

CHAPTER 5

Applications of calculus to the physical world



Derivatives with respect to time (1)

QUESTION 1 Find $\frac{dy}{dt}$ if:

a $y = 3t^2 - 8t + 4$

b $y = 9 - 5t^7$

c $y = (4t - 7)^5$

QUESTION 2 Find $f'(t)$ if:

a $f(t) = 6t^5 - 7t^3 + 2t$

b $f(t) = t^4 + t^3 - t^2 - t + 1$

c $f(t) = \sin(2t + 5)$

QUESTION 3 Find $\frac{dx}{dt}$ if:

a $x = 3t^6 + t^3 - 7t - 1$

b $x = 4e^{2t-1}$

c $x = (3t + 1)^2$

QUESTION 4 Find \dot{x} if:

a $x = 7$

b $x = 3 - 2t - 5t^2$

c $x = \ln t$

QUESTION 5 Find \ddot{x} if:

a $x = t^3 - 10t^2$

b $x = 8t$

c $x = 3 \cos \pi t$

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Derivatives with respect to time (2)

QUESTION 1 Find x if:

a $\dot{x} = 4t + 3$ and when $t = 1$, $x = 5$

b $\dot{x} = 8t^2 - 12t + 7$ and when $t = -1$, $x = 3$

QUESTION 2 Find x if:

a $\ddot{x} = 8 - t$ and when $t = 0$, $\dot{x} = 4$ and $x = 2$

b $\ddot{x} = 7$ and when $t = 2$, $\dot{x} = 2$ and $x = 58$

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Derivatives with respect to time (3)

QUESTION 1 If $Q = 20 + 8t - t^2$ ($t \geq 0$) find:

a Q when $t = 5$

b t when $Q = 0$

c $\frac{dQ}{dt}$ when $t = 7$

d t when $\frac{dQ}{dt} = 0$

QUESTION 2 $\frac{dV}{dt} = 10t^2 - t^3$

a Find $\frac{dV}{dt}$ when $t = 4$

b If $V = 300$ when $t = 0$ find V when $t = 4$

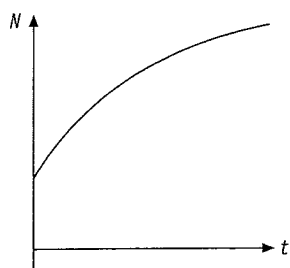
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Rates of change (1)

QUESTION 1 Fill in the correct inequality signs (< or >) for each diagram:

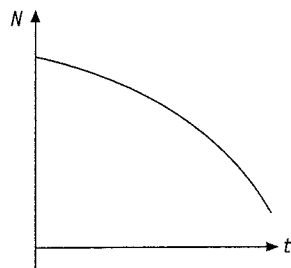
a



$$\frac{dN}{dt} \text{ _____ } 0$$

$$\frac{d^2N}{dt^2} \text{ _____ } 0$$

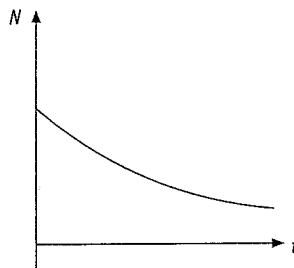
b



$$\frac{dN}{dt} \text{ _____ } 0$$

$$\frac{d^2N}{dt^2} \text{ _____ } 0$$

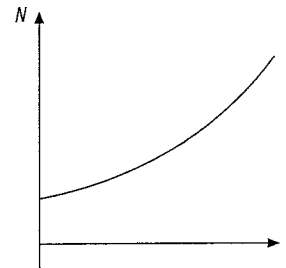
c



$$\frac{dN}{dt} \text{ _____ } 0$$

$$\frac{d^2N}{dt^2} \text{ _____ } 0$$

d



$$\frac{dN}{dt} \text{ _____ } 0$$

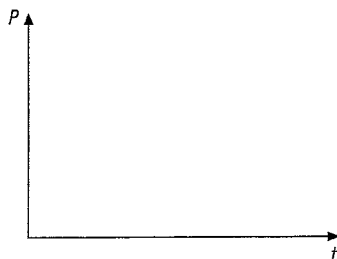
$$\frac{d^2N}{dt^2} \text{ _____ } 0$$

QUESTION 2 The number of registered pets in a town, P , was studied over a period of time. At the beginning of this period there were 25 000 registered pets.

