Mathematics

HSC Half Yearly 2008

Name/Number: _____

Teacher: _____

- Total Marks: 60
- Reading time – 5 minutes
- Working time – 90 minutes
- Write in blue or black pen
- Board approved calculators are permitted
- Show all necessary working for each question
- A table of standard integrals is provided at the back of this paper

Question 1 (12 Marks) **Marks**

a) A raffle is held and 100 tickets are sold. If I buy 12 tickets, what is the probability that I win the prize? 2

b) The probability of getting a red light at a particular traffic light is 35%.
Find the probability that I will get a green or orange light next time I drive by. 2

c) There are 26 students in a class, where 12 study Japanese and 20 study Italian. All students do at least one of these languages.

i) How many students study both Languages? 1

ii) What is the probability that a randomly chosen student studies Japanese? 2

iii) What is the probability that a randomly chosen student studies both? 1

d) Joe and Tom are playing in a tennis tournament. They will play two games together, where each of them has an equal chance of winning the first game: 0.5. If Joe wins the first game, his probability of winning the second game is increased to 0.6. If Joe loses the first game, his probability of winning the second game is reduced to 0.3.

i) Draw a tree diagram for the two games, including the probabilities of each branch. 2

ii) Find the probability that Joe wins only one game. 2

Question 2 (12 Marks) **Marks**

a) A bag contains 10 smarties. 4 are green and 6 are blue. Steve selects 3 smarties at random (without replacement). Using a tree diagram, or otherwise, find the probability that:

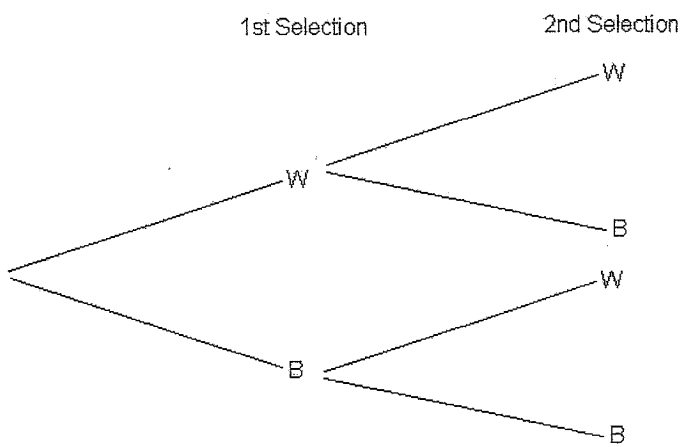
i) the first smartie Steve selects is green. 1

ii) Steve selects 3 blue smarties. 2

iii) Steve selects only one green smartie. 3

b) 7 white (W) marbles and 3 black (B) marbles are placed in a bag. One marble is selected at random, removed, then replaced by a marble of the other colour. Another marble is then randomly selected.

i) Copy the following diagram into your writing booklet. Complete the tree diagram by showing the probabilities on each branch. 2



ii) Find the probability that both marbles selected are black. 2

iii) Find the probability that at least one marble selected is black. 2

Question 3 (12 Marks) **Marks**

a) Find all values of x for which the curve $y = 3x^2 - 6x$, is decreasing. 3

b) Find all the stationary points on the curve: $y = 4x^3 - 6x^2 + 5$, and determine their nature. 3

c) Explain why the curve $y = 4x^2$ has no point of inflexion. 1

d) Find $f'(1)$ and $f''(-4)$ if $f(x) = 5x^3 - 2x + 4$. 2

e) Find all values of x for which the curve $y = x^3 + 6x^2 - 2x$ is concave downwards. 3

Question 4 (12 Marks) **Marks**

a) Find any stationary and inflexion points on the curve: $y = x^3 - 6x^2 + 9x$.

