DIFFERENTIATION

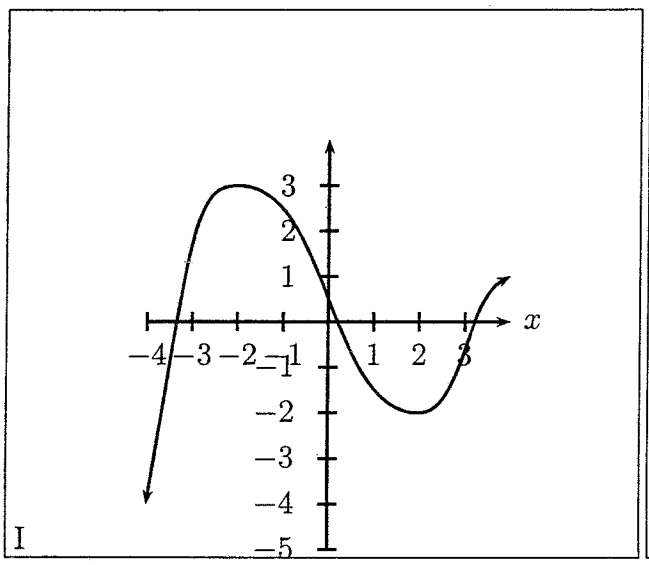
$$\frac{dy}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

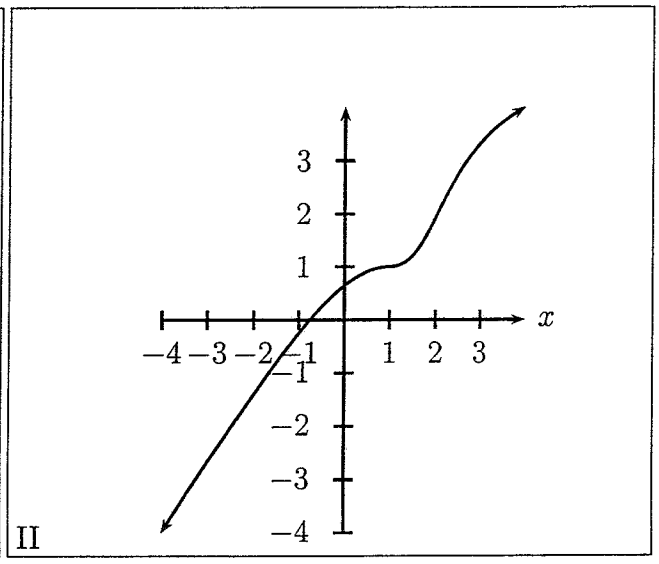
- If $f'(x) > 0$ for all x in an interval (a, b) then f is increasing on (a, b) .
- If $f'(x) < 0$ for all x in an interval (a, b) then f is decreasing on (a, b) .
- If $f'(x) = 0$ for all x in an interval (a, b) then f is constant on (a, b) .

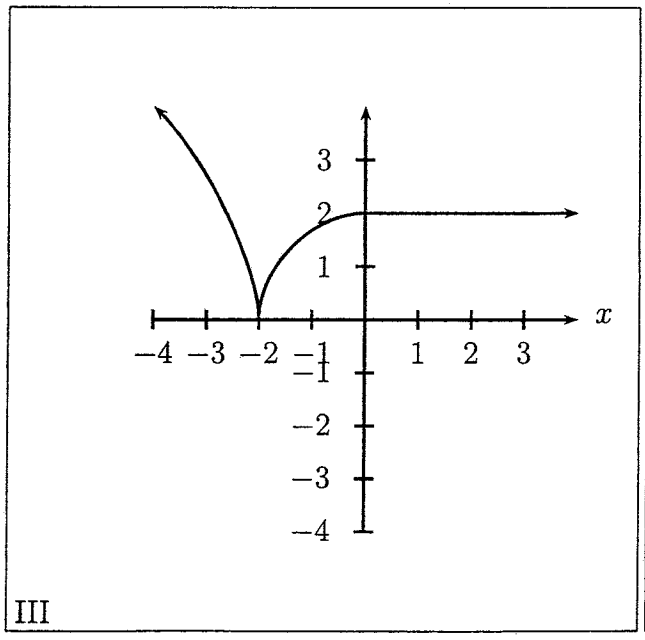
190. For each of the following graphs of $y = f(x)$ determine the value(s) of x for which:

