



Applications of arithmetic series (1)

QUESTION 1 There are 36 chairs in the front row of a theatre. There are 38 chairs in the second row, 40 in the third row and so on, two more chairs in every row. How many chairs are there:

a in the 26th row

b in total in 26 rows

QUESTION 2 Sam began work on a salary of \$28 400 p.a. After each year with the company she will receive an increase of \$1200 p.a. What is:

a Sam's salary in the 7th year with the company

b her total earnings for the first 7 years

QUESTION 3 A factory begins operation and makes a loss of \$18 000 in the first month. The manager hopes to reduce this loss by \$500 each month. If she achieves this aim:

a For how many months will the company make a loss?

b What will be the total loss over that time?

c If in fact the loss is only reduced by \$400 each month what will be the total loss?

Series and applications

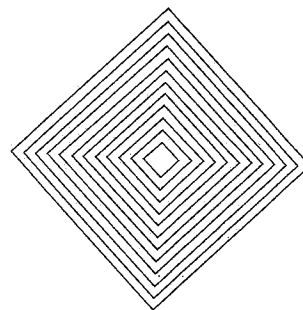
Applications of arithmetic series (2)

QUESTION 1 At a sports carnival, novelty races are held on the last afternoon. In one event, competitors must run to pick up a tennis ball, run back to the starting point to drop the ball in a bucket, and then run to get another ball and so on. The first ball is 8 metres from the start, the second ball is 14 m from the start, the third 20 m and so on, each subsequent ball being 6 m further from the start. If there are 12 balls:

a how far is the last ball from the start?

b how far must a competitor run?

QUESTION 2 A company logo consists of a series of squares as shown in the diagram. The design is to be made up in neon tubing to create a large light on top of the company building. The inside (first) square will have a side length of 50 cm, the second square a side of length 70 cm and each subsequent square will have a side 20 cm longer than the previous square.



a What is the perimeter of the first square? (in cm)

b Find an expression for the perimeter of the n^{th} square.

c Find the total length of tubing needed to construct the 12 squares.



Applications of geometric series (1)

QUESTION 1 Tom decides to exercise on a regular basis. He intends to exercise 5 minutes per day for the first week, 10 minutes per day for the second week and in each subsequent week to double the number of minutes. Kate makes a bet with Tom that he will give up the plan in 8 weeks or less. Who do you think will win the bet? Justify your answer.

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QUESTION 2 A tree is 1 metre high when it is planted. It grows 3 metres in the first year and 1 metre in the second year. If each year its growth is $\frac{1}{3}$ of that of the previous year, how tall will the tree grow?

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Series and applications

Applications of geometric series (2)



QUESTION 1 In her will a woman leaves an allocated amount to a charity to be paid over several years. The charity will receive \$10 000 in the first year, \$8000 in the second year and so on, each year receiving 80% of what it did in the previous year.

a How much, to the nearest whole dollar, will the charity receive in the 12th year?

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b What is the total amount the charity will receive?

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c The woman changes the terms of the will so that when the donation falls below \$200 per year the remaining amount allocated will be paid in full.

i When will the donation fall below \$200?

ii How much will be donated in the final payment?

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Answers

Page 141 1 a 9 b 81 c 49 d 27 e 365 f 63 g 9375 h 39 2 a 12 b 123 c 55 d 82 e 189 f 9999 g 120 h 475

3 a $\sum_{k=1}^8 (2k-1)$ or $\sum_{k=0}^7 (2k+1)$ b $\sum_{k=1}^7 2^{k-1}$ or $\sum_{k=0}^6 2^k$

Page 142 1 difference 2 a true b false c true d false e false f true g false h false 3 a 3 + 10 + 17 + 24 + 31 + 38
b 2 + 7 + 12 + 17 + 22 + 27 c -8 - 4 + 0 + 4 + 8 + 12 d 6 + 4 + 2 + 0 - 2 - 4 e -5 - 8 - 11 - 14 - 17 - 20 f -23 - 6 + 11 +
28 + 45 + 62 4 a 5 b 8 c 28 d -3 e -4 f 7 5 a 483 + 540 + 597 b 31 + 14 - 3 c 5 + 18 + 31 d -91 - 155 - 219

Page 143 1 a 31 b 50 c 34 d -26 e -4 f -41 2 a 213 b -79 3 a 9, 2 b 5, 7 c 11, -3 4 a $3n-1$ b $2n+5$ c $6-2n$

Page 144 1 $8n-4$ 2 27 3 5 4 a 111 b 13th term c not a term d $T_{90} = 1002$

Page 145 1 a 244 b 86 c 60 d 140 e -632 f 13 540 2 a 420 b -1484 3 a 250 b 41 572

Page 146 1 a $9n^2 + 25n$ b $13n - 6n^2$ 2 a $\frac{n}{2}(41n + 71)$ b $\frac{n}{2}(177 - 7n)$ 3 a 43 b 19 995

