

## EXERCISE 12D

### INDEFINITE INTEGRALS INVOLVING LOGARITHMIC FUNCTIONS

1. Find the following indefinite integrals for each of the following:

- |   |  |
|---|--|
| <p>(a) <math>\frac{6x}{x^2 + 4}</math></p>                | <p>(b) <math>\frac{x}{5 - x^2}</math></p>                                    |
| <p>(c) <math>\frac{3x}{2 - 5x^2}</math></p>               | <p>(d) <math>\frac{1}{a - bx}</math></p>                                     |
| <p>(e) <math>\frac{p}{p + qx}</math></p>                  | <p>(f) <math>\frac{2x + 1}{x^2 + x + 1}</math></p>                           |
| <p>(g) <math>\frac{2x - 3}{2x^2 - 6x + 7}</math></p>      | <p>(h) <math>\frac{ax + b}{ax^2 + 2bx + c}</math></p>                        |
| <p>(i) <math>\frac{4 - 2x}{4x - x^2}</math></p>           | <p>(j) <math>\frac{12 - 5x}{13 + 48x - 10x^2}</math></p>                     |
| <p>(k) <math>\frac{(x - 1)(x - 3)}{x^2}</math></p>        | <p>(l) <math>\frac{4}{x^3} - \frac{7}{x^2} + \frac{8}{x} + 9 + 6x</math></p> |
| <p>(m) <math>\left(x + \frac{1}{x^2}\right)^2</math></p>  | <p>(n) <math>x^{1/2} + x^{-1} + x^{-1/2}</math></p>                          |
| <p>(o) <math>(x^2 + 1 + x^3)x^{-4}</math></p>             | <p>(p) <math>\left(x^2 + \frac{2}{x\sqrt{x}} - 4\right)\sqrt{x}</math></p>   |
| <p>(q) <math>\frac{x^2 + 1}{2x}</math></p>                | <p>(r) <math>\frac{12x}{x^2 + 1}</math></p>                                  |
| <p>(s) <math>\frac{e^x - e^{-x}}{e^x + e^{-x}}</math></p> | <p>(t) <math>\frac{1}{e^{-2x} + 1}</math></p>                                |

2. Show that  $\frac{1}{(x+1)(x+2)} = \frac{1}{x+1} - \frac{1}{x+2}$ .

Hence find  $\int \frac{dx}{(x+1)(x+2)}$ .

3. (a) Show that  $\frac{x^2 + 1}{x + 1} = x - 1 + \frac{2}{x + 1}$ .

(b) Find the equation of the curve for which  $\frac{dy}{dx} = \frac{x^2 + 1}{x + 1}$  if the curve passes through the point (0, 1).

ANSWERS 12D

1.

(a)  $3\ln(x^2 + 4) + c$

(b)  $-\frac{1}{2}\ln|5 - x^2| + c$

(c)  $-\frac{3}{10}\ln|2 - 5x^2| + c$

(d)  $-\frac{1}{b}\ln|a - bx| + c$

(e)  $\frac{p}{q}\ln|p + qx| + c$

(f)  $\ln(x^2 + x + 1) + c$

(g)  $\frac{1}{2}\ln|2x^2 - 6x + 7| + c$

(h)  $\frac{1}{2}\ln|ax^2 + 2bx + c| + k$

(i)  $\ln|4x - x^2| + c$

(j)  $\frac{1}{4}\ln|13 + 48x - 10x^2| + c$

(k)  $x - 4\ln|x| - \frac{3}{x} + c$

(l)  $-\frac{2}{x^2} + \frac{7}{x} + 8\ln|x| + 9x + 3x^2 + c$

(m)  $\frac{x^3}{3} + 2\ln|x| - \frac{1}{3x^3} + c$

(n)  $\frac{2}{3}x^{\frac{3}{2}} + \ln|x| + 2x^{\frac{1}{2}} + c$

(o)  $-\frac{1}{x} - \frac{1}{3x^3} + \ln|x| + c$

(p)  $\frac{2}{7}x^{\frac{7}{2}} + 2\ln|x| - \frac{8}{3}x^{\frac{3}{2}} + c$

(q)  $\frac{x^2}{4} + \frac{1}{2}\ln|x| + c$

(r)  $6\ln|x^2 + 1| + c$

(s)  $\ln(e^x + e^{-x}) + c$

(t)  $\frac{1}{2}\ln(e^{2x} + 1) + c$

2.  $\ln\left|\frac{x+1}{x+2}\right| + c$

3. (b)  $y = \frac{1}{2}x^2 - x + 2\ln|x+1| + 1$