Applications of calculus to the physical world

TOPIC TEST

Time allowed: 1 hour **Total marks = 100**

SECTION I Multiple-choice questions

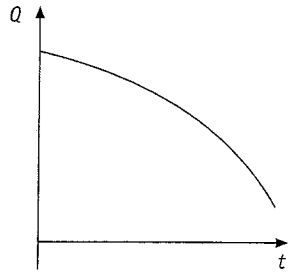
10 marks

Instructions
- This section consists of 10 multiple-choice questions
- Each question is worth 1 mark
- Fill in only ONE CIRCLE
- Calculators may be used

1 Velocity is the rate of change of:

Ⓐ displacement Ⓑ speed Ⓒ acceleration Ⓓ none of these

2 From the diagram



Ⓐ $\frac{dQ}{dt} > 0$ and $\frac{d^2Q}{dt^2} > 0$

Ⓑ $\frac{dQ}{dt} > 0$ and $\frac{d^2Q}{dt^2} < 0$

Ⓒ $\frac{dQ}{dt} < 0$ and $\frac{d^2Q}{dt^2} > 0$

Ⓓ $\frac{dQ}{dt} < 0$ and $\frac{d^2Q}{dt^2} < 0$

3 If a particle is undergoing motion such that at a particular time T, $x > 0$, $\dot{x} < 0$ and $\ddot{x} < 0$ then at T the particle is:

Ⓐ slowing down Ⓑ speeding up Ⓒ stationary

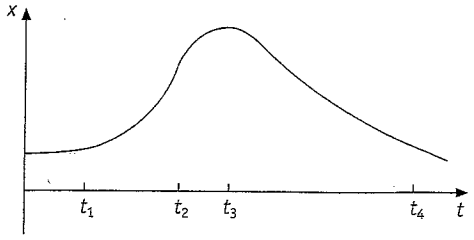
Ⓓ there is not enough information to determine what the particle is doing

4 The population P of a colony of flying foxes at time t is approximated using the expression $P = 800e^{0.5t}$. Initially the population was approximately:

Ⓐ 1320 Ⓑ 800 Ⓒ 400

Ⓓ there is not enough information to determine the initial population

5 The graph shows the distance x of a particle (which is moving in a straight line), from a fixed point at time t. At what time is the particle moving fastest?



Ⓐ t_1 Ⓑ t_2

Ⓒ t_3 Ⓓ t_4

6 If the velocity of a moving particle at time t is given by $v = 9t$ the acceleration is?

Ⓐ constant Ⓑ zero Ⓒ increasing Ⓓ decreasing

7 The rate, R kg/s at which grain is flowing from a bin is given by $R = 80t - 3t^2$ where t is the time in seconds. For which value of t is R not physically possible?

Ⓐ $t = 0$ Ⓑ $t = 10$ Ⓒ $t = 20$ Ⓓ $t = 30$

8 Which *must* be true if a particle is stationary?

Ⓐ displacement is zero Ⓑ velocity is zero

Ⓒ acceleration is zero Ⓓ all of these

9 The number of animals on an island, N, at time t is given by the formula $N = 7000e^{-kt}$ where k is a positive constant. Over time the number of animals on the island is?

Ⓐ increasing exponentially Ⓑ decreasing exponentially

Ⓒ increasing at a constant rate Ⓓ decreasing at a constant rate

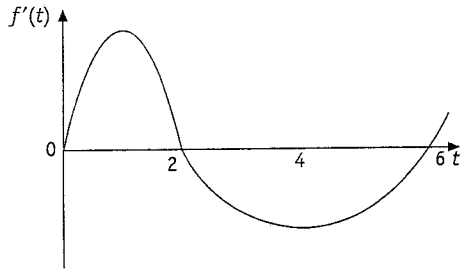
10 The diagram shows a sketch of the graph of $y = f'(t)$ at time t. When $t = 2$, $y = f(t)$ is?

Ⓐ a maximum

Ⓑ a minimum

Ⓒ zero

Ⓓ there is insufficient information



SECTION II

90 marks

Show all necessary working

11 If $y = 8 - 5t - 3t^2$ find: **2 marks each**

a $\frac{dy}{dt}$

b $\frac{d^2y}{dt^2}$

12 If $f(t) = 2\cos 4t$ find: **2 marks each**

a $f'(t)$

b $f''(t)$

13 If $x = t^3 - 3t^2 + 7e^t$ find: **2 marks each**

a $\dot{x}$

b $\ddot{x}$

14 Find x if $\dot{x} = 8t^2 - 6t + 3$ and, when $t = 2$, $x = 3$ **4 marks**

15 Find x if $\ddot{x} = -4$ and, when $t = 1$, $\dot{x} = 5$ and $x = 8$ **6 marks**

16 $Q = 7 + 20t - 2t^2$ $(t \geq 0)$. Find:

a Q when $t = 4$ **2 marks**

b t when $Q = 39$ **4 marks**

c $\frac{dQ}{dt}$ when $t = 3$ **4 marks**

d t when $\frac{dQ}{dt} = 0$ **4 marks**

17 $\frac{dP}{dt} = 17 - 4t$. When $t = 0$, $P = 12$. Find:

a $\frac{dP}{dt}$ when $t = 3$ **2 marks**

b P when $t = 3$ **5 marks**

18 Gas is pumped into a previously empty chamber at a rate given by $\frac{dV}{dt} = \frac{e^t}{5}$ where V is the volume of gas in litres and t is the time in minutes.

