Outcome 1 – Graphs **(30 Marks)**

1. Sketch each of the following and indicate on your sketch: **6**

   (i) the coordinates of the vertex

   (ii) the y-intercept.

   (a) $y = x^2$ (b) $y = (x+1)^2 - 2$

2. For each parabola, find: **6**

   (i) the equation of the axis of symmetry

   (ii) the vertex

   (iii) the minimum or maximum value of the function

   (a) $y = -x^2 + 4x + 5$ (b) $y = \frac{1}{2}x^2 + 6x - 3$

3. In each of the following circles, find: **6**

   (i) the coordinates of the centre

   (ii) the length of the radius

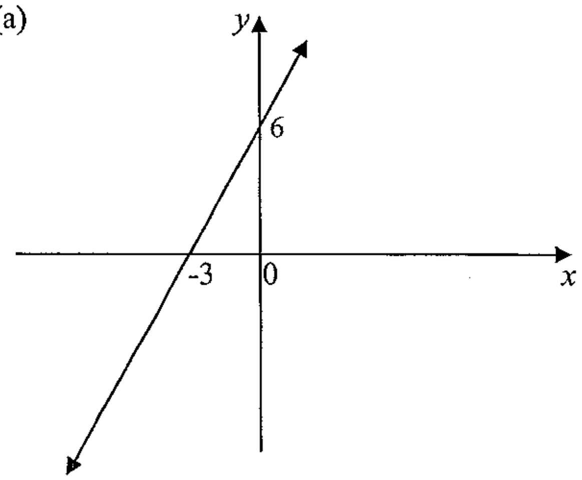
   (iii) the x-intercepts and y-intercepts (if they exist).

   (a) $x^2 + y^2 = 9$ (b) $(x-4)^2 + (y-3)^2 = 25$

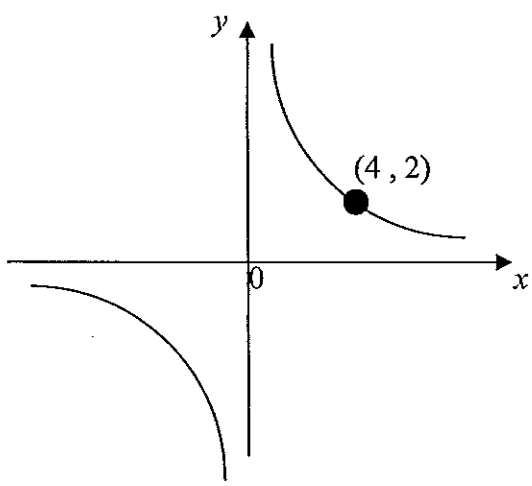
4. Find the coordinates of the point(s) of intersection between the curves $y = x^3$ and $y = \frac{1}{x}$. **2**

5. Find the equation of each graph **2**

(a)



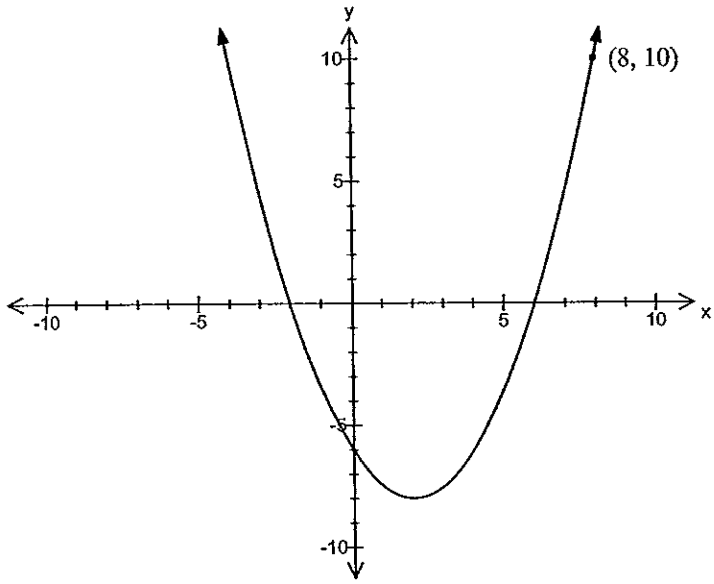
(b)



6. Sketch the following curves showing all essential features. **6**

(a) $3x+2y=6$ (b) $y=2^x$ (c) $y=\dfrac{-2}{x}$

7. Find the equation of the following parabola in the form $y=k(x-a)(x-b)$ **2**



Outcome 2 – Polynomials **(25 Marks)**

START A NEW PAGE

1. For each of these polynomials state: **6**

(i) the degree

(ii) the leading coefficient

(iii) the constant term.

