DIFFERENTIATION

&

APPLICATIONS

2-UNIT

LESSON (37) – HW

Qu ① If $f(x) = 2x^2 + 1$

(i) Find $f(5) =$

(ii) Find $f(n+1) =$

(iii) Find $f(x+h) =$

(iv) Show that: $f(x+h) - f(x) = 2h(2x+h)$

(v) Hence find: $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

Qu ② Find the "gradient formula" for the following graphs.

(i) $y = x^3$ (ii) $y = 6x^2$ (iii) $y = x^4 - 3x^2$

(iv) $y = 8x^1$ (v) $y = 5x^3 + 8x$ (vi) $y = x^2 - 5x + 4$

ANSWERS

Qu ① (i) 51 (ii) $2n^2+4n+2$ (iii) $2x^2+4xh+2h^2+1$ (iv) Show! (v) $4x$

Qu ② (i) $3x^2$ (ii) $12x$ (iii) $4x^3-6x$ (iv) 8 (v) $15x^2+8$ (vi) $2x-5$

LESSON (38) – HW

Qu ①: Find the gradient function $f'(x)$ (ie. "differentiate" the following.)

(a) $f(x) = 2x^3 - 6x$ (b) $f(x) = x^{-4}$ (c) $f(x) = x^{\frac{1}{2}}$ (d) $f(x) = \frac{1}{x^2}$

(e) $f(x) = \frac{4}{x^3}$ (f) $f(x) = 4\sqrt{x}$ *(g) $f(x) = \frac{2}{\sqrt{x}}$ *(h) $f(x) = (2x-5)^2$

Qu ②: For the equation $y = 8x - 2x^2$ find the gradient at the point where:–

(a) $x = 0$ (b) $x = 1$ (c) $x = 2$ (d) $x = -1$

Qu ③:

Find the equation of the Tangents to $y = 8x - 2x^2$ at:–

(a) The point $(1, 6)$ (b) the point where $x = 3$

Qu① a) $6x^2 - 6$ b) $-4x^{-5}$ $= -4/x^5$ c) $\frac{1}{2}x^{-1/2}$ $= \frac{1}{2\sqrt{x}}$ d) $-2x^{-3}$ $= -2/x^3$ e) $-12x^{-4}$ $= \frac{-12}{x^4}$ f) $2x^{-1/2}$ $= 2/\sqrt{x}$

g) $-x^{-3/2}$ $= -\frac{1}{\sqrt{x^3}}$ h) $4(2x-5)^1$ $= 8x - 20$ Qu② $f'(x) = 8 - 4x$ $f'(0) = 8$ $f'(1) = 4$ $f'(2) = 0$ $f'(-1) = 16$

Qu③ a) $y = 4x + 2$ b) $y = -4x + 18$ [illegible]

LESSON 39 — HW

Qu ① Use the "Product Rule" to differentiate the following

(i) $x^3(5x^2-1)$ (ii) $(4x^2+5).(2x+1)$

Qu ② Use the "Quotient Rule" to differentiate the following:

(i) $\dfrac{x^2+1}{x}$ (ii) $\dfrac{3x}{x+1}$

(iii) $\dfrac{x^2+4}{x-2}$

Qu ③ Use the "Function of a Function Rule" to differentiate

(i) $(x+6)^5$ (ii) $(4x-5)^3$ (iii) $(x^2+3x)^4$

| ANSWERS |

(1) $3x^2(5x^2-1) + x^3.10x = 25x^4-3x^2$ (2) $8x(2x+1)+2.(4x^2+5) = 24x^2+8x+10$ (3) $\dfrac{x(2x)-1(x^2+1)}{x^2} = \dfrac{x^2-1}{x^2}$

(2) Quest ② $\dfrac{(x+1).3-1(3x)}{(x+1)^2} = \dfrac{3}{(x+1)^2}$ (3) $\dfrac{(x-2).2x - 1.(x^2+4)}{(x-2)^2} = \dfrac{x^2-4x-4}{(x-2)^2}$

