

Applications of calculus to the physical world

Velocity

QUESTION 1 Complete:

- a Velocity is defined as the rate of change of _____.
- b If a particle is at rest its velocity is _____.

QUESTION 2 A particle is moving along the x -axis. Briefly explain:

- a the significance of a negative velocity _____

- b the difference between speed and velocity _____

QUESTION 3 The displacement from the origin, x m, of a moving particle at time t seconds is given by:
 $x = t^2 - 11t + 18$. Find:

- a the displacement when $t = 6$
- b the times when the particle is at the origin

- c an expression for the velocity

- d the velocity when $t = 6$

QUESTION 4 The velocity (v m s⁻¹) of a particle moving in a straight line is given by $v = 9 - 4t$. Find:

- a the velocity when $t = 5$
- b when the particle is at rest

- c an expression for the displacement if $x = 16$ when $t = 3$

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Acceleration

QUESTION 1 Complete:

Acceleration is defined as the rate of change of _____.

QUESTION 2 State whether a moving particle will be slowing down or speeding up if:

- a velocity is negative but acceleration is positive _____
- b velocity is positive but acceleration is negative _____
- c velocity and acceleration are both positive _____
- d velocity and acceleration are both negative _____

QUESTION 3 The displacement x metres at time t seconds of a moving particle is given by $x = 80t^2 - t^3$

Find:

a an expression for the acceleration

b the acceleration when $t = 2$

QUESTION 4 The acceleration, in m s^{-2} , of a particle moving along the x -axis is given by $\ddot{x} = -2$

When $t = 1$, $\dot{x} = 4$ and $x = 3$. Find:

a an expression for the velocity

b the velocity when $t = 5$

c an expression for the displacement

d the displacement when $t = 5$

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Velocity and acceleration (1)

QUESTION 1 A particle is moving in a straight line from a fixed point O . At time t seconds, its displacement from O , x m, is given by $x = 18t^2 - t^3$

a At what times is the particle stationary?

b Find the displacement at the time when the acceleration is zero.

QUESTION 2 The acceleration (a m s⁻²) of a moving particle is given by $a = 18 - 2t$
The particle starts from rest at the origin.

a Find an expression for the position, x , of the particle.

b Find the acceleration and position of the particle when it is at rest again.

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Velocity and acceleration (2)

QUESTION 1 A body, moving along the x -axis, starts from rest at the origin. Its velocity, \dot{x} m s⁻¹, is given by $\dot{x} = 24t - 3t^2$. Find:

a the velocity after 5 seconds

b the acceleration after 3 seconds

c when the particle comes to rest again

d the displacement when $t = 4$

QUESTION 2 The acceleration a m s⁻² of a moving particle at time t seconds is given by $a = 1 - \frac{8}{(t+1)^3}$. Initially it is at a position 3 metres to the left of the origin travelling to the right with a speed of 16 m s⁻¹. Find an expression for:

a the velocity

b the displacement

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Velocity and acceleration (3)

QUESTION 1 The acceleration of a moving particle, $a \text{ m s}^{-2}$, is given by $a = -6$. Initially the particle is at $x = 10 \text{ m}$ with velocity $v = 13 \text{ m s}^{-1}$.

a Find, as functions of t :

i the velocity

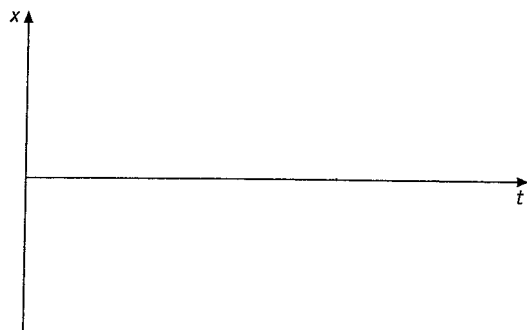
ii the displacement

b When is the particle

i at rest

ii at the origin

c Sketch x as a function of t



d Briefly describe the motion

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Velocity and acceleration (4)



QUESTION 1 A particle moves along a straight line so that its displacement, x metres, from a fixed point, O , is given by $x = 2 \sin \pi t + 1$ where t is the time in seconds ($t \geq 0$).

a What is the position of the particle initially?

