

DERIVATIVE AND THE TANGENT

1. Find the equation to the curve $y = 3x^4 + 2x^3 - x^2 + 7x - 4$ at the point where $x = 2$.
2. Find the equation of the tangent to the curve $y = x^2 + \frac{2}{x} + 4$ at the point $P(-1, 3)$.
3. A tangent to the curve $y = 2x^2$ is parallel to the line $4x - y - 3 = 0$. What is the equation of this tangent?
4. At the point $(2, 3)$ on the curve $y = px^2 + qx + 7$, the tangent is inclined at 135° to the positive x -axis. Find the values of p and q .
5. Differentiate the following with respect to x :

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|--------------------------------|--------------------------------|------------------------------|
| (i) $\frac{\sqrt{x} + x}{x^2}$ | (ii) $\frac{7}{\sqrt[4]{x^3}}$ | (iii) $(10 - 9x)^8$ |
| (iv) $\frac{1}{x^2 - 2x + 2}$ | (v) $5x^2(\sqrt{4x + 9})$ | (vi) $\frac{7x + 3}{5x - 2}$ |
6. Find the point A on the curve $y = (2x + 5)^6$ where the line $y = 12x + 25$ is a tangent.
7. (a) If $y = \frac{x^2}{x + 1}$, show that $\frac{dy}{dx} = \frac{x^2 + 2x}{(x + 1)^2}$.
 (b) Hence find the equation of the tangent to the curve $y = \frac{x^2}{(x + 1)^2}$ at the point $\left(1, \frac{1}{2}\right)$.
 (c) Find the acute angle that this tangent makes with the x -axis. (Give your answer correct to the nearest degree.)
8. Evaluate the following limits:

| | | |
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| (i) $\lim_{x \rightarrow 0} \frac{x^2 + x}{x}$ | (ii) $\lim_{x \rightarrow 2} \frac{3x^2 - x - 10}{x - 2}$ | (iii) $\lim_{x \rightarrow \infty} \frac{4x^2 - 6x + 5}{3x^2 + 8x}$ |
|--|---|---|
9. Differentiate by first principles:

| | | |
|----------------|-----------------------------|----------------------------|
| (i) $y = 3x^2$ | (ii) $f(x) = 2x^2 - 3x + 1$ | (iii) $f(x) = \frac{1}{x}$ |
|----------------|-----------------------------|----------------------------|

ANSWERS:

1. $y = 123x - 176$ 2. $y = -4x + 1$ 3. $y = 4x - 2$ 4. $p = 2, q = -9$ 5. (i) $\frac{-3}{2\sqrt{x^5}} - \frac{1}{x^2}$
- (ii) $\frac{-21}{4\sqrt{x^7}}$ (iii) $-72((10 - 9x)^7)$ (iv) $\frac{2 - 2x}{(x^2 - 2x + 2)^2}$ (v) $10x\sqrt{4x + 9} + \frac{10x^2}{\sqrt{4x + 9}}$
- (vi) $\frac{-29}{(5x - 2)^2}$ 6. $A(-2, 1)$ 7. (b) $y = \frac{3}{4}x - \frac{1}{4}$ (c) $\theta = 37^\circ$ 8. (i) 1 (ii) 11
- (iii) $\frac{4}{3}$ 9. (i) $6x$ (ii) $4x - 3$ (iii) $\frac{-1}{x^2}$