Quest ③ 1) $5(x+6)^4$ 2) $12(4x-5)^2$ 3) $(8x+12)(x^2+3x)^3$

LESSON 40 — HW

Quest 1: Differentiate the following functions:-

(a) $f(x) = \dfrac{1}{(8-3x^2)^4}$ (b) $f(x) = \dfrac{2x+2}{\sqrt{x}}$ (c) $f(x) = (x-2)^5 \cdot (5x+3)$

$f'(x) =$ $f'(x) =$ $f'(x) =$

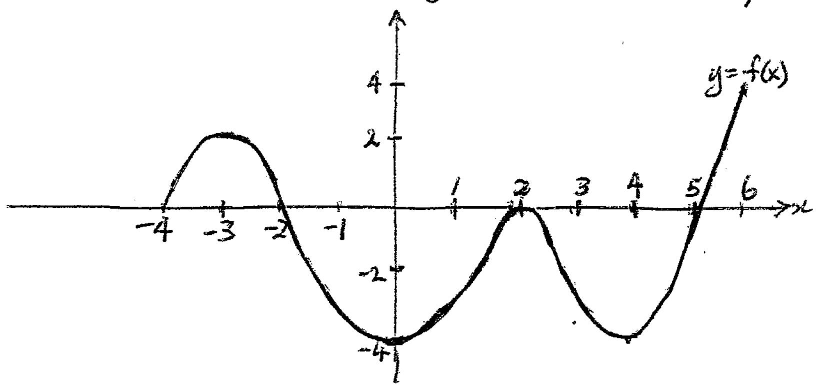
Quest 2: For the curve $y = \sqrt{25-x^2}$

(i) Find $\dfrac{dy}{dx}$ (gradient function). (ii) the gradient at $x = 3$

(iii) The equation of the Tangent at $x=3$ (iv) The equation of the Normal at $x=3$

Quest 3: In the diagram below describe which sections of the

(a) Graph (what range of x?) have $f'(x) > 0$ & $f'(x) < 0$ (ie positive slope & negative slope respectively)



(b) What is the maximum and minimum values of this function in the range $-4 \leq x \leq 4$?

LESSON 41 — HW

Quest 1 For each of the following functions find whether the curve is increasing ($f'(x) > 0$) decreasing ($f'(x) < 0$) or stationary ($f'(x) = 0$) at $x = 3$.

(a) $y = x^3 - 10x$ (b) $y = 3x^2 - 27$ (c) $y = 5x^2 - 8x^3$

Quest 2 Find the stationary points for the following curves:

(a) $y = 3x^2 - 6x$ (b) $y = 2x^3 - 3x^2 + 1$

* (c) $y = x^3 + 3x^2 - 9x$

Quest 3 Find the second derivative, $f''(x)$ of the following fns:

a) $f(x) = 5x^3 + 6x$ (b) $f(x) = (3 - 10x)^4$ (c) $f(x) = \frac{4}{x}$

LESSON (42) – HW

Quest (1) For the function $f(x) = 4x - x^2$

(i) Find $f'(x)$

(ii) Find $f''(x)$

(iii) Find the x-intercepts of $y = 4x - x^2$

(iv) Find the stationary point(s) (where $f'(x) = 0$)

(v) Find whether the stationary pt.(s) are a rel. max or rel. min.

(vi) Find any point of inflexion.

(vii) Find the value (height) of the curve at $x = -1$ and $x = 5$

(viii) Sketch the graph of this curve in the domain $-1 \leq x \leq 5$

(ix) What is a) the Absolute maximum value of $f(x) = 4x - x^2$

b) " " minimum " " "

... ~~for~~ $-1 \leq x \leq 5$

Quest (2)

For the function $g(x) = x^3 - 3x^2$

find all of the above (in quest (1))

LESSON - 43 - HW WORKSHEET

Quest ① For the function $y = \frac{1}{3}x^3 - x^2 - 3x - 6$

(i) find $f'(x)$

(ii) find $f''(x)$

(iii) Find all stationary points (ie find x when $f'(x) = 0$)

(iv) Test $f''(x)$ at these points to determine their nature (type).