(a) $P(x)=8x^5+9x^3-2x-1$ (b) $P(x)=3\sqrt{5}x^3-4x+6$

2. Perform the following division and express your answer in the form: **3**

Dividend = divisor x quotient + remainder.

$$\left(2x^3-x^2+5x+1\right)\div(x+6)$$

3. Expand and simplify $(3x-4)(5x^3+9x^2-7)$. **2**

4. Given $P(x)=2x^3+6x^2+4x+7$ and $Q(x)=x^3+3x-6$ find $P(x)-Q(x)$. **1**

3. Find the value of k given: **4**

(a) The remainder is 6 when $P(x)=2x^3+6x^2-3x+k$ is divided by $(x-1)$.

(b) $P(x)=x^3-10x^2+kx-8$ is exactly divisible by $(x-2)$.

4. $P(x)=x^3+ax^2+bx-30$ is divisible by $(x-3)$ but leaves a remainder of -8 when divided by $(x-1)$. Find a and b. **3**

5. Sketch the following polynomial functions show the x and y intercepts. **6**

(a) $y=(x+1)(x-3)(x-7)$

(b) $y=-x(x-4)^3$

(c) $y=x^3+8x^2-20x$

Outcome 3 – Functions and Logarithms (25 Marks)

START A NEW PAGE

1. Given $f(x) = x^2 - x - 3$: **2**

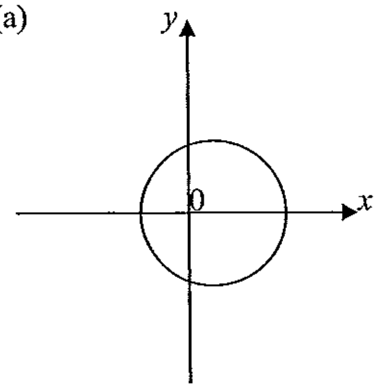
   (a) $f(2)$ (b) $f(y-3)$

2. Sketch $f(x) = x^2 + 1$ for $x \geq 0$. On the same number plane diagram, sketch $y = f^{-1}(x)$, the inverse function of $y = f(x)$. **2**

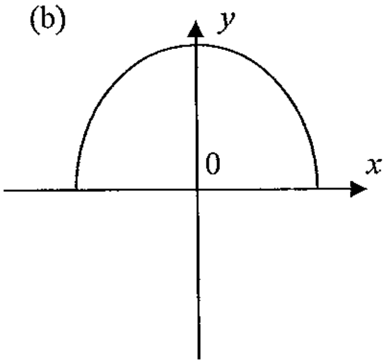
3. Find the inverse of $y = \dfrac{3x-2}{3}$. **2**

4. State whether or not the following diagrams represent the sketch of a function. **3**

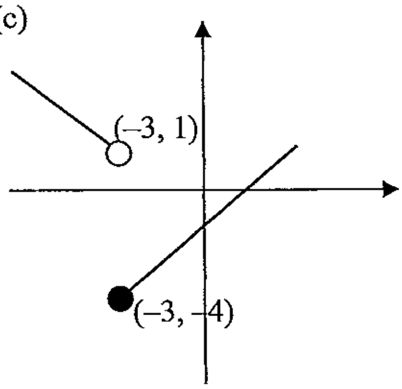
(a)



(b)



(c)



5. Write the statement $3^4 = 81$ in logarithm form: **1**

6. Write the statement $\log_2 \dfrac{1}{32} = -5$ in index form: **1**

7. Solve each of the following equations: **6**

   (a) $\log_x 9 = 2$ (b) $\log_{\sqrt{2}} 8 = x + 2$ (c) $3^{2x} = 26$

8. Simplify, fully, each expression. **6**

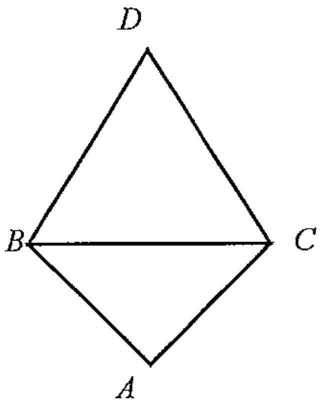
   (a) $\log_6 9 + \log_6 4$ (b) $\log_4 144 - \log_4 9$ (c) $\log_{100} 20 - \dfrac{1}{2}\log_{100} 4$

