

Logarithmic and exponential functions

The derivative of $y = \ln x$ (1)

QUESTION 1 Differentiate:

a $y = \log_e x$

b $y = \log_e 2x$

c $f(x) = \ln 6x$

d $y = \log_e(7x + 5)$

e $y = \ln(1 - 2x)$

f $y = \ln(5x + 3)$

g $y = \ln x^2$

h $f(x) = \ln x^5$

i $y = \ln x^9$

j $f(x) = \ln(x^2 + 5)$

k $y = \ln(3x^2 - 4)$

l $y = \log_e(x^3 - 7x^2)$

QUESTION 2 Find the derivative of:

a $y = (\ln x)^2$

b $f(x) = \log_e(3x - 1)^2$

Logarithmic and exponential functions

The derivative of $y = \ln x$ (2)



QUESTION 1 Find the exact value of $f'(e)$ if:

a $f(x) = \ln x$

b $f(x) = \log_e (2x - 1)$

c $f(x) = 3 \ln (x^2 + 1)$

QUESTION 2 Differentiate:

a $y = x \ln x$

b $y = x^4 \log_e x$

c $\frac{\ln x}{x^2}$

d $\frac{x+1}{\ln x}$

Answers

Page 98 1 $\log_a y$ 2 a 1.230 b 2.312 c -0.456 d 0.217 3 a 2.37 b 9.48 c -4.74 4 a 0.96 b 0.867 c 0.382 d 1.053

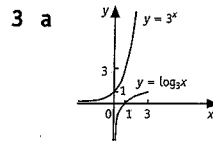
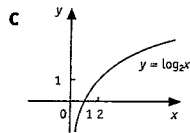
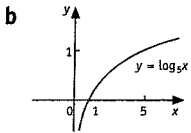
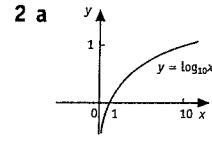
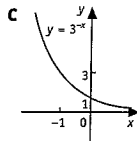
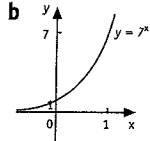
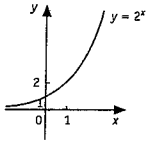
5 a $x = \frac{1}{3}$ b $x = 56$

Page 99 1 $\log_b a$ 2 a $1\frac{1}{2}$ b $\frac{2}{3}$ c $2\frac{1}{2}$ d $2\frac{2}{3}$ e $2 \log_2 3$ 3 a 2.0959 b 1.3917 c 3.1699 d 1.7297 e 0.6275 f 2.5052

g -0.4650 h -0.4150 i -1.1495

Page 100 1 a $\frac{1}{2} \log_2 11$ b $\frac{1}{2} \log_5 6$ c $\frac{1}{3} \log_3 32$ 2 a 2.579 b 6.229 c -0.802 d 1.738 e 2.425 f 1.395

Page 101 1 a



b $y = x$

Page 102 1 $10^{x+h} - 10^x$, $10^h - 1$, $\frac{10^h - 1}{h}$ 2 a 2.30 b 0.69 c 1.10 3 a 2.30 b 0.69 c 1.10 4 a 2.72 5 1

6 a $(\ln 5)5^x$ b $(\ln 7)7^x$ c $(\ln 4)4^x$ d $(\ln 11)11^x$ e $(\ln 6)6^x$ f $(\ln 9)9^x$ g $(\ln 8)8^x$ h $(\ln 15)15^x$

Page 103 1 a 1 b 0 c 1 d 2 e 2 f 7 g 7 h 5 2 a 7.3891 b 54.5982 c 296.8263 d 0.3679 e 0.2231 f 2.0541

g 1.2809 h -1.4397 i 8.3178 j 81.3421 k 2.7183 l 3.3944 3 a 0.470 b 6.686 c 0.671 d -0.077 e 0.128 f -1.363

Page 104 1 a e^x b $3e^x$ c $2e^{2x}$ d $4e^x$ e $10e^{5x}$ f $-e^{-x}$ g $1 - e^x$ h $12e^{2x+5}$ i $-32e^{-8x}$ j $18x^2 - 9e^{3x}$ k $e^x + e^{-x}$ l $63e^{-9x}$

