Series and applications

TOPIC TEST

Time allowed: 1 hour **Total marks = 100**

SECTION I Multiple-choice questions

10 marks

Instructions
- This section consists of 10 multiple-choice questions
- Each question is worth 1 mark
- Fill in only ONE CIRCLE
- Calculators may be used

1 The series $72 + 108 + 162 + 243 + 364.5 + \ldots$ is?

Ⓐ arithmetic Ⓑ geometric

Ⓒ both arithmetic and geometric Ⓓ neither arithmetic nor geometric

2 The common difference of the series $68 + 85 + 102 + 119 + 136 + \ldots$ is?

Ⓐ 1.25 Ⓑ 12.5 Ⓒ 13 Ⓓ 17

3 The common ratio of the series $46.875 + 75 + 120 + 192 + 307.2 + \ldots$ is?

Ⓐ 0.625 Ⓑ 1.35 Ⓒ 1.6 Ⓓ 28.125

4 Which series will have a limiting sum?

Ⓐ $8 + 8.5 + 9 + 9.5 + 10 + 10.5 + \ldots$

Ⓑ $8 + 8 + 8 + 8 + 8 + \ldots$

Ⓒ $6000 + 6600 + 7260 + 7986 + 8784.6 + \ldots$

Ⓓ $700 + 630 + 567 + 510.3 + 459.27 + \ldots$

5 The fortieth term of the series for which $T_n = 5n - 9$ is?

Ⓐ 25 Ⓑ 61 Ⓒ 155 Ⓓ 191

6 The simple interest on \$4000 at 6% p.a. for 3 years is?

Ⓐ \$240 Ⓑ \$476 Ⓒ \$720 Ⓓ \$764

7 Which formula is incorrect?

Ⓐ $S_n = \frac{n}{2}[2a + (n-1)d]$ Ⓑ $S_n = \frac{a(r^{n-1})}{r-1}$

Ⓒ $S_n = \frac{n}{2}(a + l)$ Ⓓ $S = \frac{a}{1-r}$

8 The amount to which \$1000 accumulates if invested for 2 years at 10% compound interest paid annually is?

Ⓐ \$1020.10 Ⓑ \$1100 Ⓒ \$1200 Ⓓ \$1210

9 The limiting sum of the series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots$ is?

Ⓐ $\frac{1}{2}$ Ⓑ 1 Ⓒ $1\frac{1}{2}$ Ⓓ 2

10 The number of terms in the series $1 + 1.0825 + 1.0825^2 + \ldots + 1.0825^{27} + 1.0825^{28}$ is?

Ⓐ 5 Ⓑ 27 Ⓒ 28 Ⓓ 29

SECTION II

90 marks

Show all necessary working

11 Fill in the missing term in each series: **1 mark each**

a $7 + 10 + \_\_\_\_\_ + 16 + 19 + \ldots$

b $768 + 384 + 192 + \_\_\_\_\_ + 48 + 24 + \ldots$

12 Find the value of: **2 marks each**

a $\sum_{k=1}^{5} (3k + 1)$

b $\sum_{n=1}^{4} 500(0.3)^{n-1}$

13 Determine whether the series is arithmetic, geometric, neither or both: **1 mark each**

a $32 + 60 + 88 + 116 + 144 + \ldots$

b $15 + 7.5 + 3.5 + 1.25 + \ldots$

c $8 + 8 + 8 + 8 + 8 + \ldots$

d $56 - 196 + 686 - 2401 + 8403.5 - \ldots$

14 Find the seventh term of the arithmetic series with first term 8 and common difference 11. **3 marks**

15 Find an expression, in simplest form, for the n^{th} term of the arithmetic series with first term 4 and common difference 7. **3 marks**

16 The fourth term of an arithmetic series is 31 and the ninth term is 61. Find the twelfth term. **5 marks**

17 Find the sum of the first eight terms of the arithmetic series with first term 3 and common difference 10. **3 marks**