9. At the beginning of 2007, David deposited \$150 000 in an account which will pay an interest rate of 6% per annum, compounding monthly. During which year will David's investment be worth twice the original deposit? **2**

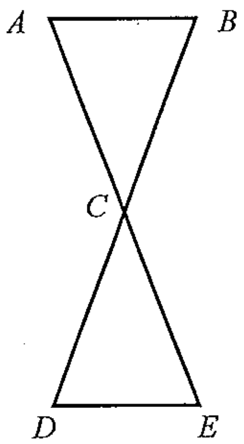
Outcome 4 – Geometry and Similarity (25 Marks)

1. **5**

(a) ΔABC is a right-angled isosceles triangle. ΔDBC is equilateral. Find the size of $\angle ABD$.



(b) $AC = BC = DC = EC$. Prove that AB and DE are parallel.

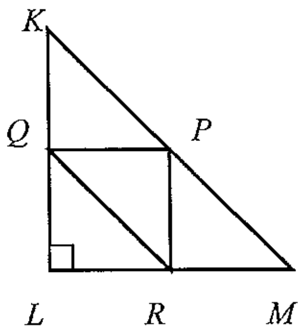


2. ΔKLM is right-angled at L. Q, P and R are the midpoints of the sides of ΔKLM. $KL = 15$ cm, $LM = 10$ cm. Find the: **4**

(i) area of ΔKLM

(ii) area of ΔPQR

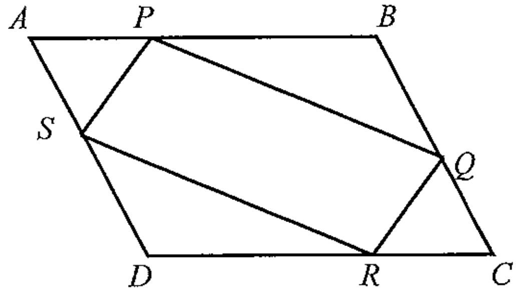
(iii) ratio of area ΔLRQ to area trapezium $KQRM$. Show all reasons.



3. $ABCD$ is a parallelogram. $AP = AS = CQ = CR$. By using congruent triangles, or otherwise: **5**

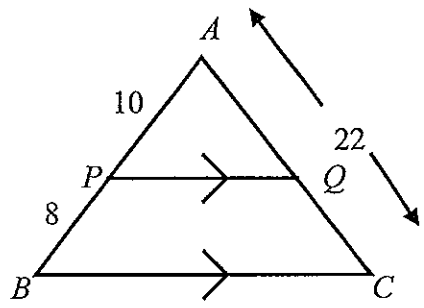
(i) Prove $QR = PS$ and $PQ = SR$.

(ii) What shape is $PQRS$? Justify your answer.



4. (i) Prove that $\triangle APQ$ and $\triangle ABC$ are similar. **4**

(ii) Hence find the length of PQ.



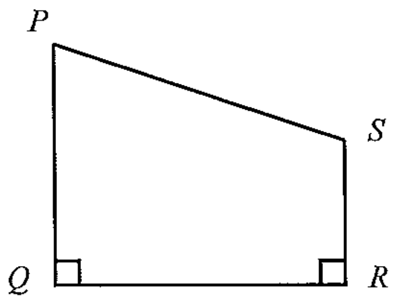
5. Two similar cones have surface areas in the ratio 256:81. What is the ratio of the corresponding: **2**

(i) slant heights (ii) volumes.

6. In the quadrilateral $PQRS$, $PQ \perp QR$ and $SR \perp QR$ **5**

(i) Prove that $PR^2 - QS^2 = PQ^2 - RS^2$

(ii) Hence, prove that $PS^2 - QR^2 = (PQ - RS)^2$

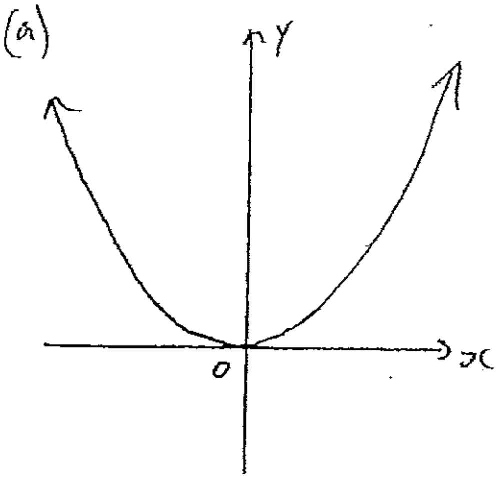


END OF EXAM.

