EXERCISES IN MATH READINESS

EMR

FOR UNIVERSITY STUDY

MODERATE EXERCISES - Logarithms and Solving Equations

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(1) Simplify.

a. $\ln \sqrt{x}$

b. $\ln e^{2x}$

c. $e^{\ln 3}$

(Click On HINT To Get Some Help , Click On SOLUTION To See The Answers)

(2) Express without e or ln: $e^{t \ln 2}$.

(HINT , SOLUTION)

(3) Rewrite so there are no logarithms of products, powers or quotients.

a. $\ln (x^3 e^x)$

b. $\log (\sqrt{x} \sin x) / (x+4)$

(HINT , SOLUTION)

(4) Solve each equation.

a. $100 = 50e^{-x}$

b. $1/4 = 5^{2t-1}$

c. $\ln (2x+5) = 0$

d. $\log_x 6 = 1/3$

(HINT , SOLUTION)

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EXERCISES IN MATH READINESS

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MODERATE EXERCISES-Logarithms

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(1) Simplify.

a. $\ln \sqrt{x}$

b. $\ln e^{2x}$

c. $e^{\ln 3}$

SOLUTION:

(a) We can write $\sqrt{x}=x^{1/2}$, so $\ln \sqrt{x}= \ln x^{1/2}$.
Then we can use the power property of logarithms ($\log_a x^r = r \log_a x$):
$\ln x^{1/2} = (1/2)\ln x$. So the solution is $(1/2)\ln x$, or $(\ln x)/2$.

(b) Since $\log_a a^y = y$, when using natural logarithms, the analogous statement would be: $\ln e^y = y$. So we have:
$\ln e^{2x} = 2x$.

(c) From another property of logarithms, we have $e^{\ln y} = y$, since $\ln = \log_e$. So $e^{\ln 3} = 3$.

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MODERATE EXERCISES-Logarithms

(2) Express without e or ln: $e^{t \ln 2}$.

SOLUTION: First, note that we can write $e^{t \ln 2}$ as $e^{(\ln 2)(t)}$. Then, by the properties of exponents, $e^{(\ln 2)(t)} = (e^{(\ln 2)})^t$. But $e^{(\ln 2)} = 2$ (by a property of logarithms). So:

$e^{t \ln 2} = e^{(\ln 2)(t)} = (e^{(\ln 2)})^t = 2^t$.

Alternatively, you can write the exponent "t ln2" as: $\ln(2^t)$, by a property of logarithms. So then: $e^{t \ln 2} = e^{\ln(2t)} = 2^t$, by another property of logarithms.

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MODERATE EXERCISES-Logarithms

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(3) Rewrite so there are no logarithms of products, powers or quotients.

a. $\ln(x^3 e^x)$
b. $\log(\sqrt{x}\sin x)/(x+4)$

(HINT , SOLUTION)

SOLUTION:

(a) This is a product inside a logarithm, so:
$\ln(x^3 e^x) = \ln(x^3) + \ln(e^x)$.

Now each of these logs can be rewritten using other properties of logs:
$\ln(x^3) = 3\ln x$ and $\ln(e^x) = x$.
So $\ln(x^3 e^x) = 3\ln x + x$.

(b) This is a quotient inside a logarithm, so:
$\log(\sqrt{x}\sin x)/(x+4) = \log(\sqrt{x}\sin x) - \log(x+4)$.

The first term on the right hand side is a product within a log, so we have:
$\log(\sqrt{x}\sin x) - \log(x+4) = \log(\sqrt{x}) + \log(\sin x) - \log(x+4)$.

But we can write $\log(\sqrt{x}) = \log x^{1/2}$, and then $\log x^{1/2} = (1/2)\log x$. So the final answer is:

$\log(\sqrt{x}\sin x)/(x+4) = (1/2)\log x + \log(\sin x) - \log(x+4)$

(Point to ponder: for what values of x is this expression defined?)

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MODERATE EXERCISES-Logarithms

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(4) Solve each equation.

a. $100 = 50\ e^{-x}$

b. $1/4 = 5^{2t-1}$

c. $\ln(2x+5) = 0$

d. $\log_x 6 = 1/3$

SOLUTION:

(a) First, isolate, or solve for, the factor with the variable by dividing both sides by 50:
$100 = 50\ e^{-x} \implies 2 = e^{-x}$

To solve for x, take natural logarithms (ln) of both sides, since the base of natural logarithms, e, is involved:
$\ln(2) = \ln(e^{-x})$

Now, by the properties of logarithms (with respect to the base "e"), $\ln(e^{-x}) = -x$. So we have:

$\ln(2) = -x$, or $x = -\ln(2) = -0.693$ (to three decimal places).

(b) In this case, we can take logs right away. We could use natural logs or common logs here. Using common logs, we get:
$1/4 = 5^{2t-1} \implies \log(1/4) = \log(5^{2t-1})$

To solve for t, note that we can use a property of logarithms to rewrite the right hand side:

$\log(5^{2t-1}) = (2t-1)\log(5)$

So then we have:

$\log(1/4) = (2t-1)\log(5)$ or dividing by $\log(5)$, $\log(1/4) / \log(5) = 2t-1$

Solving for t, we get:

```
      1        log(1/4)
t =  ---   +   --------
      2         2log(5)
```

Using a calculator, we obtain t=0.06932 to 5 decimal places.

(c) To solve this equation, think of ln as $\log_e$ and write the statement in exponential form.

So $\ln(2x+5) = 0 \implies \log_e(2x+5)=0$

Writing this in exponential form gives us:
$e^0 = 2x+5 \implies 1 = 2x+5$

Solving for x gives the answer x=-2.

(d) To solve this equation, again write it in exponential form:

$\log_x 6 = 1/3 \implies x^{(1/3)}= 6$

Next, cube both sides to solve for x:

$x^{(1/3)}= 6 \implies [x^{(1/3)}]^3 = 6^3$

Multiplying the exponents on the left hand side gives us x there. So after evaluating on the right hand side we get:

x = 216 as the solution.

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