18 Find the sum of the first fifteen terms of the arithmetic series with first term 2 and fifteenth term 100. **3 marks**

19 Find an expression for the sum to n terms of an arithmetic series with first term 6 and common difference 8. **3 marks**

20 Find the seventh term of the geometric series with first term 4 and common ratio 3. **3 marks**

21 The first term of a geometric series is 400 and the second term is 260. Find:

a the common ratio **2 marks**

b the fourth term **3 marks**

22 Find the sum to 6 terms of the geometric series with first term 78 125 and common ratio 1.2 **3 marks**

23 Find the limiting sum of the series 500 + 200 + 80 + 32 + ... **3 marks**

24 Jo begins work on a salary of $23 000 p.a. Each year her salary will increase by $1250.

a What is Jo's salary in her 12^{th} year with the company? **4 marks**

b How much will Jo have earned in total at the end of 12 years? **4 marks**

25 Charles begins work on a salary of $24 000 p.a. Each year his salary will increase by 5%. (Give answers to the nearest dollar.)

a What is his salary in his 12^{th} year? **4 marks**

b How much will Charles have earned in total at the end of 12 years? **4 marks**

26 To how much, to the nearest cent, will $12 500 accumulate if it is invested for 4 years at 6% p.a. interest compounded annually? **3 marks**

27 Find the compound interest, to the nearest cent, earned if $8000 is invested for 5 years, interest compounded quarterly at 1.5% per quarter. **4 marks**

28 Express $0.7\dot{2}$ as an infinite geometric series and hence as a fraction in simplest form. **4 marks**

29 Sally invested $3000 into an account where it will earn 5% p.a. interest compounded annually. She intends to add $3000 to the account on the same day each year for another 10 years and then on the same day in the following year withdraw the whole amount.

a Write an expression for the value, on the day of withdrawal, of:

i the last amount invested **1 mark**

ii the first amount invested **1 mark**

iii the total of all the investments **1 mark**

b Find the total amount Sally will withdraw. (Give the answer to the nearest dollar.) **5 marks**

30 Nasser borrows $150 000. Compound interest of 0.4% per month is charged monthly and Nasser agrees to repay $\$M$ each month. A_n is the amount owing at the end of n months.

a Find an expression for:

i A_1 **1 mark**

ii A_2 **2 marks**

continued ...

30 continued ...

iii A_3 **2 marks**

iv A_n **1 mark**

b Find $\$M$, to the nearest whole dollar, if the loan is to be taken over 25 years. **5 marks**

Answers

Pages 170-176 **1** B **2** D **3** C **4** D **5** D **6** C **7** B **8** D **9** D **10** D **11** **a** 13 **b** 96 **12** **a** 50 **b** 708.5 **13** **a** arithmetic **b** neither **c** both **d** geometric **14** 74 **15** $7n - 3$ **16** 79 **17** 304 **18** 765 **19** $4n^2 + 2n$ **20** 2916 **21** **a** 0.65 **b** 109.85 **22** 775 775 **23** $833\frac{1}{3}$ **24** **a** \$36 750 **b** \$358 500 **25** **a** \$41 048 **b** \$382 011 **26** \$15 780.96 **27** \$2774.84 **28** $0.7 + 0.02 + 0.002 + 0.0002 + \ldots$, $\frac{13}{18}$ **29** **a** **i** $\$3000(1.05)$ **ii** $\$3000(1.05)^{11}$ **iii** $\$[3000(1.05) + 3000(1.05)^2 + 3000(1.05)^3 + \ldots 3000(1.05)^{11}]$ **b** \$44 751 **30** **a** **i** $\$150\,000(1.004) - M$ **ii** $\$150\,000(1.004)^2 - M(1 + 1.004)$ **iii** $\$150\,000(1.004)^3 - M(1 + 1.004 + 1.004^2)$ **iv** $\$150\,000(1.004)^n - M(1 + 1.004 + 1.004^2 + \ldots + 1.004^{n-1})$ **b** \$859