Exercise 12E Exam Practice

1 The third and fifth terms of an arithmetic series are $^{-}3$ and 5 respectively.

a Find the first term and common difference the of the series. **(4 marks)**

b Find the sum of the first 50 terms of the series. **(2 marks)**

2 Natasha decides to go swimming on three evenings each week.

In the first week she swims 8 lengths on each visit to the pool. In each subsequent week she increases by two the number of lengths that she swims on each of her visits to the pool during that week.

a Show that in the first three weeks she swims 90 lengths in total. **(2 marks)**

b In the nth week of her visits the total number of lengths that she has completed passes 2000. Find the value of n. **(5 marks)**

3 **a** The sixth term of an arithmetic series is 38.

Given that the common difference of the series is k, find expressions in terms of k for

i the first term of the series,
ii the sum of the first 20 terms of the series. **(5 marks)**

b Evaluate $\sum_{r=1}^{18}(16-\frac{3}{2}r)$. **(4 marks)**

4 The first three terms of an arithmetic series are $(4x-5)$, $3x$ and $(x+13)$ respectively.

a Find the value of x. **(3 marks)**

b Given that the sum of the first n terms of the series is 126, find the two possible values of n. **(5 marks)**

5 The sum of the first four terms of an arithmetic series is 96.

The sum of the fifth, sixth and seventh terms of the series is 219.

a Find the first term and common difference of the series. **(6 marks)**

b Find and simplify an expression in terms of n for the sum of the first n terms of the series. **(4 marks)**

6 **a** Evaluate $\sum_{r=8}^{32}7r-11$. **(4 marks)**

b The rth term of an arithmetic series is given by $(3r+5)$.

Given that the sum of the first n terms is 665, find the value of n. **(6 marks)**

Exercise 12E Exam Practice

1 **a** -11, 4 **b** 4350

2 **b** 23

3 **a** **i** $38-5k$ **ii** $760+90k$ **b** 31.5

4 **a** 8 **b** 7, 12

5 **a** 3, 14 **b** $7n^2-4n$

6 **a** 3225 **b** 19

Exercise 12E Exam Practice

Qu1 $a+2d=-3$ — ①

$a+4d=5$ — ②

(a) ② − ①: $2d=8$

$d=4$ sub in ①

$a+8=-3$

$a=-11$

(b) $T_n=-11+(n-1)4$

$=4n-15$

$T_{50}=185$

$\therefore S_{50}=25(-11+185)$

$=4350$

Qu2 ~~8, 10, 12, ...~~

~~$T_n=8+(n-1)2$~~

~~$=2n+6$~~

~~(a) $T_9=18+6=24$~~

~~$S_9=4.5(8+24)$~~

~~$=$~~

(a) $24, 30, 36, \ldots$

$T_n=24+(n-1)6$

$=6n+18$

So $T_3=36$

$S_3=1.5(36+24)$

$=90$

$\therefore$ 90 lengths in 3 weeks.

(b) $2000 < \frac{n}{2}(24+18+6n)$

$4000 < 6n^2+42n$

$2000 < 3n^2+21n$

$3n^2+21n-2000>0$

$n > \dfrac{-21 \pm \sqrt{21^2-4(3)(-2000)}}{6}$

$> \dfrac{-21 \pm \sqrt{24441}}{6}$

$n>22.56$ or $n>-29.56$

$\therefore n$ is 23 as n must be positive integer.

Qu3 (a) $a+5d=38$.

$a+5k=38$

(i) $a=38-5k$

(ii) $T_n=38-5k+(n-1)k$

$=nk+38-6k$

~~$S_{20}=10(38-5k+nk+38-6k)$~~

$T_{20}=14k+38$

$\therefore S_{20}=10(38-5k+14k+38)$

$=760+90k$

(b) $\sum_{r=1}^{18}(16-\frac{3}{2}r)$

$T_1=14.5$

$T_{18}=-11$

$\therefore S_{18}=9(14.5-11)$

$=31.5$

Qu4 (a) $3x-4x+5=x+13-3x$

$-x+5=13-2x$

$\therefore x=8$.

(b) $T_1=a=27$

$T_2=a+d=24$

$T_3=21$

$d=-3$

So $T_n=27+(n-1)(-3)$

$=-3n+30$

$126=\frac{n}{2}(27+30-3n)$

$252=57n-3n^2$

$\therefore 3n^2-57n+252=0$

$n^2-19n+84=0$

$(n-7)(n-12)=0$

$n=7$ or 12

Qu5 $S_4 = 96$

$$96 = 2(2a + (n-1)d)$$
$$a + a + d + a + 2d + a + 3d = 96$$
$$4a + 6d = 96$$
$$2a + 3d = 48 \quad — ①$$
$$3a + 15d = 219$$
$$a + 5d = 73 \quad — ②$$

(a). $② \times 2 : 2a + 10d = 146 \quad — ③$

$$③ - ① \quad 7d = 98$$
$$d = 14 \quad \text{sub in } ①$$
$$2a + 42 = 48$$
$$a = 3$$

(b) $T_n = 3 + (n-1)14$
$$= 14n - 111$$
$$\therefore S_n = \frac{n}{2}(6 + (n-1)14)$$
$$= \frac{n}{2}(14n - 8)$$
$$= 7n^2 - 4n$$

Qu6 (a) $\sum_{r=8}^{32} 7r - 11$

$$a = 45$$
$$d = 7$$
$$l = 213$$
$$\therefore S_{25} = \frac{25}{2}(45 + 213)$$
$$= 3225$$

(b) $T_r = 3r + 5$
$$S_n = 665$$
$$a = 8$$
$$d = 3$$
$$665 = \frac{n}{2}(8 + 8 + (n-1)3)$$

$\cancel{1330 = n(16 + 3n - 3)}$ $\quad \therefore 665 = \frac{n}{2}(13 + 3n)$

$\cancel{= 16\ 2n^2 14n}$ $\quad 1330 = 3n^2 + 13n$

$\cancel{\therefore 2n^2 + 14n - 1330 = 0}$ $\quad 3n^2 + 13n - 1330 = 0$

$\cancel{(2n \quad n)}$ $\quad (3n + 70)(n - 19) = 0$

$$\therefore n = 19 \text{ or } -70/3$$

$\therefore$ 19th term as n must be positive integer.