_____	_____
_____	_____
_____	_____
_____	_____

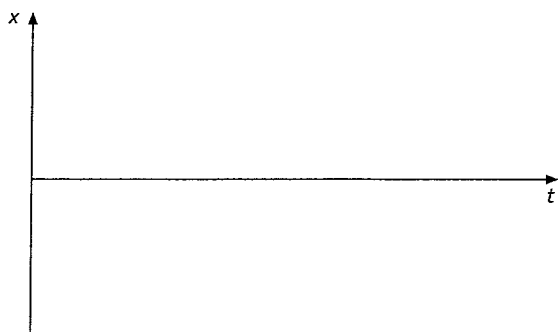
b When and where does the particle first come to rest?

_____	_____
_____	_____
_____	_____
_____	_____
_____	_____
_____	_____
_____	_____
_____	_____

c Find the acceleration when the particle is next at rest.

_____	_____
_____	_____
_____	_____
_____	_____
_____	_____
_____	_____
_____	_____
_____	_____

d Sketch a graph of displacement as a function of time.



ANSWERS

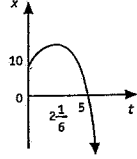
Page 130 1 a displacement b zero 2 a the particle is moving to the left b the velocity gives the direction as well as the magnitude; speed = | velocity | 3 a -12 m b at 2 seconds and at 9 seconds c $v = 2t - 11$ d 1 m s^{-1} 4 a -11 m s^{-1} b 2.25 s c $x = 9t - 2t^2 + 7$

Page 131 1 velocity 2 a slowing down b slowing down c speeding up d speeding up 3 a $a = 160 - 6t$ b 148 m s^{-2} 4 a $v = -2t + 6$ b -4 m s^{-1} c $x = -t^2 + 6t - 2$ d 3 m

Page 132 1 a 0 s and 12 s b 432 m 2 a $x = 9t^2 - \frac{1}{3}t^3$ b -18 m s^{-2} , 972 m

Page 133 1 a 45 m s^{-1} b 6 m s^{-2} c 8 s d 128 m 2 a $v = t + \frac{4}{(t+1)^2} + 12$ b $x = \frac{t^2}{2} - \frac{4}{t+1} + 12t + 1$

Page 134 1 a i $v = -6t + 13$ ii $x = -3t^2 + 13t + 10$ b i $2\frac{1}{6} \text{ s}$ ii 5 s

c  d The particle is initially at a position 10 m to the right of the origin travelling right at a speed of 13 m s^{-1} . It stops after $2\frac{1}{6}$ seconds, then moves left passing through the origin after 5 seconds, and continues to travel left at increasing speed.

Page 135 1 a 1 m to the right of the origin b $\frac{1}{2} \text{ s}$, 3 m c $2\pi^2 \text{ m s}^{-2}$ d (see right)

Pages 136-140 1 A 2 D 3 B 4 B 5 B 6 A 7 D 8 B 9 B 10 A 11 a $-5 - 6t$ b -6 12 a $-8 \sin 4t$ b $-32 \cos 4t$ 13 a $3t^2 - 6t + 7e^t$ b $6t - 6 + 7e^t$

14 $x = \frac{8t^3}{3} - 3t^2 + 3t - 12\frac{1}{3}$ 15 $x = -2t^2 + 9t + 1$ 16 a 55 b 2 or 8 c 8 d 5 17 a 5 b 45 18 a 0.2 L/min b $V = \frac{e^t - 1}{5}$

19 a 10 000 b 0.1386 c 278 576 [nearest whole number] d 34th hour 20 a $\frac{1}{3} \text{ m s}^{-1}$ b $-\frac{1}{9} \text{ m s}^{-2}$ 21 a 4 s b 4 s c The particle

is stationary when $t = 4$, and because $\ddot{x} < 0$, the maximum displacement occurs when $t = 4$. So the maximum displacement is 0 m and the particle never moves right of the origin.