Hence, sketch the curve. 4

b) Find the absolute maximum and minimum values of the curve $y = x^4 - 2x^2 + 24$ in the domain: $0 \le x \le 4$. 2

c) The perimeter of a rectangle is 120cm, and its length is x cm. Show that the area of the rectangle is given by the equation $A = 60x - x^2$. Hence find the maximum area of the rectangle. 3

d) The population, P, of a town over t years is given by: $P = -t^2 + 12t + 30$, for $0 \le t \le 8$.

i) After how many years was the population at its maximum? 2

ii) What was the maximum population? 1

Question 5 (12 Marks) **Marks**

a) Find the indefinite integral of:

i) $\int 9x^2 + 6x \; dx$ 1

iii) $\int \frac{x^5 - 6x^4}{x^3} \; dx$ 1

b) Evaluate:

i) $\int_0^2 4x^3 - 5x^2 + 1 \, dx$ 2

ii) $\int_0^1 \frac{1}{4(2x-1)^3} \, dx$ 2

c) Find the area enclosed between the curve $y = 4 - x^2$ and the x-axis. 2

d) Find the area enclosed between the curve $y = x^2$ and the line $y = 3x + 4$. 2

e) Find the volume of the solid formed when the curve $y = x^2 + 3$, is rotated around the x-axis, between the values $x = 0$ and $x = 6$. 2

End of Paper

2unit HSC Half Yearly Answers 2008

Q1

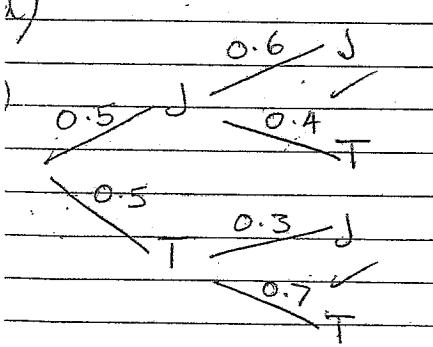
a) $P(\text{win}) = \frac{12}{100} = \frac{3}{25}$ ✓

b) $P(\text{not red}) = 1 - 35\%$ ✓
$= 65\%$ ✓

c)
i) $12 + 20 - 26 = 6$.
$\therefore$ 6 do both ✓
$P(J) = \frac{12}{26} = \frac{6}{13}$ ✓

ii) $P(\text{Both}) = \frac{6}{26} = \frac{3}{13}$ ✓

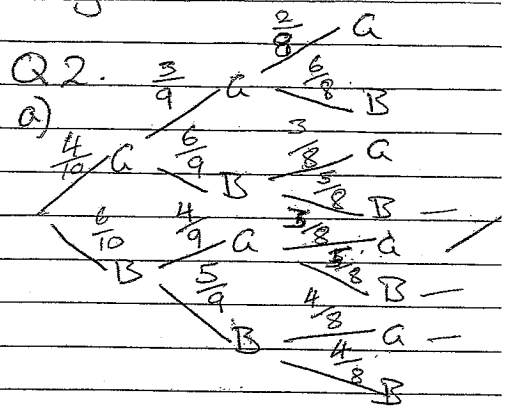
d)



iv) $P(1J) = 0.5 \times 0.4$
$+ 0.5 \times 0.3$
$= 0.35$. ✓

Q2.

a)

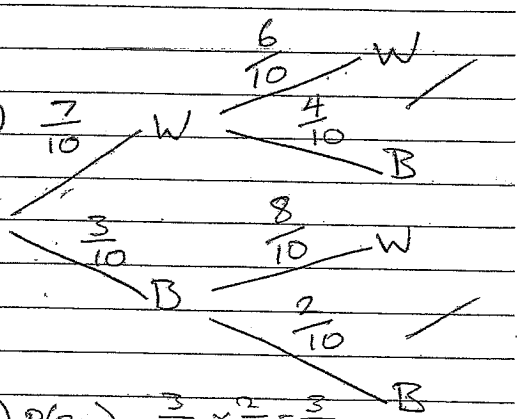


(i) $P(G) = \frac{4}{10} = \frac{2}{5}$ ✓

(iii) $P(BBB) = \frac{6}{10} \times \frac{5}{9} \times \frac{4}{8} = \frac{1}{6}$ ✓