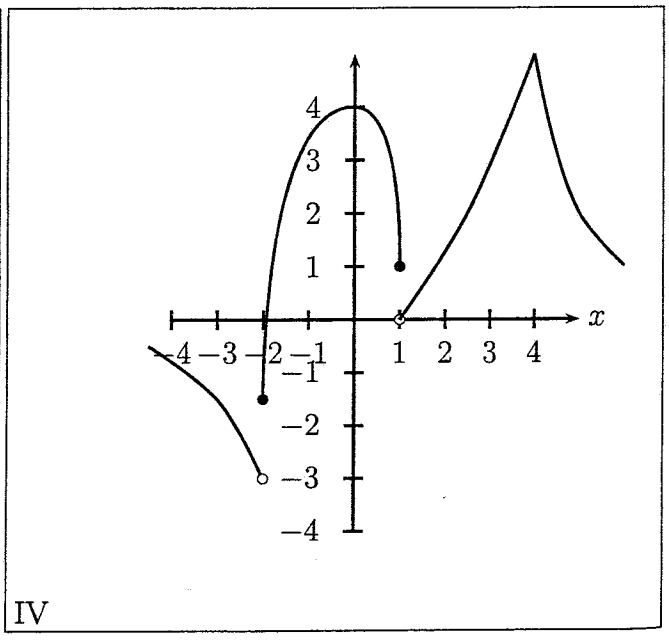
a) $\frac{dy}{dx}$ is positive b) $\frac{dy}{dx}$ is negative

c) $\frac{dy}{dx}$ is zero d) $\frac{dy}{dx}$ is undefined









191. A function $y = f(x)$ is defined for $-2 \leq x \leq 1$ and has the property that:

a) $\dfrac{dy}{dx} < 0$ for $-2 < x < -1$. b) $\dfrac{dy}{dx} = 0$ for $x = -1$.

c) $\dfrac{dy}{dx} > 0$ for $-1 < x < 1$.

Draw a possible sketch of the graph of f .

192. A function $y = f(x)$ defined for $2 \leq x \leq 5$ has the property that:

a) $f'(x) > 0$ for $2 < x < 3$. b) $f'(3) = 0$.

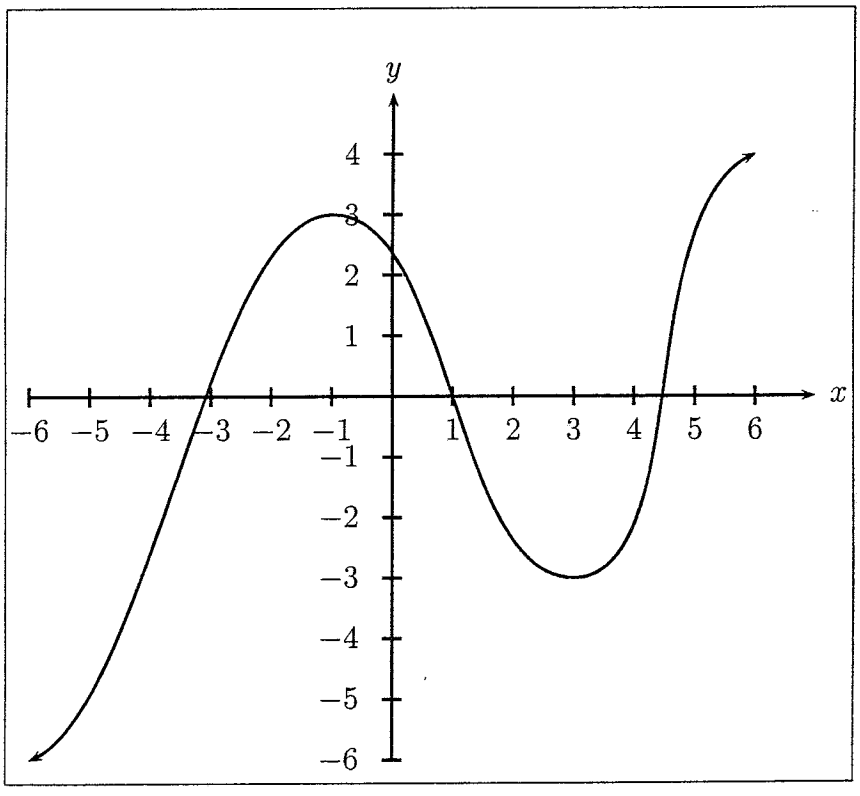
c) $f'(x) < 0$ for $3 < x < 4$. d) $f'(4)$ is undefined; and

