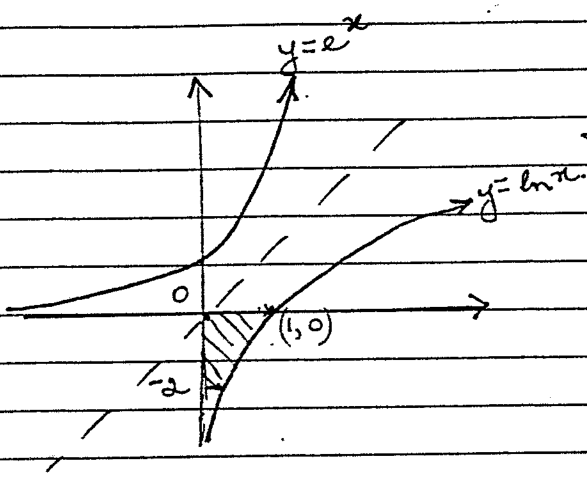


$$A = \int_{-2}^{0} e^{x}\,dx$$

$$= \left[e^{x}\right]_{-2}^{0} \quad u^2$$

$$= \left[e^{0} - e^{-2}\right] \quad u^2$$

$$= \left[1 - \frac{1}{e^{2}}\right] \quad u^2$$



The sh. area here is identical ~~to~~ that above because $y = \ln x$ is the reflection of $y = e^x$ in the line $y = x$.