Geometrical applications of differentiation

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Problems involving maxima and minima (1)

QUESTION **1** The height (in metres), of a ball thrown straight up from the ground is given by $h = 27t - 5t^2$ where t is the time in seconds. Find:

a the time when the height is a maximum

b the maximum height

QUESTION **2** The perimeter, in metres, of any rectangle with area 100 m^2 is given by $P = 2x + \frac{200}{x}$ where x is the length of one of the sides. Find:

a the value of x for which P will be a minimum

b the minimum perimeter

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Problems involving maxima and minima (2)

QUESTION **1** The product P of any two numbers whose sum is 34 is given by $P = 34a - a^2$ where a is one of the numbers. Find:

a the value of a for which P is a maximum

b the maximum product

QUESTION **2** When n items are produced at a certain factory, the cost per item ($\$C$) is given by $C = \frac{8788}{n} + 211 + 2n^2$. Find:

a the value of n for which C will be a minimum

b the minimum cost per item

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Problems involving maxima and minima (3)

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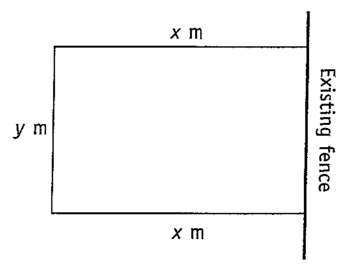
QUESTION 1 The volume of a closed cylindrical aluminium can is 128π cm^3.

a Find an expression for the height (h) in terms of the radius (r).

b Find an expression for the surface area (A) in terms of r.

c Find the least amount of aluminium needed for a can of this size. (Give the answer in terms of π)

QUESTION 2 Ty wants to establish a rectangular vegetable garden. One side will be against an existing fence and he will need to fence the other 3 sides. He has enough materials to fence an additional 10 m. If the length of the garden is x m and the width y m as shown in the diagram:



a show that the area is given by $A = 10x - 2x^2$

b find the value of x if the garden is to have maximum area

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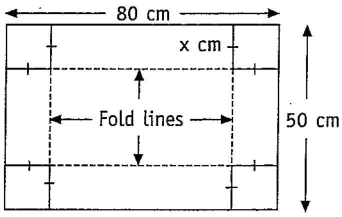
roblems involving maxima and minima (4)

UESTION **1** A piece of cardboard 80 cm long and 50 cm wide will be used to make an open box. A square of side x cm will be cut from each corner and the sides then folded up to form the box.



What condition must be placed on x in order for the box to exist? Justify your answer.

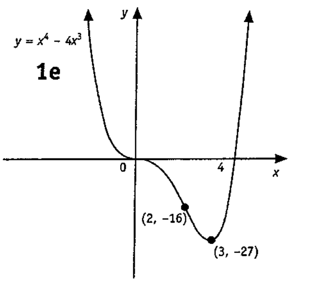
Show that the volume of the box is given by $V = 4000x - 260x^2 + 4x^3$

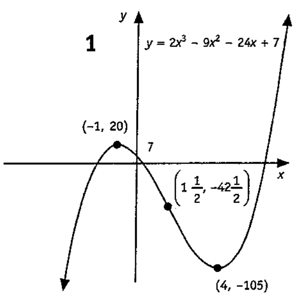
Find the value of x for which the box will have maximum volume.

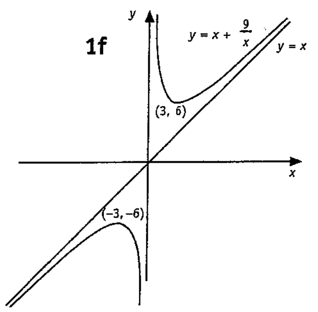
Find the maximum volume of the box.