e) $f'(x) = 0$ for $4 < x < 5$.

Draw possible sketch of the graph of f.

193. (a) A function $y = f(x)$ is sketched below. By considering the slope of the curve draw a possible sketch of $\dfrac{dy}{dx}$.

(b) By considering the gradient of your answer in a) draw a possible sketch of $\dfrac{d^2y}{dx^2}$.



194. Find $\dfrac{dy}{dx}$ for:

a) $y = x^7$.

b) $y = 4x^7 - 13x^2$.

c) $y = 3x + 3/x^4 + 5$.

d) $y = 5x + 6\sqrt{x}$.

e) $y = \dfrac{3x^2 + 5x}{\sqrt{x}}$.

f) $y = \pi^3$.

195. Find the gradient of $y = x + \dfrac{1}{x}$ at $x = 3$.

196. Prove that $y = f(x) = x^3 + x$ is an increasing function.

197. (*) The population P of a country has been increasing over the last seven years, however during this period its rate of increase has been falling. Draw a possible sketch of P vs t.

198. The population P of a small mining town varies over a five year period according to the formula $P(t) = 3000 - 100t^2$; $0 \leq t \leq 5$.

(a) Find the initial and final population.

(b) How many people live in the town after 2 years?

(c) What is the rate of change of population after 2 years?

(d) Verify that the population is decreasing over the whole period.

(e) At which point in time is the population decreasing at a rate of 300 people per year?

199. Inspectors arrive at an oil spill 3 hours after an accident and discover that the radius R (metres) of the spill is given by

$$R(t) = 3000 - 4000/t; \quad t \geq 3$$

where t is measured in hours after the accident.

(a) What is the radius of the spill 4 hours after the accident?

(b) What is the rate of change of the radius of the spill 4 hours after the accident?

(c) When is the radius of the spill increasing at 10m/hour?

(d) What is the limit on the size of the spill?

200. (*) Greg wishes to fill his swimming pool and has borrowed an electric pump from his friend Alex in order to pump water from a nearby creek. Greg starts the pump at 9.00am and notices that the pump does not operate at a constant rate, it starts slowly and then gradually increases the rate at which it pumps water into the pool. In fact the volume V (m^3) of water in the pool at time t (mins) is given by

$$V = \frac{t^2}{36} + \frac{t}{3}; \quad t \geq 0.$$

(a) How much water is in the pool at 10.00am?

(b) At what rate is the water being pumped into the pool at 10.00am?