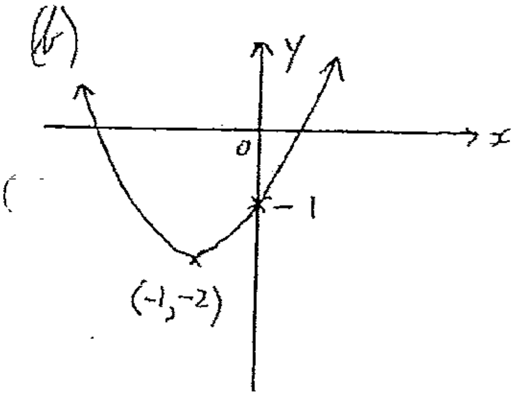
Year 10 Pathway A
Yearly 2007

Outcome ①

(a)



(b)



② (a) $y = -x^2 + 4x + 5$

(i) $x = 2$

(ii) $(2, 9)$

(iii) 9

(b) $y = \frac{1}{2}x^2 + 6x - 3$

(i) $x = -6$

(ii) $(-6, -21)$

(iii) -21

③ (a) $x^2 + y^2 = 9$

(i) $(0, 0)$

(ii) 3

(iii) $x = \pm 3$, $y = \pm 3$

(b) $(x-4)^2 + (y-3)^2 = 25$

(i) $(4, 3)$

(ii) 5

(iii) $x = 8$, $x = 0$, $y = 6$, $y = 0$

④ $x^3 = \frac{1}{x}$

$x^4 = 1$, $x^4 - 1 = 0$

$(x-1)(x+1)(x^2+1) = 0$

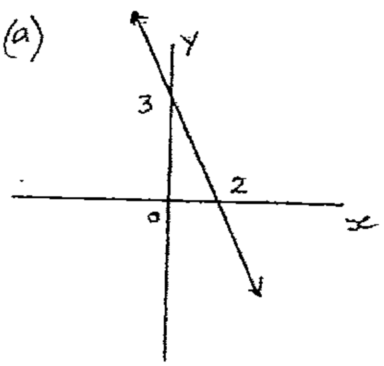
$x = \pm 1$

∴ curves intersect at $(1, 1)$ and $(-1, -1)$

⑤ (a) $y = 2x + 6$

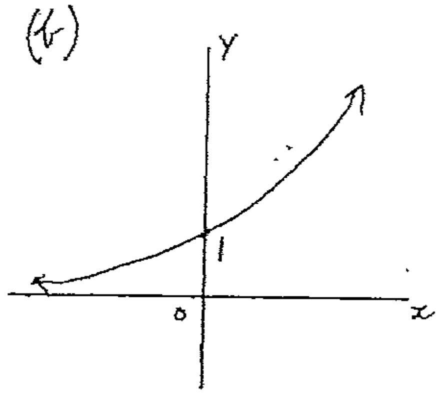
(b) $xy = 8$

⑥ (a)



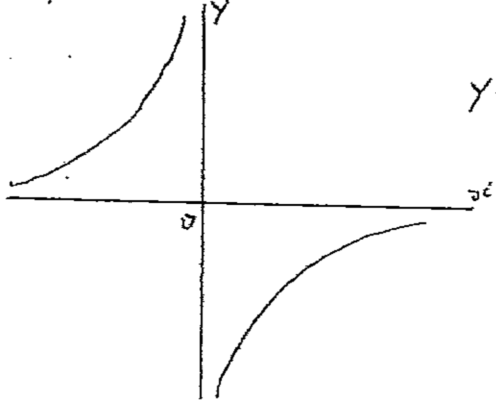
$3x + 2y = 6$

(b)



$y = 2^x$

(c)



$y = -\frac{2}{x}$

⑦ $a = 6$, $b = -2$

$y = k(x-6)(x+2)$

$-6 = k(0-6)(0+2)$

$k = \frac{1}{2}$

∴ $y = \frac{1}{2}(x-6)(x+2)$

Outcome ②

① (a) (i) 5
(ii) 8
(iii) −1

(b) (i) 3
(ii) $3\sqrt{5}$
(iii) 6

②

$$\begin{array}{r} 2x^2-13x+83 \\ x+6\overline{)2x^3-x^2+5x+1} \\ 2x^3+12x^2 \\ \hline -13x^2 \\ -13x^2-78x \\ \hline 83x \\ 83x+498 \\ \hline -497 \end{array}$$