a Throughout the period $\frac{dP}{dt} > 0$. What does this say about the number of registered pets during the period?

b At the same time $\frac{d^2P}{dt^2} < 0$. What does this say about the number of pet registrations?

c Draw a possible sketch of P against t



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Rates of change (2)

QUESTION 1 The rate of change of volume V is given by $R = \frac{dV}{dt} = 1600t - t^3$ ($t \geq 0$). Find:

a R when $t = 0$

b t when $R = 0$

c an expression for V if $V = 1000$ when $t = 0$

d V when $t = 5$

QUESTION 2 Water is flowing through a filter at a variable rate given by $\frac{dV}{dt} = 90 - 5t$, where V is the volume in litres at time t minutes.

a At what rate is the water flowing after 10 minutes?

b When will the water cease flowing?

c How much water flows through the filter in this time?

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Rates of change (3)

QUESTION 1 In a factory, flour is poured into a biscuit mixture. The rate, in grams per second, at which the flour pours into the mixture is given by $R = 900t - t^3$ where t is the time in seconds.

a What is the rate at which the flour is being poured when $t = 5$?

_____	_____
_____	_____
_____	_____

b After how many seconds is there no longer any flour being poured?

_____	_____
_____	_____
_____	_____
_____	_____

c How much flour is poured during the first 20 seconds?

_____	_____
_____	_____
_____	_____
_____	_____
_____	_____

d What is the maximum rate at which the flour pours into the mixture?

_____	_____
_____	_____
_____	_____
_____	_____
_____	_____

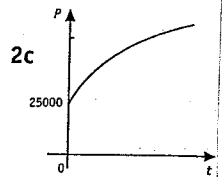
ANSWERS

Page 120 1 a $6t - 8$ b $-35t^6$ c $20(4t - 7)^4$ 2 a $30t^4 - 21t^2 + 2$ b $4t^3 + 3t^2 - 2t - 1$ c $2 \cos(2t + 5)$ 3 a $18t^5 + 3t^2 - 7$
 b $8e^{2t-1}$ c $6(3t + 1)$ 4 a 0 b $-2 - 10t$ c $\frac{1}{t}$ 5 a $6t - 20$ b 0 c $-3\pi^2 \cos \pi t$

Page 121 1 a $2t^2 + 3t$ b $\frac{8t^3}{3} - 6t^2 + 7t + 18\frac{2}{3}$ 2 a $4t^2 - \frac{t^3}{6} + 4t + 2$ b $\frac{7t^2}{2} - 12t + 68$

Page 122 1 a 35 b 10 c -6 d 4 2 a 96 b $449\frac{1}{3}$

Page 123 1 a $>$, $<$ b $<$, $<$ c $<$, $>$ d $>$, $>$ 2 a The number of registered pets is increasing over the period. b The number of pet registrations is increasing at a decreasing rate. c (see right)



Page 124 1 a 0 b 0 or 40 c $V = 800t^2 - \frac{t^4}{4} + 1000$ d 20 843.75 2 a 40 L/min b after 18 minutes c 810 litres

Page 125 1 a 4375 grams per second b 30 seconds c 140 kg d $6000\sqrt{3}$ g/s

Page 126 1 a 9113 b 20 c 442 943 2 a 51.8 b 7.7 c -3.0

Page 127 2 a 400 b 0.0260 c 264 [nearest whole number] d -6.9 [1 d.p.]

Page 128 1 a 59 g [nearest g] b 1.2 grams per year [1 d.p.] 2 b In the 10th hour

Page 129 1 a 0.03466 b 1345 2 a 0.0080 [4 d.p.] b 174 hours