Page 147 1 a 3817 b 6222 2 a 184 b 147 c 37 3 $8n-1$

Page 148 1 multiplying, ratio 2 a false b true c false d false e true f true g false h true 3 a 2 + 6 + 18 + 54
b 3 - 6 + 12 - 24 c 3072 + 1536 + 768 + 384 d 1875 + 2625 + 3675 + 5145 e 448 + 224 + 112 + 56 f -2 + 10 - 50 + 250

g 288 - 1008 + 3528 - 12 348 h 50 421 + 14 406 + 4116 + 1176 i 1000 + 300 + 90 + 27 4 a 6 b 1.5 c 0.7 d $\frac{3}{4}$ e -2 f $-\frac{1}{3}$

5 a 160 + 320 + 640 b -6250 + 31 250 - 156 250 c 0.1 + 0.02 + 0.004 d 20 - 5 + 1.25

Page 149 1 a 320 b 6561 c -131 072 d 1 114 112 e 819.2 f 0.0512 2 a 2734.375 b 797 161.5 3 a $7(2)^{n-1}$
b $6(-3)^{n-1}$ c $1.25(2)^{n-1}$ 4 a 2 b 6 c 3 5 a 0.9 b 5832 c $8000(0.9)^{n-1}$

Page 150 1 a $1\frac{1}{3}$ b 43 690 $\frac{2}{3}$ 2 9th term 3 $T_{10} = 0.625$ 4 797 161.5

Page 151 1 a 7161 b 161 425 205 c 312.4992 d 6 725 601 e 13.75 f 8333.333 2992

2 a 49 149 b 32.171 875 c 10 235 d -1 743 392 200

Page 152 1 a $\frac{5^n - 1}{2}$ b $4(1 - 0.25^n)$ c $\frac{8(1 - (-2)^n)}{3}$ 2 a 2 b 2046 c 1024 3 960 799 4 6

Page 153 1 $-1 < r < 1$ 2 a true b false c true d false e true f true 3 a 2500 b 562.5 c $33\frac{1}{3}$ d 4374 e 78 125

f 2340 $\frac{4}{7}$ g 2222 $\frac{2}{9}$ h -4266 $\frac{2}{3}$ i 2

Page 154 1 a 1536 b 58 824.5 c 9722 $\frac{2}{9}$ d 27 993.6 2 3000 3 15

Page 155 1 a 86 b 1586 2 a \$35 600 b \$224 000 3 a 36 months b \$ 333 000 c \$414 000

Page 156 1 a 74 m b 984 m 2 a 200 cm b $(80n + 120)$ cm c 76.8 m

Page 157 1 Kate. Tom would need to exercise for 10 hours and 40 minutes per day in the 8th week. 2 5.5 m

Page 158 1 a \$859 b \$50 000 c i 19th year ii \$901

Page 159 1 a \$3787 b \$10 285 c \$49 195 d \$2511 e \$275 903 f \$533 875 2 a \$3317.12 2 \$3634.45

Page 160 1 \$14 307 2 \$7609.45 3 \$532 4 \$4989.15 [nearest cent] 5 a \$2690 b \$3219

Page 161 1 11 years 2 9 years 3 a \$3292.65 b 10.3% [1 d.p.]

Page 162 1 a \$508 810 b \$175 383 2 a $\$4000(1.05)$ b $\$4000(1.05)^2$ c $\$4000(1.05)^3$ d $\$4000(1.05)^{10}$
e $\$4000(1.05 + 1.05^2 + 1.05^3 + \dots + 1.05^{10})$

Page 163 1 a i \$1000 ii $\$1000(1.04)$ iii $\$1000(1.04)^2$ iv $\$1000(1.04)^{18}$ v $\$1000(1 + 1.04 + 1.04^2 + 1.04^3 + \dots + 1.04^{18})$

b \$27 671 2 a i $\$2500(1.075)$ ii $\$2500(1.075)^2$ iii $\$2500(1.075)^{15}$ iv $\$2500(1.075 + 1.075^2 + 1.075^3 + \dots + 1.075^{15})$

b \$70 193

Page 164 1 \$194 964 2 \$6100

Page 165 1 a 8036.97 b 1074.65 2 a $A_{60} = 0$. The loan will be repaid after 5 years (60 months), so the amount owing at

that time will be zero. b \$409.37 3 $\frac{1.004^n - 1}{0.004}$

Page 166 1 a $\$120\ 000(1.005) - M$ d $\$[120\ 000(1.005)^n - M(1 + 1.005 + 1.005^2 + \dots + 1.005^{n-1})]$ f 1171

Page 167 1 a $\$40\ 000(1.065) - Y$ c 5564

Page 168 1 \$646.13

Page 169 1 a $0.8 + 0.08 + 0.008 + 0.0008 + \dots$ b $0.35 + 0.0035 + 0.000\ 035 + \dots$ 2 a $0.2 + 0.02 + 0.002 + 0.0002 + \dots$, $\frac{2}{9}$

b $0.42 + 0.0042 + 0.000\ 042 + \dots$, $\frac{14}{33}$ c $0.105 + 0.000\ 105 + 0.000\ 000\ 105 + \dots$, $\frac{35}{333}$ d $0.2 + 0.07 + 0.007 + 0.0007 + \dots$, $\frac{5}{18}$

e $1 + 0.87 + 0.0087 + 0.000\ 087 + \dots$, $1\frac{29}{33}$ f $0.8235 + 0.000\ 082\ 35 + 0.000\ 000\ 082\ 35 + \dots$, $\frac{915}{1111}$