EXERCISE 12I

RATES OF CHANGE

1. Water is pouring into a container so that the volume flow rate, $\dfrac{dv}{dt}$, is given by $\dfrac{dv}{dt} = 3t^2$, where v is in cubic cm and t is in seconds. How much water has poured into the container in the first 5 seconds?

2. The liquid in a vessel flows out through a tap at a rate given by $\dfrac{dv}{dt} = -10(40 - t)$ where v cm^3 is the volume of liquid in the vessel at time t seconds. Initially the vessel contained 8000 cm^3 of liquid. Find the amount of liquid in the vessel after 10 seconds.

3. (a) Find a formula for the rate of change of the area of a circle with respect to its radius.

   (b) Find this rate when the radius is 2 cm.

4. A cylinder has volume V, height h and radius of base r. Find

   (a) the rate of change of V with respect to h if r is constant,

   (b) the rate of change of V with respect to r if h is constant.

5. The pressure P of a given mass of gas, kept at constant temperature, and its volume V are connected by the formula $PV = 500$. Find the rate of change of P with respect to V when $V = 10$.

6. A circular metal plate is being heated so that the rate of increase of the area A after t hours is given by $\dfrac{dA}{dt} = \dfrac{\pi t}{50}$ m^2 / h.. Initially, the plate had radius 5m. Find the area after 10 hours.

7. A balloon is being filled with helium. Its volume V increases at a rate given by $\dfrac{dV}{dt} = \dfrac{\pi t^2}{4}$ m^3 / min. Find how much its volume increases in the first 6 minutes. Find also how much its volume increases between $t = 6$ and $t = 9$.

8.   The rate of fall of the price $\$P$ of an article after t months is given by $\dfrac{dP}{dt} = \dfrac{-48}{t^3}$, where $t > 0$. After 1 month the price was $\$30$.

     a)   Find the price after 4 months.

     b)   In what time will the price be $\$12$?

     c)   What is the lowest price which the article can ever reach?

9.   Two girls walk along 2 roads OA, OB at right angles to one another. They start together from O. One girl walks at a speed of 5km/h and the other girl walks at 4 km/h.

     a)   Find their distance apart after 2 hours.

     b)   Find their distance apart after t hours.

     c)   Show that the rate at which the distance between them changes is constant.

10.   The rate of descent of a submarine into an ocean, starting from the surface, is given in terms of the time of descent t seconds by $\dfrac{dh}{dt} = 1 - (1+t)^{-2}$ m/s, where h is the depth of the submarine in metres

     a)   Find a formula for h in terms of t.

     b)   Find (to 1 decimal place) the depth of the submarine after 1 minute.

ANSWERS

1. 125 cm^3

2. 4500 cm^3

3.   (a) $2\pi r \text{ cm}^2/\text{cm}$ (b) $4\pi \text{ cm}^2/\text{cm}$

4.   (a) $\dfrac{dV}{dh} = \pi r^2$ (b) $\dfrac{dV}{dr} = 2\pi rh$

5. -5

6. $26\pi \text{ m}^3$

7. $18\pi \text{ m}^3$, $42.75\pi \text{ m}^3$

8.   (a) $\$7.50$ (b) 2 months   (c) $\$6$

9.   (a) $2\sqrt{41} \text{ km}$ (b) $\sqrt{41}\, t \text{ km}$

10.   (a) $h = \dfrac{t^2}{1+t} \text{ m}$ (b) 59.0 m