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EXTENSION 1 TEST 28-5-03

Trigonometric Functions, Inverse Functions and Inverse Trigonometric Functions.

Name________________________Class________

Instructions: Show all necessary working throughout the test on A4 paper.

Begin a new page as specified.

Time allowed: 45 minutes

[Begin a new page] **SKB**

1. (a) Simplify to a single termed expression: $\dfrac{2\sin(\frac{\pi}{2}-2\theta)-1}{\sec\theta}$ [4]

(b) Find the general solution of: $\sqrt{3}\sin x-\cos x=\sqrt{3}$ [4]

(c) Find $\int \cos^2 x+\dfrac{1}{\cos^2 x}\,dx$ [3]

(d) Find the exact volume generated when the area bounded by the curve $y=\sin 2x$, the x-axis and lines $x=0$ and $x=\dfrac{\pi}{6}$, is rotated about the x-axis. [4]

[Begin a new page] **HRK**

2. (a) A function is given by the rule $f(x)=\dfrac{x+1}{x+2}$. Find the rule for the inverse function $f^{-1}(x)$. [2]

(b) Find the value of the expression $\cos^{-1}(-\frac{1}{2})-\sin^{-1}(-\frac{1}{2})$ in terms of π. [2]

(c) (i) Sketch the graph of $y=4\sin^{-1}(\frac{x}{2})$. [2]

(ii) Find the exact equation of the tangent to the curve $y=4\sin^{-1}(\frac{x}{2})$ at the point where $x=1$. [3]

(d) Find $\dfrac{d}{dx}[e^{\tan^{-1}(\ln[\cos 2x])}]$ in its simplest form. [3]

(e) Show that $\displaystyle\int_1^{\sqrt{2}}\frac{dx}{\sqrt{4-2x^2}}=6\int_{\sqrt{2}}^{\sqrt{6}}\frac{dx}{4+2x^2}$. [3]

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a) $\dfrac{2\sin(\frac{\pi}{2}-2\theta)-1}{\sec\theta}$

$= \dfrac{2\cos 2\theta - 1}{\sec\theta}$

$= \dfrac{2(2\cos^2\theta - 1) - 1}{\sec\theta}$

$= \dfrac{4\cos^2\theta - 2 - 1}{\sec\theta}$

$= (4\cos^2\theta - 3)\cos\theta$

$= 4\cos^3\theta - 3\cos\theta$

$= \cos 3\theta$

) $\sqrt{3}\sin x - \cos x = \sqrt{3}$

$r = \sqrt{3+1}$ $\tan\alpha = \frac{1}{\sqrt{3}}$

$r = 2$ $\alpha = \frac{\pi}{6}$

$\therefore 2\sin(x - \frac{\pi}{6}) = \sqrt{3}$

$\sin(x - \frac{\pi}{6}) = \frac{\sqrt{3}}{2}$

$x - \frac{\pi}{6} = \frac{\pi}{3}$

$x = \pi n + (-1)^n \frac{\pi}{3} + \frac{\pi}{6}$

where $n \in J$

) $\int \cos^2 x + \frac{1}{\cos^2 x}\, dx$

$= \int \cos^2 x + \sec^2 x\, dx$

$= \int \frac{1}{2}(\cos 2x + 1) + \sec^2 x\, dx$

$\frac{1}{2}(\frac{1}{2}\sin 2x + x) + \tan x + c$

$V = \pi \int_0^{\frac{\pi}{6}} \sin^2 2x\, dx$

$= \frac{\pi}{2}\int_0^{\frac{\pi}{6}} (1 - \cos 4x)\, dx$

$= \frac{\pi}{2}\left[x - \frac{1}{4}\sin 4x\right]_0^{\frac{\pi}{6}}$

$= \frac{\pi}{2}\left[\left(\frac{\pi}{6} - \frac{1}{4}\sin\frac{4\pi}{6}\right) - 0\right]$