(iv) $P(1G) = \frac{4}{10} \times \frac{6}{9} \times \frac{5}{8} + \frac{6}{10} \times \frac{4}{9} \times \frac{5}{8}$
$+ \frac{6}{10} \times \frac{5}{9} \times \frac{4}{8} = \frac{1}{2}$ ✓

b)



(ii) $P(BB) = \frac{3}{10} \times \frac{2}{10} = \frac{3}{50}$ ✓

(iii) $P(\text{atleast 1 B}) = 1 - P(WW)$
$= 1 - \frac{7}{10} \times \frac{6}{10}$ ✓
$= \frac{29}{50}$ ✓

Q3.

a) $y = 3x^2 - 6x$
$y' = 6x - 6$ ✓
Decreasing: $y' < 0$ ✓
$6x - 6 < 0$
$6x < 6$
Decreasing for $x < 1$ ✓

b) $y = 4x^3 - 6x^2 + 5$
$y' = 12x^2 - 12x$ ✓
$y' = 0$ for Stationary
$\therefore 12x^2 - 12x = 0$
$12x(x - 1) = 0$.
$\therefore x = 0$, or 1.

$x = 0$	$x = 1$
$y = 5$	$y = 3$.

Stationary pts: $(0,5)$, $(1,3)$ ✓

$(0,5)$	$(1,3)$
$y'' = 24x - 12$	$y'' > 0$
$y'' < 0$	

$\therefore$ $(0,5)$ is a Maximum.
$(1,3)$ is a Minimum. ✓

c) $y = 4x^2$
$y' = 8x$
$y'' = 8. > 0$ for all x.
$\therefore y = 4x^2$ has no point of inflexion because it is always concave up. ✓

d) $f(x) = 5x^3 - 2x + 4$
$f'(x) = 15x^2 - 2$
$f''(x) = 30x$.

$\therefore f'(1) = 15 - 2 = 13$ ✓
$f''(-4) = 30 \times -4 = -120$ ✓

e) $y = x^3 + 6x^2 - 2x$
$y' = 3x^2 + 12x - 2$ ✓
$y'' = 6x + 12$
Concave downwards: $y'' < 0$ ✓
$\therefore 6x + 12 < 0$
$6x < -12$
$x < -2$ ✓

Q4.

a) $y = x^3 - 6x^2 + 9x$

$y' = 3x^2 - 12x + 9$ ✓

$y'' = 6x - 12$.

Stationary pts: $y' = 0$

$3x^2 - 12x + 9 = 0$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$\therefore x = 1 \quad x = 3$

$y = 4 \quad y = 0$.

∴ Stationary pts:

$(1, 4), (3, 0)$. ✓

Inflexion: $y'' = 0$

$6x - 12 = 0$.

$6x = 12$

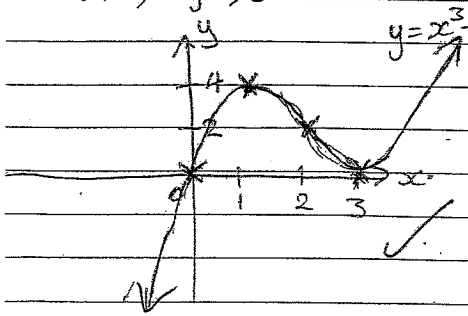
$x = 2$.

$y = 2$.

Inflexion: $(2, 2)$. ✓

At $(1, 4)$ $y'' < 0$ Max

At $(3, 0)$ $y'' > 0$ Min.



✓

b) $y = x^4 - 2x^2 + 24$.

$x=0$: $y = 24$

$x=4$: $y = 248$

$y' = 4x^3 - 4x$

$y' = 0$: $4x(x^2 - 1) = 0$

$4x(x-1)(x+1) = 0$.

$\therefore x = 0, \; x = 1, \; x = -1$

for Stationary.