Answers

Page 1 1 a negative b positive c negative 2 a increasing b decreasing c increasing 3 a all real values of x b $x > -4$ c all real values of x except $x = 0$
Page 2 1 a $x < 3$ b $-2 < x < 2$ c no values of x 2 a positive b negative c negative d positive e zero f negative
Page 3 1 a x-axis b 0 c maximum, minimum 2 a true b false 3 a absolute maximum b local maximum c local minimum d absolute minimum 4 a $x = -4$ b $x = 0$ c $x = -1$ or $x = 3$ 5 a true b false c false
Page 4 1 a $(-3, -13)$ b $(0, 2)$ c $(-3, 38)$ 2 a $(-1, 14)$ and $(2, -13)$ b $(0, 0)$ and $\left(\frac{2}{3}, \frac{4}{27}\right)$
Page 5 1 a 0 b minimum c maximum d horizontal point of inflexion 2 a maximum b horizontal point of inflexion c minimum 3 a maximum (at $x = -1$) b horizontal point of inflexion (at $x = -2$) c maximum (at $x = 0$)
Page 6 1 a maximum at $(0, 8)$, minimum at $(5, -117)$ b maximum at $\left(-4, 28\frac{2}{3}\right)$, minimum at $\left(2, -7\frac{1}{3}\right)$
Page 7 1 a 14 b $6x + 4$ c $42x^5 - 72x^7$ d $150x^4 + 48x^2$ e 0 f -2 g $180(3x - 2)^3$ h $12x^{-5}$ i $-\frac{1}{4}x^{-\frac{3}{2}}$ 2 a 16 b $12x^2 - 18x$ c $336x^6$ d $4x^{-3}$ e $72(2 - x)^7$ f $20x^{-6} - 56x^{-9}$
Page 8 1 a $80x^3 - 126x^5$ b $12 - 18x^{-4}$ c $896(4x + 1)^6$ 2 a 72 b -32 c -10 3 a 14 b $\frac{1}{4}$ 4 a $(3x - 5)^5(21x - 5)$ b $18(3x - 5)^4(21x - 10)$
Page 9 1 a up b down 2 a concave up b concave down c concave up 3 a concave down b concave up c concave up 4 a $x < -1\frac{2}{3}$ b $x > -\frac{1}{2}$
Page 10 1 a minimum b maximum 2 a minimum when $x = 2$ b maximum when $x = 4$ c maximum when $x = 1$, minimum when $x = 3$ d minimum when $x = -2$, maximum when $x = 0$, minimum when $x = 2$
Page 11 1 a maximum, minimum, horizontal point of inflexion b test the sign of either the first or second derivative either side of the point 2 a horizontal point of inflexion at $(0, -7)$ b minimum at $(0, 2)$ c minimum at $(0, -1)$, horizontal point of inflexion at $\left(2, \frac{1}{3}\right)$
Page 12 1 a concavity b $\frac{d^2y}{dx^2}$, $\frac{d^2y}{dx^2}$ has a different sign either side of the point c $\frac{dy}{dx}$ 2 a $(-2, 30)$ b $(1, -9)$
Page 13 1 a $(-2, -91)$ and $(2, -59)$ b no points of inflexion
Page 14 1 a true b false 2 a 76 b 100 c minimum at $(1, 4)$ d $4 \le y \le 100$ 3 greatest value is 18 (when $x = -1$) and least value -9 (when $x = 2$)
Page 15 1 a $(0, 0)$ and $(4, 0)$ b horizontal point of inflexion at $(0, 0)$, minimum at $(3, -27)$ c $(2, -16)$ d i ∞ ii ∞ e (see left)
Page 16 1 (see centre)
Page 17 1 a $x \neq 0$ b minimum at $(3, 6)$, maximum at $(-3, -6)$ d as $x \to 0^+$, $y \to \infty$, as $x \to 0^-$, $y \to -\infty$ e as x gets large, $\frac{9}{x} \to 0$ f (see right)







Page 18 1 a $y = -2x$ b $y = -2x + 1$ c $4x - 3y + 25 = 0$ d $x + 4y - 4 = 0$
Page 19 1 $(3, 2)$ 2 a $y = x - 3$ b $(0, -3)$ 3 a $y = 2ax - a^2$ b $a = \pm 3$
Page 20 1 a $x - 6y + 19 = 0$ b $x + 8y - 2 = 0$ c $27x - 6y - 52 = 0$ d $4x + y - 18 = 0$
Page 21 1 $(3, 15)$ 2 $Q\left(-18, 40\frac{1}{2}\right)$
Page 22 1 a 2.7 seconds b 36.45 m 2 a $x = 10$ b 40 m
Page 23 1 a $a = 17$ b 289 2 a $n = 13$ b $1225
Page 24 1 a $h = \frac{128}{r^2}$ b $A = 2\pi r^2 + \frac{256\pi}{r}$ c 96π cm^2 2 b $x = 2.5$
Page 25 1 a $x < 25$, x cannot be longer than half the shortest side c $x = 10$ d 18 000 cm^3