EXCEL HSC MATHEMATICS
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Derivatives of sin x, cos x and tan x (1)

QUESTION 1 Complete:

As $h \to 0$,

a $\sin h \to$

b $\cos h \to$

c $\tan h \to$

QUESTION 2 Differentiate:

a $y = \sin x$

b $y = \cos x$

c $f(x) = \tan x$

d $y = \sin 3x$

e $y = 4 \cos x$

f $y = \tan \frac{x}{2}$

g $f(x) = 2 \sin 5x$

h $y = \frac{1}{2} \cos 2x$

i $y = \sin \pi x$

j $y = \sin (2x + 3)$

k $f(x) = 5 \tan \left(3x - \frac{\pi}{4}\right)$

l $y = x - \cos x$

m $f(x) = \sin (x^2)$

n $y = x + \tan (x - 1)$

o $y = \cos (2 - 5x)$

The trigonometric functions

Derivatives of sin x, cos x and tan x (3)

QUESTION **1** Find the derivative of:

a $y = x \cos x$

b $f(x) = x^2 \sin x$

c $f(x) = \tan^2 x$

d $y = \cos(x^2)$

e $y = \dfrac{\sin 2x}{x}$

f $y = \dfrac{1 - \cos x}{2x}$

g $y = \dfrac{1}{\cos x}$

h $f(x) = \operatorname{cosec} x$

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Integrals (1)

QUESTION **1** Find:

a $\int \sin x \, dx$

b $\int \cos x \, dx$

c $\int \sec^2 x \, dx$

d $\int 3 \cos x \, dx$

e $\int \sin 2x \, dx$

f $\int \frac{1}{2} \sec^2 4x \, dx$

g $\int -2 \sin \frac{x}{2} \, dx$

h $\int \sec^2 (x + 1) \, dx$

i $\int (1 - \cos x) \, dx$

j $\int \cos(3x - 2) \, dx$

k $\int \sin \pi x \, dx$

l $\int \frac{\sec^2 x}{2} \, dx$

QUESTION **2** Find the exact value of:

a $\int_0^{\pi} \sin x \, dx$

b $\int_0^{\frac{\pi}{2}} \cos x \, dx$

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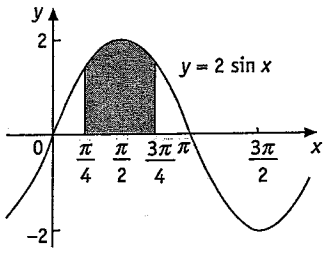
Applications of integration (1)

QUESTION **1** Find the exact area bounded by the curve $y = 2 \sin x$ and the lines $x = \frac{\pi}{4}$, $x = \frac{3\pi}{4}$ and $y = 0$



QUESTION **2**

a Complete the table of values for $f(x) = \sin^2 x$

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$f(x)$				

b Use the trapezoidal rule and these four function values to approximate $\int_0^{\frac{\pi}{2}} \sin^2 x \, dx$. Give the answer in terms of π.

QUESTION **3**

a Differentiate $x \sin x$

b Hence find the exact value of $\int_0^{\frac{\pi}{2}} (x \cos x + \sin x) \, dx$

Answers

Page 81 1 a 0 b 1 c 0 2 a $\cos x$ b $-\sin x$ c $\sec^2 x$ d $3\cos 3x$ e $-4\sin x$ f $\frac{1}{2}\sec^2\frac{x}{2}$ g $10\cos 5x$ h $-\sin 2x$ i $\pi\cos$;

j $2\cos(2x+3)$ k $15\sec^2\left(3x-\frac{\pi}{4}\right)$ l $1+\sin x$ m $2x\cos(x^2)$ n $1+\sec^2(x-1)$ o $5\sin(2-5x)$

Page 82 1 a $-4\sin 2x$ b $-\frac{1}{4}\cos\frac{x}{2}$ c $4\pi^2\sin \pi x$ 2 a 6 b -1.5 c 0 3 a 1.5 b 16 c $\frac{\sqrt{3}}{6}$

Page 83 1 a $-x\sin x+\cos x$ b $x^2\cos x+2x\sin x$ c $2\tan x\sec^2 x$ d $-2x\sin(x^2)$ e $\frac{2x\cos x-\sin 2x}{x^2}$ f $\frac{x\sin x+\cos x-}{2x^2}$

g $\tan x\sec x\left(=\frac{\sin x}{\cos^2 x}\right)$ h $-\cot x\,\mathrm{cosec}\,x\left(=-\frac{\cos x}{\sin^2 x}\right)$

Page 84 1 $3x-y+\sqrt{3}-\frac{\pi}{3}=0$ 2 $x+y-\frac{1}{2}-\frac{\pi}{4}=0$ 3 $x=\frac{\pi}{6}$ or $\frac{5\pi}{6}$ 4 ($\cos x=2$ has no solutions)

Page 85 1 a $-\cos x+C$ b $\sin x+C$ c $\tan x+C$ d $3\sin x+C$ e $-\frac{1}{2}\cos 2x+C$ f $\frac{1}{8}\tan 4x+C$ g $4\cos\frac{x}{2}+C$

h $\tan(x+1)+C$ i $x-\sin x+C$ j $\frac{1}{3}\sin(3x-2)+C$ k $-\frac{1}{\pi}\cos\pi x+C$ l $\frac{\tan x}{2}+C$ 2 a 2 b 1

Page 86 1 a 1 b $-1+\sqrt{3}$ c 0 d 0 e 2 f $\frac{\pi^2}{2}$

Page 87 1 $2\sqrt{2}$ units^2 2 a 0, 0.25, 0.75, 1 b $\frac{\pi}{4}$ 3 a $x\cos x+\sin x$ b $\frac{\pi}{2}$

Page 88 1 4 units^2 2 $\sqrt{2}$ units^2 3 π units^3