$= \frac{\pi}{2}\left[\frac{\pi}{6} - \frac{\sqrt{3}}{8}\right]$

$= \frac{\pi}{48}\left[4\pi - 3\sqrt{3}\right] u^3.$

2 a) let $y = \dfrac{x+1}{x+2}$

$\therefore$ inverse function is

$x = \dfrac{y+1}{y+2}$

$xy + 2x = y + 1$

$xy - y = 1 - 2x$

$y = \dfrac{1-2x}{x-1}$

$\therefore f^{-1}(x) = \dfrac{1-2x}{x-1}$

b) $\cos^{-1}(-\frac{1}{2}) - \sin^{-1}(-\frac{1}{2})$

$= \frac{2\pi}{3} + \frac{\pi}{6}$

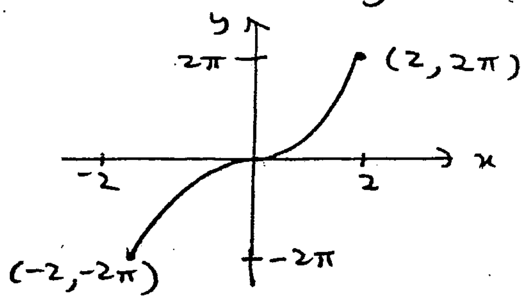
$= \frac{5\pi}{6}$

c) (i) $D: -1 \le \frac{x}{2} \le 1$

$\therefore -2 \le x \le 2$

$R: -\frac{\pi}{2} \le \frac{y}{4} \le \frac{\pi}{2}$

$-2\pi \le y \le 2\pi$



(ii) $y = 4\sin^{-1}(\frac{x}{2})$

$y' = 4 \cdot \dfrac{\frac{1}{2}}{\sqrt{1 - \frac{x^2}{4}}}$

$y' = \dfrac{2}{\sqrt{\frac{4-x^2}{4}}}$

$y' = \dfrac{4}{\sqrt{4-x^2}}$

at $x = 1$ $y' = \frac{4}{\sqrt{3}}$

at $x = 1$ $y = 4\sin^{-1}\frac{1}{2}$

$= \frac{4\pi}{6} = \frac{2\pi}{3}$

$\therefore$ eqn. of tangent is

$y - \frac{2\pi}{3} = \frac{4}{\sqrt{3}}(x - 1)$

$3\sqrt{3}y - 2\sqrt{3}\pi = 12x - 12$

$\therefore 12x - 3\sqrt{3}y + 2\sqrt{3}\pi - 12 = 0$

1) $\frac{d}{dx}\left[e^{\tan^{-1}(\ln[\cos 2x])}\right]$

$$e^{\tan^{-1}(\ln[\cos 2x])} \cdot \frac{1}{1+[\ln(\cos 2x)]^2} \cdot \frac{1}{\cos 2x} \cdot -2\sin 2x$$

$$= -2e^{\tan^{-1}(\ln(\cos 2x))} \cdot \frac{\tan 2x}{1+[\ln(\cos 2x)]^2}$$

) $LHS = \int_1^{\sqrt{2}} \frac{dx}{\sqrt{4-2x^2}}$

$$= \int_1^{\sqrt{2}} \frac{dx}{\sqrt{2(2-x^2)}}$$

$$= \frac{1}{\sqrt{2}} \int_1^{\sqrt{2}} \frac{dx}{\sqrt{2-x^2}}$$

$$= \frac{1}{\sqrt{2}} \left[\sin^{-1}\frac{x}{\sqrt{2}}\right]_1^{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} \left[\sin^{-1} 1 - \sin^{-1}\frac{1}{\sqrt{2}}\right]$$

$$= \frac{1}{\sqrt{2}} \left[\frac{\pi}{2} - \frac{\pi}{4}\right]$$

$$= \frac{\pi}{4\sqrt{2}}$$

$RHS = 6\int_{\sqrt{2}}^{\sqrt{6}} \frac{dx}{4+2x^2}$

$$= 6\int_{\sqrt{2}}^{\sqrt{6}} \frac{dx}{2(2+x^2)}$$

$$= 3\int_{\sqrt{2}}^{\sqrt{6}} \frac{dx}{2+x^2}$$

$$= 3\left[\frac{1}{\sqrt{2}} \tan^{-1}\frac{x}{\sqrt{2}}\right]_{\sqrt{2}}^{\sqrt{6}}$$

$$= \frac{3}{\sqrt{2}}\left(\tan^{-1}\sqrt{3} - \tan^{-1} 1\right)$$

$$= \frac{3}{\sqrt{2}}\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$

$$= \frac{3\pi}{12\sqrt{2}}$$

$$= \frac{\pi}{4\sqrt{2}}$$

$\therefore LHS = RHS$