EXERCISE 12G

EXPONENTIAL GROWTH AND DECAY 2

1. The number of bacteria N in a colony after t minutes is given by $N = 10\,000\,e^{0{\cdot}05t}$. Find
   (a) the number of bacteria after 10 minutes;
   (b) the time required for the original number to double;
   (c) the rate at which the colony increases when (i) $t = 10$ (ii) $N = 20000$

2. The charge, Q units, on the plate of a condenser t seconds after it starts to discharge is given by the formula
$$Q = Ae^{-kt}.$$
   (a) If the original charge is 5 000 units, find the value of A;
   (b) If $\frac{dQ}{dt} = -2\,000$ when Q = 1 000, find the value of k;
   (c) Find the rate of discharge when $Q = 5\,000$.

3. The rate of increase in the number N of bacteria in a certain culture is given by $\frac{dN}{dt} = 0{\cdot}15N$ where t is the time in hours.
   (a) If the original number of bacteria is 1 000, express N as a function of t;
   (b) After how many hours has the original number of bacteria doubled and what is the rate of increase at this time?

4. A population of size N is decreasing according to the law $\frac{dN}{dt} = -\frac{N}{100}$ where t denotes the time in days. If initially the population is of size N_0, find to the nearest day how long it takes for the size to be halved.

5. A radioactive substance decomposes at a rate that is proportional to the mass present at any time. If 10 per cent decomposes in 200 years, what percentage of the original mass will remain after 1 000 years?

6. A vessel filled with liquid is being emptied and the volume V cubic metres remaining after t minutes is given by $V = V_0e^{-kt}$.
   (a) Show that $\frac{dV}{dt} = -kV$.
   (b) If one quarter of the vessel is emptied in the first 5 minutes, what fraction remains after 10 minutes ?
   (c) At what rate is the liquid flowing out
      (i) after 10 minutes
      (ii) when one quarter of the vessel is empty ?

7. A heated body is cooling and the excess of its temperature, θ, above that of its surroundings is given by $\theta = Ae^{-kt}$ where t is the time in minutes.

(a) If at time $t = 0, \theta = 80$, find A.

(b) If the temperature of the surroundings is $20^o C$ and the body cools to $70^o C$ in 10 minutes, find :

(i) its temperature after 20 minutes;

(ii) the time taken to reach $60^o C$.

8. Sunlight transmitted into water loses intensity as it penetrates to greater depths according to the law

$$I(d) = I(0)\, e^{-kd}$$

where $I(d)$ is the intensity at depth d metres below the surface.

If $I(300) = 0 \cdot 3\, I(0)$ find :

(a) the value of k;

(b) the depth at which the intensity would be decreased by one half.

ANSWERS

1. (a) 16 487
   (b) 13.86 min
   (c) (i) 824/min
   (ii) 1 000/min

2. (a) 5 000
   (b) 2
   (c) $\frac{dQ}{dt} = -10\,000$ units/s

3. (a) $N = 1\,000\, e^{0 \cdot 15t}$
   (b) $4 \cdot 6$; 300/h

4. 70 days

5. 59%

6. (b) $\frac{9}{16}$
   (c) (i) $0 \cdot 032\, V_0$ m^3/min
   (ii) $0 \cdot 043\, V_0$ m^3/min

7. (a) 80
   (b) (i) $51 \cdot 25^0 C$
   (ii) $14 \cdot 8$ min

8. (a) $0 \cdot 004$
   (b) 173 m