(v) Find any points of inflexion (when $f''(x) = 0$)

(vi) Find the y-intercept of this curve (when $x = 0$)

(v) Sketch the Curve

Quest ②: For the function $y = 3x^4 - 12x^3 + 12x^2 + 2$

– find all of the above results (as in quest ①)

LESSON 44 — HWORKSHEET

Quest 1: For the curve $y = x^3(x-4)$

Find $y' =$

Find $y'' =$

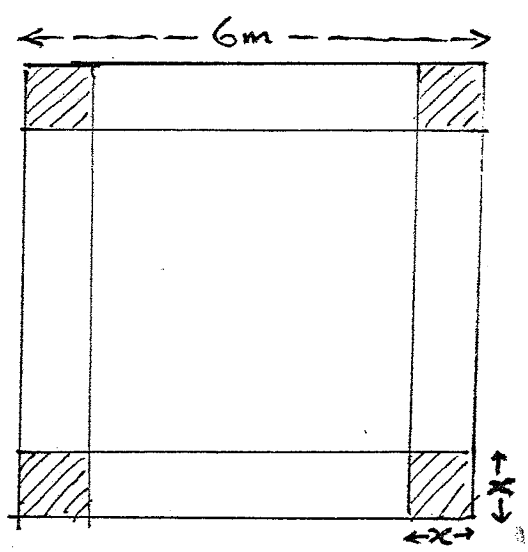
Find any stationary points.

Determine their nature using $f''(x)$.
or Test $f'(x)$ just before and after each stationary pt.

Sketch the graph of this function.

Quest 2

A square sheet of metal, whose edges are 6 metres in length has a square of side x cut out of each of its 4 corners.
The sides are then folded up to form an open tank with base $(6-2x)$ metres long.



Show that the Volume of the tank is given by: $V = 4x^3 - 24x^2 + 36x$

Find the value of x for which the Volume is a Maximum.

LESSON (45) — HW

Quest (1)

For the curve $y = x + \frac{1}{x}$

(i) Find y' and y''

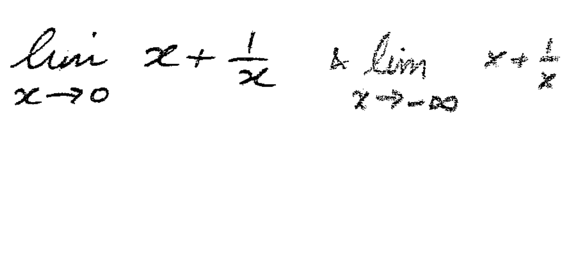
(ii) Find any stationary points & determine their nature

(iii) What value(s) of x cannot belong to the domain of $y = x + \frac{1}{x}$?

(iv) Investigate $\lim_{x \to \infty} x + \frac{1}{x}$ and $\lim_{x \to 0} x + \frac{1}{x}$ & $\lim_{x \to -\infty} x + \frac{1}{x}$

(v) Sketch the curve.

Quest (2): A piece of wire 20π cm in length is cut in two, and 2 circles of radius R and r are formed:



(i) Show that the equation relating r and R is:- $R + r = 10$

(ii) Use this result the show that the total area of the two circles is:

$$A = \pi r^2 + \pi R^2 = 2\pi R^2 - 20\pi R + 100\pi$$

(iii) Find $\frac{dA}{dR}$ and hence the value of R which gives a minimum area

Quest (3): The cost of running a pleasure cruiser on a 200 km journey is known to be:- COST: $\$y = \frac{5}{x}(x^2 + 400)$ - where x is the speed of the ship.

Find the most economical speed for this trip.

(ie minimum cost $\$y$)