Applications of calculus to the physical world

EXCEL HSC MATHEMATICS
pages 139–140

Exponential growth and decay (1)

QUESTION **1** Given $M = 75e^{0.4t}$ find (to the nearest whole number)

a M when $t = 12$

b t when $M = 250\ 000$

c $\frac{dM}{dt}$ when $t = 24$

QUESTION **2** Given $N = 200e^{-0.09t}$ find, correct to one decimal place:

a N when $t = 15$

b t when $N = 100$

c $\frac{dN}{dt}$ when $t = 20$

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Exponential growth and decay (2)

QUESTION **1**

a If $N = N_0e^{kt}$ where N_0 and k are constants, show that $\frac{dN}{dt} = kN$

b If $P = P(0)e^{-kt}$ where $P(0)$ and k are constants show that $\frac{dP}{dt} = -kP$

QUESTION **2** $Q = Q_0e^{-kt}$. When $t = 0$, $Q = 400$ and when $t = 8$, $Q = 325$. Find the value of:

a Q_0

b k (correct to 4 decimal places)

c Q when $t = 16$

d $\frac{dQ}{dt}$ when $t = 16$

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Exponential growth and decay (3)

QUESTION **1** The mass, M, in grams of a radioactive substance is expressed as $M = M(0)e^{-0.02t}$, where t is the time in years and $M(0)$ is the original mass. If the original mass was 75 grams, find:

a the mass of the substance after 12 years

b the rate at which the mass is decreasing after 12 years

QUESTION **2** The number of bacteria in a colony is given by $N = N_0e^{kt}$, where N_0 and k are constants and t is the time in hours.

a Show that N satisfies the equation $\frac{dN}{dt} = kN$

b If the number of bacteria is initially 20 000, and 70 000 three hours later, after how many hours will the number of bacteria exceed 1 million?

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Exponential growth and decay (4)

QUESTION **1** The rate of increase of a population of birds nesting in a wildlife sanctuary is governed by the equation $\frac{dP}{dt} = kP$ where k is a constant and t the time in years.

a If the population doubles every 20 years find a value for k, correct to five decimal places.

b If the original population was 100 birds find the expected population after 75 years.

QUESTION **2** The half-life of a radioactive element is the time for half the mass of that element to disintegrate. If the mass present at time t hours is given by $M = M_0e^{-kt}$ and if a particular element has a half-life of 87 hours, find:

a the value of k

b the time for $\frac{3}{4}$ of the element to disintegrate

ANSWERS

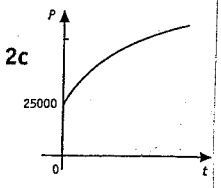
Page 120 1 a $6t - 8$ b $-35t^6$ c $20(4t - 7)^4$ 2 a $30t^4 - 21t^2 + 2$ b $4t^3 + 3t^2 - 2t - 1$ c $2\cos(2t + 5)$ 3 a $18t^5 + 3t^2 - 7$ b $8e^{2t-1}$ c $6(3t + 1)$ 4 a 0 b $-2-10t$ c $\frac{1}{t}$ 5 a $6t - 20$ b 0 c $-3\pi^2 \cos \pi t$

Page 121 1 a $2t^2 + 3t$ b $\frac{8t^3}{3} - 6t^2 + 7t + 18\frac{2}{3}$ 2 a $4t^2 - \frac{t^3}{6} + 4t + 2$ b $\frac{7t^2}{2} - 12t + 68$

Page 122 1 a 35 b 10 c -6 d 4 2 a 96 b $449\frac{1}{3}$

Page 123 1 a >,< b <, < c <, > d >, > 2 a The number of registered pets is increasing over the period. b The number of pet registrations is increasing at a decreasing rate. c (see right)

2c



Page 124 1 a 0 b 0 or 40 c $V = 800t^2 - \frac{t^4}{4} + 1000$ d 20 843.75 2 a 40 L/min b after 18 minutes c 810 litres

Page 125 1 a 4375 grams per second b 30 seconds c 140 kg d $6000\sqrt{3}$ g/s

Page 126 1 a 9113 b 20 c 442 943 2 a 51.8 b 7.7 c -3.0

Page 127 2 a 400 b 0.0260 c 264 [nearest whole number] d -6.9 [1 d.p.]

Page 128 1 a 59 g [nearest g] b 1.2 grams per year [1 d.p.] 2 b In the 10^{th} hour

Page 129 1 a 0.03466 b 1345 2 a 0.0080 [4 d.p.] b 174 hours