EXERCISE 12I(P)   PRELIMINARY EXERCISES

RATES OF CHANGE

1. A trench is being dug by a team of labourers who remove V cubic metres of soil in t minutes, where $V = 10t - \dfrac{t^2}{20}$.

    a) At what rate is the soil being removed at the end of 40 minutes ?

    b) Are the labourers working at a constant rate ?

    c) What is their initial rate of work (i.e. when $t = 0$)?

    d) At what time are they removing soil at the rate of 5 m^3 per minute ?

2. A water tank is being emptied and the quantity of water, Q litres, remaining in the tank at any time, t minutes, after it starts to empty is given by
    $Q = 1000(20 - t)^2$, $t \geq 0$.

    a) Find a formula for the rate at which the tank is being emptied.

    b) How long does it take to empty the tank ?

    c) At what time is the water flowing out at the rate of 20,000 litres/minute?

3. A machine manufactures items at a variable rate given by
    $\dfrac{dQ}{dt} = 2t + 1, t \geq 0$
    where Q is the number of items manufactured after t minutes.

    a) At what rate is the machine working
        i)   initially,
        ii) after 10 minutes ?

    b) What is the total number of items produced in the first 10 minutes?

4. A cubical block of ice has an edge of 10 cm initially. It melts so that its volume decreases at a constant rate of 25 cm^3 /hour and the block remains cubical. Find

    a) the volume V at any time t,

    b) the time required to completely melt the ice.

ANSWERS

1.   a) $6 \, \mathrm{m}^3$ /min   b) no   c) $10 \mathrm{m}^3$ /min   d) 50 min

2.   a) $\dfrac{dQ}{dt} = -2000(20 - t)$ litres/min   b) 20 min   c) 10 min

3.   a) (i) 1 item/min   (ii) 21 items/min   b) 110

4.   a) $V = -25t + 1000$ after t hours   b) 40 hours