a What is the initial rate at which the gas is pumped into the chamber? **3 marks**

b Find an expression for V in terms of t. **5 marks**

19 The number of bacteria in a culture is given by $N = 10\,000e^{kt}$ where k is a positive constant and t the time in hours. After 5 hours the number of bacteria has increased to 20 000.

a What was the initial number of bacteria? **2 marks**

b Find the value of k correct to four decimal places. **4 marks**

continued ...

19 continued ...

c How many bacteria are in the colony after 24 hours? **4 marks**

d How long is it before the number of bacteria reaches 1 million? **5 marks**

20 The displacement, x m, of a moving particle at time t seconds is given by $x = \ln(1 + t)$

a Find the velocity when $t = 2$ **4 marks**

b Find the acceleration when $t = 2$ **5 marks**

21 The acceleration (a m s^{-2}) of a particle moving along the x-axis is given by $a = -1$. Originally the particle is moving with velocity 4 m s^{-1} at a position 8 m to the left of the origin.

a When is the particle stationary? **5 marks**

b When is the particle at the origin? **6 marks**

c Briefly explain why the particle will never move to the right of the origin. **4 marks**

ANSWERS

Page 130 **1** a displacement **b** zero **2** **a** the particle is moving to the left **b** the velocity gives the direction as well as the magnitude; speed = | velocity | **3** **a** −12 m **b** at 2 seconds and at 9 seconds **c** $v = 2t - 11$ **d** 1 m s^{-1} **4** **a** −11 m s^{-1} **b** 2.25 s **c** $x = 9t - 2t^2 + 7$

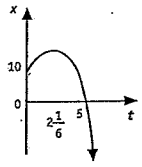
Page 131 **1** velocity **2** **a** slowing down **b** slowing down **c** speeding up **d** speeding up **3** **a** $a = 160 - 6t$ **b** 148 m s^{-2} **4** **a** $v = -2t + 6$ **b** −4 m s^{-1} **c** $x = -t^2 + 6t - 2$ **d** 3 m

Page 132 **1** **a** 0 s and 12 s **b** 432 m **2** **a** $x = 9t^2 - \frac{1}{3}t^3$ **b** −18 m s^{-2}, 972 m

Page 133 **1** **a** 45 m s^{-1} **b** 6 m s^{-2} **c** 8 s **d** 128 m **2** **a** $v = t + \frac{4}{(t+1)^2} + 12$ **b** $x = \frac{t^2}{2} - \frac{4}{t+1} + 12t + 1$

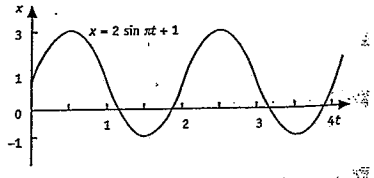
Page 134 **1** **a** **i** $v = -6t + 13$ **ii** $x = -3t^2 + 13t + 10$ **b** **i** $2\frac{1}{6}$ s **ii** 5 s

c



d The particle is initially at a position 10 m to the right of the origin travelling right at a speed of 13 m s^{-1}. It stops after $2\frac{1}{6}$ seconds, then moves left passing through the origin after 5 seconds, and continues to travel left at increasing speed.

Page 135 **1** **a** 1 m to the right of the origin **b** $\frac{1}{2}$ s, 3 m **c** $2\pi^2$ m s^{-2} **d** (see right)



Pages 136-140 **1** A **2** D **3** B **4** B **5** B **6** A **7** D **8** B **9** B **10** A **11** **a** $-5 - 6t$ **b** −6 **12** **a** $-8 \sin 4t$ **b** $-32 \cos 4t$ **13** **a** $3t^2 - 6t + 7e^t$ **b** $6t - 6 + 7e^t$

14 $x = \frac{8t^3}{3} - 3t^2 + 3t - 12\frac{1}{3}$ **15** $x = -2t^2 + 9t + 1$ **16** **a** 55 **b** 2 or 8 **c** 8 **d** 5 **17** **a** 5 **b** 45 **18** **a** 0.2 L/min **b** $V = \frac{e^t - 1}{5}$

19 **a** 10 000 **b** 0.1386 **c** 278 576 [nearest whole number] **d** 34th hour **20** **a** $\frac{1}{3}$ m s^{-1} **b** $-\frac{1}{9}$ m s^{-2} **21** **a** 4 s **b** 4 s **c** The particle is stationary when $t = 4$, and because $\ddot{x} < 0$, the maximum displacement occurs when $t = 4$. So the maximum displacement is 0 m and the particle never moves right of the origin.