2 a $e^x(x+1)$ b $2xe^{2x}(x+1)$ c $e^{-x}(7-3x)$ d $5e^{7x}(7x^2 - 61x + 5)$

Page 105 1 a $4e^x(e^x + 5)^3$ b $3(4x - e^x)^2(4 - e^x)$ 2 a $\frac{1-x}{e^x}$ b $\frac{1-x}{e^x}$ 3 a $\frac{xe^x}{(x+1)^2}$ b $\frac{3e^x(x^2 - 2x - 5)}{(x^2 - 5)^2}$

Page 106 1 a $e^x + C$ b $5e^x + C$ c $\frac{1}{3}e^{3x} + C$ d $\frac{1}{2}e^{2x+3} + C$ e $-4e^{-x} + C$ f $-\frac{1}{2}e^{3-2x} + C$ g $e^x + x^2 + C$ h $\frac{e^{4x}}{8} + C$

i $\frac{x^3}{3} - 4x^2 + 3e^{-2x} + C$ 2 a $e^2 - 1$ b $6(e-1)$ c $\frac{1}{4}(e^{12} - 1)$ d $\frac{e}{2}(e^4 - 1)$ e $\frac{1}{4}(1 - e^{-4})$ f $e(e^2 - 1)$ g $e^2 + e^{-2} - e - e^{-1}$

17 i $\frac{1}{3}e^6 - \frac{1}{3}e^3 + 1\frac{1}{2}$

Page 107 1 a $\frac{1}{x}$ b $\frac{1}{x}$ c $\frac{1}{x}$ d $\frac{7}{7x+5}$ e $\frac{-2}{1-2x}$ f $\frac{5}{5x+3}$ g $\frac{2}{x}$ h $\frac{5}{x}$ i $\frac{9}{x}$ j $\frac{2x}{x^2+5}$ k $\frac{6x}{3x^2-4}$ l $\frac{3x-14}{x^2-7x}$

2 a $\frac{2 \ln x}{x}$ b $\frac{6}{3x-1}$

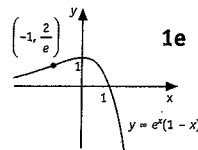
Page 108 1 a $\frac{1}{e}$ b $\frac{2}{2e-1}$ c $\frac{6e}{e^2+1}$ 2 a $1 + \ln x$ b $x^3(1 + 4 \log_e x)$ c $\frac{1-2 \ln x}{x^3}$ d $\frac{x \ln x - x - 1}{x(\ln x)^2}$

Page 109 1 a $\ln x + C$ b $6 \ln x + C$ c $3 \ln(x+2) + C$ d $\ln(x^2+5) + C$ e $\ln(x^3-2) + C$ f $\ln(3x-7) + C$

g $4 \ln(x^2-3) + C$ h $\frac{1}{4} \ln(4x-1) + C$ i $-\frac{7}{2} \ln(1-2x) + C$ 2 a $\ln 4$ b $\frac{1}{2}$ c $\ln 4$ d $\ln 6.8$

Page 110 1 $2x - ey = 0$ 2 $e^{2x} - 2ey - e^2 + 4 = 0$ 3 (0, 1) 4 $\frac{1}{e}$

Page 111 1 a (1, 0) b maximum at (0, 1) c $(-1, \frac{2}{e})$ d i $-\infty$ ii 0 e



Page 112 1 a 2 units² b $(3 \ln 3 - 2)$ units² 2 $y = e^x + e^{-x} + 2$

Page 113 1 $4 \ln 2$ units² 2 $y = 3x^2 - \ln(2x-1) + 4$ 3 $2 \ln 2$ units²

Pages 114-119 1 A 2 D 3 D 4 C 5 A 6 B 7 A 8 B 9 D 10 C 11 C 12 A 13 a 2^{10x-1} b 2 14 a 6 b 0.2 c 2 15 a 0.462 1.563 c 3.459 16 a 0.921 b 1.13 c 0.209 17 a 3.759 b -0.768 c 1.233 18 a 9 b 4 c 8 19 a b (see next page) 20 a $(\ln 7)^{7^x}$