$2x^3-x^2+5x+1 = (x+6)(2x^3-x^2+5x+1) - 497$

③ $15x^4+27x^3-21x-20x^3-36x^2+28$

$= 15x^4+7x^3-36x^2-21x+28$

④ x^3+6x^2+x+13

⑤ (a) $2\times1^3+6\times1^2-3\times1+k=6$

$2+6-3+k=6$

$k=1$

(b) $2^3-10\times2^2+2k-8=0$

$8-40+2k-8=0$

$k=20$

⑥ $3^3+9a+3b-30=0$

$3a+b=1$ —— ①

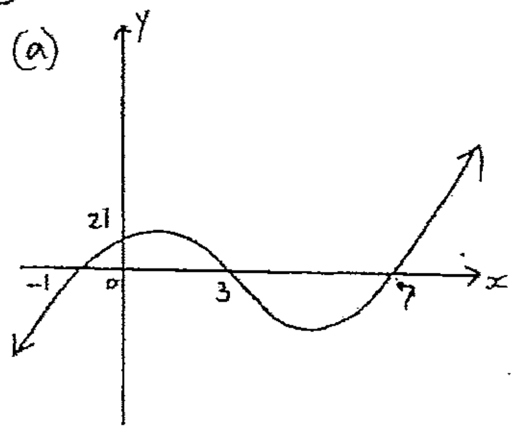
$1+a+b-30=-8$

$a+b=21$ —— ②

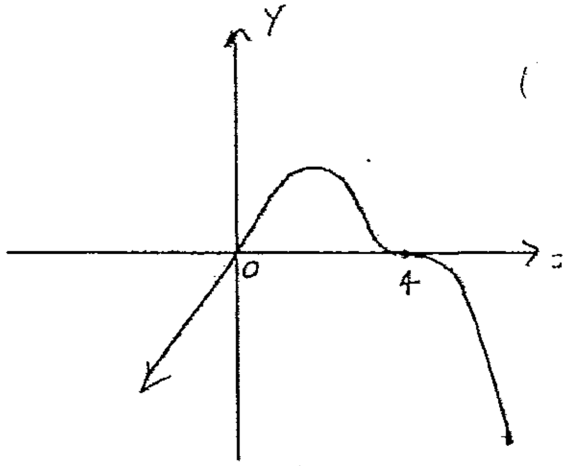
$a=-10,\ b=31$

⑦

(a)

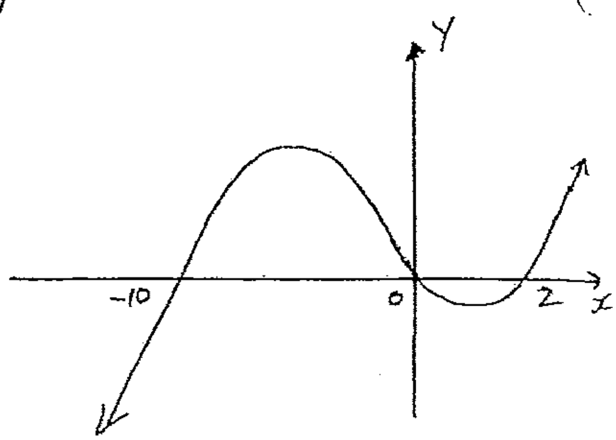


(b)



