EXERCISE 13C(P) PRELIMINARY EXERCISES

VOLUMES OF SOLIDS OF REVOLUTION

1. Find the volume of the solid of revolution formed by rotating the arc of the parabola $y = x^2$ between $x = 0$ and $x = 3$ about the X-axis.

2. Find the volume of the solid formed by rotating the line $y = 2x$ between $x = 0$ and $x = 4$ about the X-axis.

3. A cone is formed by rotating about the X-axis the segment of the line joining the points (1,0) and (3,4). Find the volume of the cone.

4. The semicircle $y = \sqrt{9 - x^2}$ is rotated about the X-axis. Calculate the volume of the sphere generated.

5. The region bounded by the parabola $y = (x-2)^2$ and the coordinate axes is rotated about the X-axis. Find the volume of the solid formed.

ANSWERS

(1) $\dfrac{243\pi}{5}$ units^3 (2) $\dfrac{256\pi}{3}$ units^3 (3) $\dfrac{32\pi}{3}$ units^3

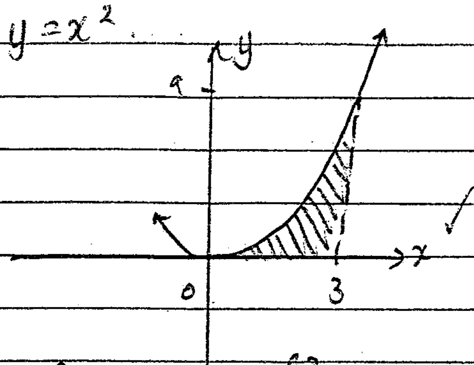
(4) 36π units^3 (5) $\dfrac{32\pi}{5}$ units^3

Volumes of Solids of Revolution

Excellent work!

Angelina

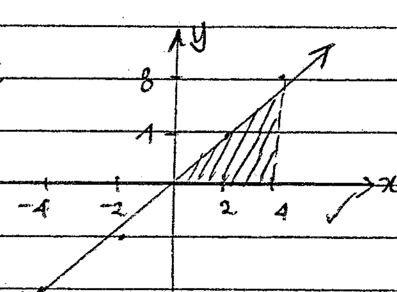
1. $y = x^2$



$V = \pi\int y^2\,dx = \pi\int_0^3 x^4\,dx = \pi\left[\frac{x^5}{5}\right]_0^3$

$= 48\frac{3}{5}\pi$ units^3

2. $y = 2x$

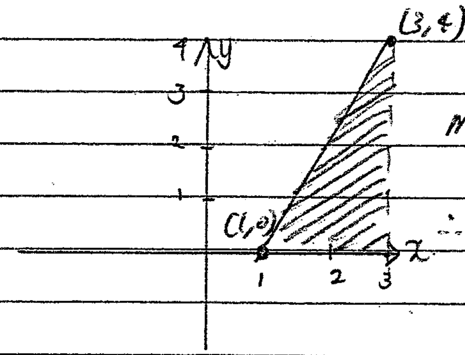


$V = \pi\int y^2\,dx = \pi\int_0^4 4x^2\,dx$

$= \pi\left[\frac{4}{3}x^3\right]_0^4$

$= 85\frac{1}{3}\pi$ units^3

3.



$m = \frac{4}{2}$

$= 2.$

$\therefore y = 2(x-1)$

$= 2x - 2$

$V = \pi\int_1^3 4x^2 - 8x + 4\,dy$

$= \pi\left[\frac{4}{3}x^3 - 4x^2 + 4x\right]_1^3$

$= \pi\left(12 - 1\frac{1}{3}\right) = 10\frac{2}{3}\pi$ units^3

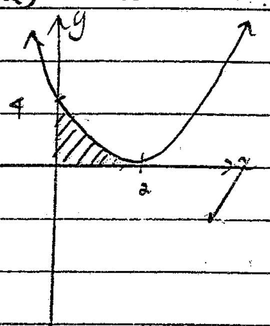
4. $V = \pi\int_{-3}^{3} 9 - x^2\,dx$

$= \pi\left[9x - \frac{1}{3}x^3\right]_{-3}^{3}$

$= \pi(18 + 18)$

$= 36\pi$ units^3

5. $y = (x-2)^2 = x^2 - 4x + 4$



$V = \pi\int_0^2 (x-2)^4\,dx$

$= \pi\left[\frac{(x-2)^5}{5}\right]_0^2$

$= \pi\left[0 - \left(\frac{-32}{5}\right)\right]$

$= \frac{32\pi}{5}$ units^3