

EXERCISE 10G

LIMITS AT INFINITY

1. Evaluate the following limits:

(a) $\lim_{x \rightarrow \infty} \frac{3}{x}$

(b) $\lim_{x \rightarrow \infty} \frac{5}{x+6}$

(c) $\lim_{x \rightarrow \infty} (3x - 2)$

(d) $\lim_{x \rightarrow \infty} \left(5 + \frac{2}{3x-1} \right)$

(e) $\lim_{x \rightarrow \infty} \frac{2x}{x+2}$

(f) $\lim_{x \rightarrow \infty} \frac{2x^2}{5x^2+1}$

(g) $\lim_{x \rightarrow \infty} \frac{5x^2 - 4x}{3x^3 + 2}$

(h) $\lim_{x \rightarrow \infty} \frac{9x^3 - 5x^2}{3x^2 - 2x - 1}$

(i) $\lim_{x \rightarrow \infty} \frac{px+q}{mx+n}$

(j) $\lim_{x \rightarrow \infty} \frac{ax^2 + bx + c}{(px+q)^2}$

(k) $\lim_{x \rightarrow \infty} \frac{x^3(x-6)}{5x^4 - 11x}$

2. Describe the geometric meaning of your answers in question 1 (f), (g),(h).

3. Evaluate the following limits:

(a) $\lim_{x \rightarrow -\infty} \frac{6x+1}{x^2+1}$

(b) $\lim_{x \rightarrow -\infty} \frac{x^2+4}{3x^2+7}$

(c) $\lim_{x \rightarrow -\infty} \frac{x^3+5x}{3x^2-1}$

(d) $\lim_{x \rightarrow -\infty} \frac{|x-3|}{x-3}$

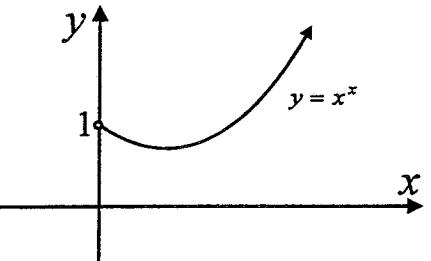
(e) $\lim_{x \rightarrow -\infty} \frac{(x-2)(x^2+3)}{7x^3-2}$

(f) $\lim_{x \rightarrow -\infty} \left(x + \frac{5}{x+10} \right)$

4. This is a graph of the function

$$y = x^x \quad (x > 0)$$

Use the graph to find $\lim_{x \rightarrow +\infty} x^x$



5. This is a graph of the function

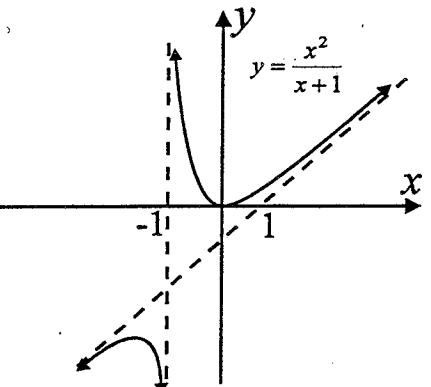
$$y = \frac{x^2}{x+1}$$

Use the graph to find:

(a) $\lim_{x \rightarrow +\infty} f(x)$

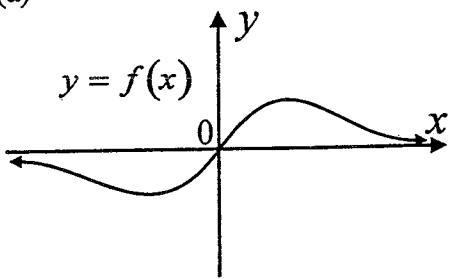
(b) $\lim_{x \rightarrow -\infty} f(x)$

(c) $\lim_{x \rightarrow -1} f(x)$

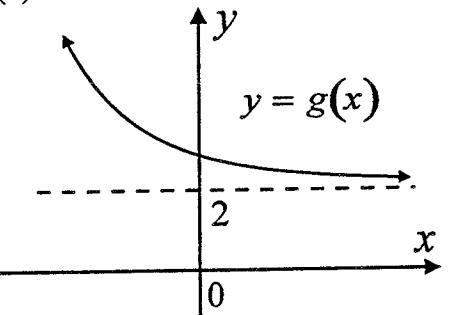


6. For each of the following functions find (i) $\lim_{x \rightarrow +\infty} y$ and (ii) $\lim_{x \rightarrow -\infty} y$

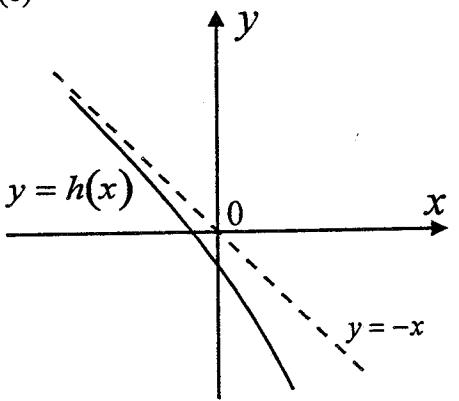
(a)



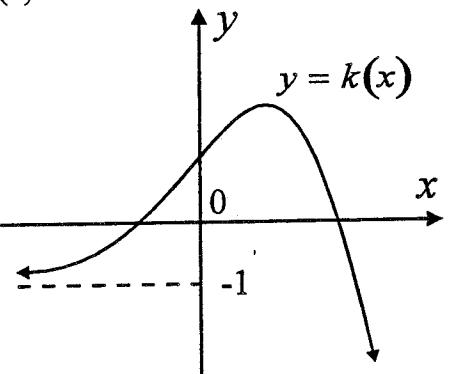
(b)



(c)



(d)



ANSWERS

1. (a) 0 (b) 0 (c) does not exist (d) 5 (e) 2 (f) $\frac{2}{5}$ (g) 0
(h) does not exist (i) $\frac{p}{m}$ (j) $\frac{a}{p^2}$ (k) $\frac{1}{5}$
2. (f) Horizontal asymptote $y = \frac{2}{5}$ (g) Horizontal asymptote $y = 0$ i.e. x-axis
(h) no horizontal asymptote. Oblique asymptote: $y = 3x + \frac{1}{3}$.
3. (a) 0 (b) $\frac{1}{3}$ (c) does not exist (d) -1 (e) $\frac{1}{7}$ (f) does not exist
4. does not exist
5. (a) does not exist (b) does not exist (c) does not exist
6. (a) 0, 0 (b) 2, does not exist (c) does not exist, does not exist
(d) does not exist, -1