

## EXERCISE 12F

### EXPONENTIAL GROWTH AND DECAY

1. If  $P = P_0 e^{kt}$  find  $P$  when  $P_0 = 10$ ,  $k = 3\%$ ,  $t = 2$ .
2. If  $M = M_0 e^{-kt}$ , find  $M$  if  $M_0 = 1$ ,  $k = 0.03$ ,  $t = 10$ .
3. If  $M = \frac{1}{2} M_0$  and  $k = 0.1$ , find  $t$  given  $M = M_0 e^{-kt}$ .
4. The number of bacteria in a container is given by  $N = N_0 e^{kt}$ . If initially there are 2000 bacteria in the container which increase at the rate of 2% per hour, find the number in the container after 100 hours.
5. A radioactive substance decomposes from 100g to 90g in 2 hours according to the formula  $M = M_0 e^{-kt}$ , where  $t$  is the number of hours. Find:
  - (a)  $k$  correct to 3 decimal places,
  - (b) the half life of the substance (ie. the time taken for it to decay to half its original mass). Give your answer to the nearest hour.
6. The population of a city struck by a plague dies off at the rate of 8% per annum according to the formula  $P = P_0 e^{-kt}$ . Initially the population is 100 000.
  - (a) What will the population be in 2 years ?
  - (b) How long (to the nearest year) will it take for the population to be reduced to 1000 ?
7. A ship slows down with a deceleration proportional to its velocity,  $v$ . That is,  $\frac{dv}{dt} = -kv$ . If its initial velocity is 10m/s and 8m/s ten seconds later, find when its velocity will be 1m/s.
8. A germ population grows at a rate proportional to the size of that population. That is,  $\frac{dp}{dt} = kp$ . If initially the population is  $10^8$ , and it grows to  $10^9$  in 20 seconds, find the population after 30 seconds.

### ANSWERS

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|-----------------------|------------------------|
| (1) 10.618            | (2) 0.7408             |
| (3) 6.931             | (4) 14 778             |
| (5) (a) 0.053 (b) 13  | (6) (a) 85 214 (b) 58  |
| (7) After 103 seconds | (8) $3.16 \times 10^9$ |

## EXERCISE 12F(P) PRELIMINARY EXERCISES

### EXPONENTIAL GROWTH AND DECAY

1. If  $\frac{dy}{dx} = 2y$  and  $y = 5$  when  $x=0$ , express  $y$  as a function of  $x$ .
2. If  $\frac{dQ}{dt} = 0.4Q$  and  $Q = 50$  when  $t=0$ , express  $Q$  as a function of  $t$ .
3. If  $\frac{dM}{dt} = -0.01M$  and  $M = 10$  when  $t=0$ , express  $M$  as a function of  $t$ .
4. If  $y = Ae^{-kt}$  and  $y=1000$  when  $t = 0$ , and  $y=368$  when  $t=2$ , find the values of  $A$  and  $k$ .
5. If  $P = Ae^{-kh}$  and  $P = 76$  when  $h = 0$ , and  $P = 28$  when  $h = 5$ , find the values of  $A$  and  $k$ .
6. If the population of a city increases at a rate which is proportional to the current population, and if the population of that city was 100 000 in 1970 and 120 000 in 1980, express in terms of  $t$  the population  $t$  years after 1970.

### ANSWERS

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|------------------------|---------------------------|
| (1) $y = 5e^{2x}$      | (2) $Q = 50e^{0.4t}$      |
| (3) $M = 10e^{-0.01t}$ | (4) $A=1000, k=0.5$       |
| (5) $A=76, k=0.2$      | (6) $P = 100000e^{0.02t}$ |