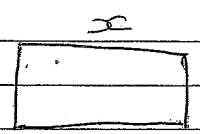
$x = 1$: $y = 23$

$x = -1$: $y = 23$ ← out of range

∴ Abs Max = $(4, 248)$ ✓

Abs Min = $(1, 23)$. ✓

c)



$P = 2x + 2y$.

$\therefore 120 = 2x + 2y$ ✓

$x + y = 60$.

$\therefore y = 60 - x$.

$A = xy$

$= x(60 - x)$ ✓

$A = 60x - x^2$.

Max: $A' = 0, \; A'' < 0$.

$A' = 60 - 2x$

$A'' = -2. < 0$ for all x.

$60 - 2x = 0$

$2x = 60$

$x = 30$.

$\therefore A = 60 \times 30 - 30^2$

$= 900$

Max Area $= 900 cm^2$. ✓

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1) $P = -t^2 + 12t + 30, \; 0 \le t \le 8$.

a) $P' = -2t + 12$ ✓

$P'' = -2. < 0$ for all x.

For Max: $P' = 0, \; P'' < 0$.

$-2t + 12 = 0$

$\frac{-2t}{-2} = \frac{-12}{-2}$

$t = 6$. ✓

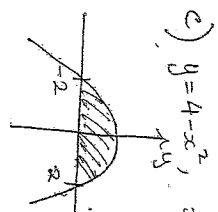
∴ After 6 years, the Population is at its Maximum.

b) $P = -6^2 + 12 \times 6 + 30$

$= 66$.

Max population was 66 people. ✓

2) a) $y = 4 - x^2$, x-axis



x-intercept: $y = 0$.

$\therefore 4 - x^2 = 0$

$(2 - x)(2 + x) = 0$

$\therefore x = \pm 2$. ✓

$A = \int_0^2 4 - x^2 \, dx$

$= 2 \left[4x - \frac{x^3}{3} \right]_0^2$

$= 2 \left(8 + \frac{8}{3} \right) = 10\frac{2}{3}$ units^2 ✓

3) a) $\int 9x^2 + 6x \, dx = 3x^3 + 3x^2 + c$ ✓

(i) $\int \frac{x^5 - 6x^4}{x^3} \, dx = \int x^2 - 6x \, dx$

$= \frac{x^3}{3} - 3x^2 + C$ ✓

(ii) $\int_0^2 4x^3 - 5x^2 + 1 \, dx = \left[x^4 - \frac{5x^3}{3} + x \right]_0^2$ ✓

$= 16 - \frac{40}{3} + 2$

$= 4\frac{2}{3}$.

b) $\int_0^1 \frac{1}{4(2x+1)^3} \, dx = \frac{1}{4} \int_0^1 (2x+1)^{-3} \, dx$ ✓

$= \frac{1}{4} \left[\frac{(2x+1)^{-2}}{-2 \times 2} \right]_0^1$

$= \frac{1}{4} \left(\frac{1}{36} - \frac{1}{4} \right)$

Q5.

1) $y = x^2$ — ①

$y = 3x + 4$ — ②

Sub. ① into ②.

$x^2 = 3x + 4$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$\therefore x = -1, 4$ ✓

$A = \int_{-1}^{4} 3x + 4 - x^2 \, dx$.

$= \left[\frac{3x^2}{2} + 4x - \frac{x^3}{3} \right]_{-1}^{4}$

$= \left(24 + 16 - \frac{64}{3} \right) - \left(\frac{3}{2} - 4 + \frac{1}{3} \right)$

$= 20\frac{5}{6}$ units^2

2) a) $y = 3x + 4$

$y = x^2$

b) $V = \pi \int y^2 \, dx$

$y^2 = (x^2 + 3)^2$

$= x^4 + 6x^2 + 9$

$= \pi \int_0^6 (x^2 + 3)^2 \, dx$

$= \pi \int_0^6 x^4 + 6x^2 + 9 \, dx$

$= \pi \left(\frac{6^5}{5} + 2 \times 6^3 + 54 \right)$

$= 2041.2\pi$ units^3.