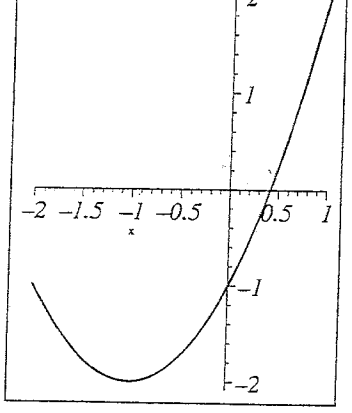
SOLUTIONS

Differentiation

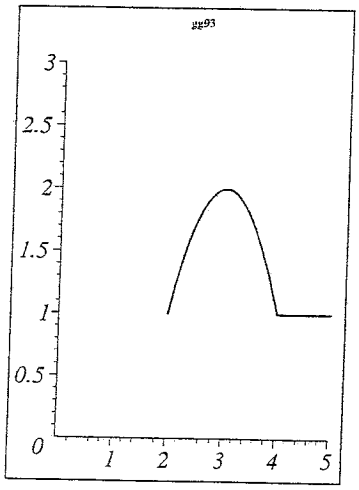
190. For I (a) $-\infty < x < -2$ or $x > 2$ (b) $-2 < x < 2$ (c) $x = \pm 2$ (d) None
For II (a) All $x \in \mathbb{R}$ except $x = 1$ (b) None c) $x = 1$ (d) None

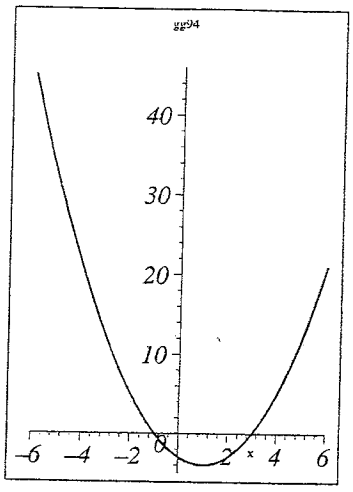
For III (a) $-2 < x < 0$ (b) $x < 2$ (c) $x \geq 0$ (d) $x = -2$
For IV (a) $-2 < x < 0$ or $-1 < x < 4$ (b) $-\infty < x < -2$ or $0 < x < 1$ or $x > 4$
(c) $x = 0$ (d) $x = -2, 1, 4$

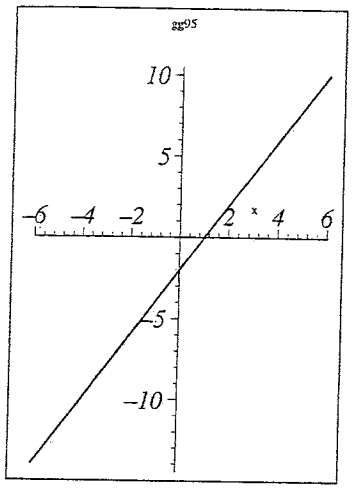
191.



192.



193. a) 

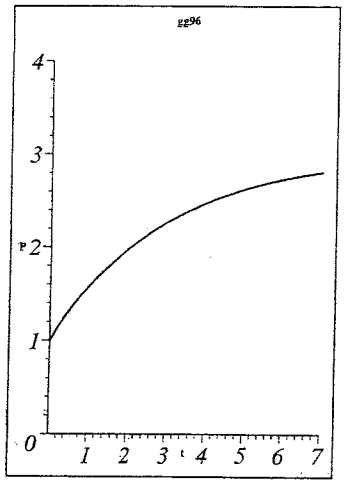
b) 

194. (a) $7x^6$ $28x^6 - 26x$ (c) $3 - \frac{12}{x^5}$

(d) $5 + \frac{3}{\sqrt{x}}$ (e) $\frac{9}{2}\sqrt{x} + \frac{5}{2\sqrt{x}}$ (f) 0

195. 8/9

196. $\frac{dy}{dx} > 0$

197. 

198. (a) 3000, 500 (b) 2600 (c) -400 (d) $P' \leq 0$ (e) 1.5 years.

199. (a) 2000m (b) 250 m/hr c) 20 hours d) 3000m

200. (a) 120m ^{3} (b) $3\frac{2}{3}$ m ^{3} /min (c) 12 mins.