(c) $y = x(x^2+8x-20)$

$= x(x+10)(x-2)$



Outcome ③

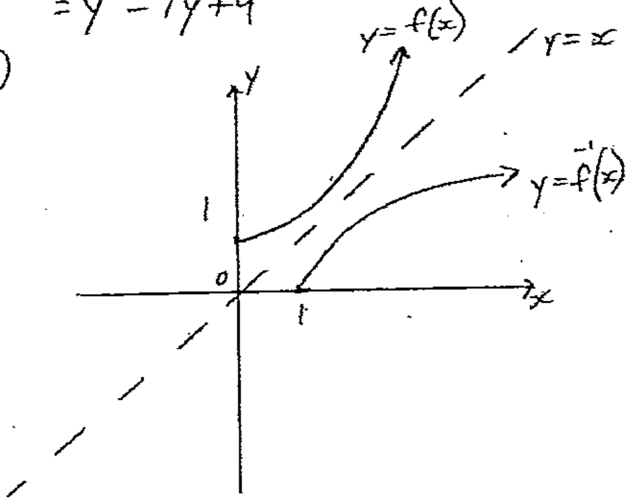
① (a) -1

(b) $(y-3)^2-(y-3)-3$

$= y^2-6y+9-y+3-3$

$= y^2-7y+9$

②



③ $x = \frac{3y-2}{3}$

$3x = 3y-2$

$3y = 3x+2$

$y = \frac{1}{3}(3x+2)$

or $f^{-1}(x) = \frac{3x+2}{3}$

④ (a) No (b) Yes (c) Yes

⑤ $\log_3 81 = 4$

⑥ $2^{-5} = \frac{1}{32}$

⑦ (a) $x^2=9$

$x=\pm 3$

but $x \neq -3$

$\therefore x = 3$

(b) $(\sqrt{2})^{x+2} = 8$

$2^{\frac{x+2}{2}} = 2^3$

$\frac{x+2}{2} = 3$

$x = 4$

(c) $\log_3 26 = 2x$

$x = \frac{1}{2}\log_3 26$

⑧ (a) $\log_6 36 = 2$

(b) $\log_4\left(\frac{144}{9}\right) = 2$

(c) $\log_{100}\left(\frac{20}{\sqrt{4}}\right)$

$= \log_{100} 10$

$= \frac{1}{2}$

⑨ $A = P\left(1+\frac{0{\cdot}5}{100}\right)^n$

$150000(1{\cdot}005)^n = 300000$

$1{\cdot}005^n = 2$

$n \log_{10} 1{\cdot}005 = \log_{10} 2$

$n = \frac{\log_{10} 2}{\log_{10} 1{\cdot}005}$

≈ 139

After approx. 139 months, the amount doubles i.e. during 2020.

Outcome ④

① (a) $\angle ABC = \frac{1}{2}(180^\circ - 90^\circ)$ (equal base ∠ of right isos △)
$= 45^\circ$

$\angle DBC = 60^\circ$ (△DBC equilateral)

$\therefore \angle ABD = 105^\circ$

(b) $AC = EC$ (Given)
$BC = DC$ (Given)

$\angle ACB = \angle DCE$ (vert. opp. ∠s)

$\therefore \triangle ABC \equiv \triangle DCE$ (SAS)

$\therefore \angle BAC = \angle DEC$ (base ∠s of congruent isosceles △s)

$\therefore AB \parallel DE$ (alternat ∠s equa)

② (a) $75\ cm^2$
(b) $\frac{75}{4}\ cm^2$
(c) $\triangle LRQ \equiv \triangle PRQ \equiv \triangle QPK \equiv \triangle RMP$

Area Trapezium = Area △PRQ + Area △QPK + Area △RMP
= 3 × Area △LRQ

∴ Ratio = 1:3

③ (i) $\angle PAS = \angle QCR$ (opp. ∠s of ∥gram)
$AP = QC$ (Given)
$AS = RC$ (Given)
$\therefore \triangle APS \equiv \triangle QCR$ (SAS)
$\therefore QR = PS$

$AB = DC$ (Opp sides of ∥gram)

$\therefore BP = AB - AP$
$= DC - RC$ (AP = RC → given)
$= DR$
Similarly $BQ = DS$

$\angle SDR = \angle QBP$ (opp ∠s of ∥gram)

$\therefore \triangle SDR \equiv \triangle QBP$ (SAS)

(ii) Parallelogram (Opposite sides are equal)

④ (i) $\angle APQ = \angle ABC$ (corresp. ∠s in ∥ lines)

$\angle AQP = \angle ACB$ (")

$\therefore \triangle APQ \parallel\!| \triangle ABC$

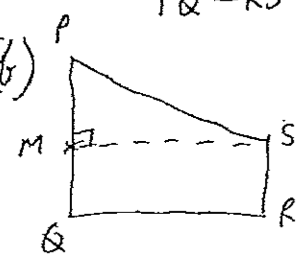
(ii) $\frac{AQ}{22} = \frac{10}{18}$ (sides of sim △s)

$AQ = 12.\dot{2}$

⑤ (a) 16 : 9
(b) $16^3 : 9^3$
$= 4096 : 729$

⑥ (a) $PR^2 = PQ^2 + QR^2$
$PR^2 - QS^2 = PQ^2 + QR^2 - QS^2$
$= PQ^2 - (QS^2 - QR^2)$
$= PQ^2 - RS^2$

(b)



$PS^2 = QR^2 + PM^2$
$PS^2 - QR^2 = PM^2$
$= (PQ - QM)^2$
$= (PQ - RS)^2$